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Estimation of Finite Population Total under Super Population Model when Variables are Subject to Measurement Error

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SUMMARY

In the present paper, an attempt has been made to examine the effect of measurement error in the study variate on the efficiency of the model-based estimators of finite population total under super population model when variance of the study variate, y, is a function of the auxiliary variable x, related to y, and included as an independent variable in the model. Simulation results show that there is considerable loss in the precision of the estimators due to measurement error. However, such losses are marginal if the variability in the measurement errors as compared to variability in model errors is small.

Keywords: Super population model, Measurement errors, Regression, Finite population, Prediction.

1. INTRODUCTION

A general treatment for inference problem in regression models with measurement errors is considered in Fuller (1987). Bolfarine (1991) considered finite population prediction under error-in-variables super population model. He considered two models: (i) location error-in-variables super population model, i.e.

$$y_i = \mu + e_i\,,\ Y_i = y_i + v_i\,,\ \text{i=1, 2,……., N} \qquad (1.1)$$

where Y_i and y_i are the observed and true value of y, respectively, $e_i \sim iidN\left(0, \sigma_e^2\right)$, $v_i \sim iidN\left(0, \sigma_v^2\right)$ and $\operatorname{cov}\left(e_i, v_i\right) = 0$, and (ii) regression model with measurement errors as

$$y_i = \beta_0 + \beta_1 x_i + e_i\,,\ \text{i=1, 2, …., N}$$

$$Y_i = y_i + v_i\,,\ X_i = x_i + u_i \qquad (1.2)$$

where Y_i and X_i are observed values and, y_i and x_i are true values of y and x, respectively. $e_i \sim iidN\left(0, \sigma_e^2\right)$, $v_i \sim iidN\left(0, \sigma_v^2\right)$, $u_i \sim iidN\left(0, \sigma_u^2\right)$ and $v(y_i) = \sigma_e^2$. e_i, v_i and u_i are mutually independent. However, he assumed that the random sample comes from a bivariate normal population of (y, x). He developed prediction estimators for finite population mean $\bar{y} = \sum_{i=1}^{N} y_i / N$ and $S_y^2 = \sum_{i=1}^{N} \left(y_i - \bar{y}\right)^2 \Big/ \left(N-1\right)$, population mean square, under the model (1.1), an optimal predictor for $\bar{y}$ under the model (1.2). Chattopadhyay and Datta (1994) have extended the work of Bolfarine (1991) to the stratified sampling under the location error-in-variables super-population model. Various authors have made contribution on this aspect in recent past. Notably among them are Battese *et al.* (1988), Eltinge (1994), Mukhopadhyay (1994), Stefanski (2000), Ghosh and Sinha (2007), Ma and Li (2010), West (2010) etc.

When we consider the model-based/model assisted estimation of finite population total or mean of the study variate y, it has been found generally in most of socio-economic surveys that variance of y is a function of the auxiliary variable x related to y, when x is included as an independent variable in the model. The structure of the variance function is

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generally observed as $V(y)=\sigma^2 x^g$, $\frac{1}{2} \le g \le 2$, σ^2 is variance of error term in the model, for most of the data encountered in practice (See the work of Smith; 1938, Jessen; 1942, Desh Raj; 1958, Rao and Bayless; 1969, Bayless and Rao, 1970). Also for instance, Royall (1970, 1971) and Royall (1973a, 1973b) have assumed g=1. Some of the authors (Mukhopadhya, 1994, Chattopadhyay and Datta, 1994, etc.) have taken g=0, i.e. the variance of y is not a function of x. Scott *et al* (1978) have considered g=2. Therefore, there is need to examine the effect of measurement error in variables on the precision of the estimators of finite population total or mean under a super population model where variance of y is a function of x.

It has been generally conceived that the auxiliary information on the auxiliary variables (x) related to the study variate (y) are obtained from administrative records and various other sources in most of the socio-economic surveys. Therefore, error in the magnitude of x may not be always found and even if there is little bit error, it may not pose serious problems, particularly, in estimating model parameters. However, response errors and/ or measurement error are likely to occur in the study variable which would certainty affects the estimate of finite population parameters as well as its standard error. Therefore, in the present paper, an attempt has been made to develop model based estimators for the finite population total when only the study variable is subject to the measurement error under super population model with $V(y_i)=\sigma_e^2$ and $V(y_i)=\sigma_e^2 x_i$. Section-2 deals with the development of model-based estimators and derivation of model variance of the estimators etc. A limited simulation study to examine the effect of measurement error on the standard error of the estimators under these two models has been conducted in Section-3.The section-4 has dealt with discussion and concluding remarks.

2. ESTIMATION OF FINITE POPULATION TOTAL UNDER SUPER POPULATION MODEL WHEN STUDY VARIABLE IS SUBJECT TO MEASUREMENT ERROR

We consider the following two super population models.

Model- I:

$$y_i=\beta x_i+e_i\,,\; Y_i=y_i+v_i\,,\; i=1,2,...,N\,. \qquad (2.1)$$

where Y_i and y_i are the observed and true values of y, respectively. v_i's and e_i's are assumed to be independently distributed with mean zero and variances σ_v^2 and σ_e^2, respectively. It is also assumed that $\text{cov}(v_i,e_i)$ is zero. x_i's (i=1,2,…,N) are the values of the auxiliary variable x related to y, and it is assumed that they are correctly known.

Model- II:

$$y_i=\beta x_i+e_i x_i^{1/2}\,,\; Y_i=y_i+v_i\,,\; i=1,2,\ldots,N \qquad (2.2)$$

with $v(y_i)=\sigma_e^2 x_i$.

where the notations, terms and assumptions are same as defined in Model- I. The model (2.2) is also referred to as ξ - model (Royall & Herson, 1973a).

2.1 Estimation of finite population total under Model-I:

The objective is to estimate $T=\sum_{i=1}^{N} y_i$. Consider that a sample of size n units from the population consisting of N units is drawn, not necessarily by probability sampling. The population total T can be decomposed as follows

$$T=\sum_{i\in s} y_i+\sum_{i\in \bar{s}} y_i \qquad (2.1.1)$$

where $\bar{s}$ is complement to s, i.e. it contains non-sampled units of the population. An estimator of T is, therefore, given by

$$\hat{T}_1=\sum_{i\in s} Y_i+\sum_{i\in \bar{s}} \hat{y}_i \qquad (2.1.2)$$

where $\hat{y}_i=\hat{\beta}x_i$ and $\hat{\beta}=\frac{\sum_{i\in s} Y_i x_i}{\sum_{i\in s} x_i^2}$ is the best linear unbiased estimator (blue) of β that is obtained after the fitting of the Model-I with data contained in s by least square technique. We state and prove the following theorem.

Theorem 2.1.1: The estimator $\hat{T}_1$ is model-unbiased estimator of T with model variance.

$$V\left(\hat{T}_1\right)=\sigma_e^2\left[\left(\frac{\left(\sum_{i\in \bar{s}} x_i\right)^2}{\left(\sum_{i\in s} x_i^2\right)}+N-n\right)+\delta\left(\frac{\left(\sum_{i\in \bar{s}} x_i\right)^2}{\left(\sum_{i\in s} x_i^2\right)}+2\sum_{i\in s} x_i\frac{\sum_{i\in \bar{s}} x_i}{\sum_{i\in s} x_i^2}+n\right)\right],$$

$\delta = \dfrac{\sigma_v^2}{\sigma_e^2}$.

Proof:

We take the model expectation of the estimator $\hat{T}_1$, as follows

$$E\left(\hat{T}_1 - T\right) = E\left[\sum_{i\in s} Y_i + \left(\frac{\sum_{i\in s} Y_i x_i}{\sum_{i\in s} x_i^2}\right)\sum_{i\in\bar{s}} x_i - \sum_{i=1}^{N} y_i\right]$$

$$= E\left[\sum_{i\in s}(\beta x_i + e_i + v_i) + \frac{\sum_{i\in s}(\beta x_i + e_i + v_i)x_i}{\sum_{i\in s} x_i^2}.\sum_{i\in\bar{s}} x_i - \sum_{i=1}^{N}(\beta x_i + e_i)\right]$$

$$= \left[\beta\left(\sum_{i\in s} x_i + \sum_{i\in\bar{s}} x_i\right) - \beta\sum_{i=1}^{N} x_i\right] = 0$$, as per assumption under model-I (2.1.3)

This shows that the estimator is model unbiased estimator of T.

We derive the model variance of $\hat{T}_1$ as

$$V\left(\hat{T}_1\right) = E\left(\hat{T}_1 - T\right)^2 = E\left[\sum_{i\varepsilon s} Y_i + \frac{\sum_{i\varepsilon s} Y_i x_i}{\sum_{i\varepsilon s} x_i^2}\sum_{i\varepsilon s} x_i - \sum_{i=1}^{N} y_i\right]^2$$

$$= E\left[\sum_{i\in s}(\beta x_i + e_i + v_i) + \frac{\sum_{i\in s}(\beta x_i + e_i + v_i)x_i}{\sum_{i\in s} x_i^2}.\sum_{i\in\bar{s}} x_i - \sum_{i=1}^{N}(\beta x_i + e_i)\right]$$

$$= E\left[\sum_{i\varepsilon s}(e_i + v_i) + A\sum_{i\in s}(e_i + v_i)x_i - \sum_{i=} e_i\right], \mathbf{A} = \sum_{\mathbf{i}\varepsilon\bar{\mathbf{s}}} \mathbf{x_i} \Big/ \sum_{\mathbf{i}\varepsilon\mathbf{s}} \mathbf{x_i}$$

Squaring the above expression and taking model expectation as per model assumptions, we get the required expression of the variance after little algebraic simplification as fallows

$$= \sigma_e^2\left[\frac{\left(\sum_{i\in\bar{s}} x_i\right)^2}{\left(\sum_{i\in s} x_i^2\right)} + N - n\right] + \sigma_v^2\left[\frac{\left(\sum_{i\in\bar{s}} x_i\right)^2}{\left(\sum_{i\in s} x_i^2\right)} + 2\sum_{i\in s} x_i \frac{\sum_{i\in\bar{s}} x_i}{\sum_{i\in s} x_i^2} + n\right] \quad (2.1.4)$$

It can easily be verified that the first term of the above expression in (2.1.4) is the model variance of $\hat{T}_1$ when there is no measurement error in y under model-I.

The above variance expression (2.1.4) can further be written as

$$V\left(\hat{T}_1\right) = \sigma_e^2\left[\left[\frac{\left(\sum_{i\in\bar{s}} x_i\right)^2}{\left(\sum_{i\in s} x_i^2\right)} + N - n\right] + \delta\left[\frac{\left(\sum_{i\in\bar{s}} x_i\right)^2}{\left(\sum_{i\in s} x_i^2\right)} + 2\sum_{i\in s} x_i \frac{\sum_{i\in\bar{s}} x_i}{\sum_{i\in s} x_i^2} + n\right]\right],$$

$$\delta = \frac{\sigma_v^2}{\sigma_e^2} \quad (2.1.5)$$

This proves the theorem.

Remarks: It is obvious that the model variance of $\hat{T}_1$ has increased by the second term of the expression (2.1.4), when there is measurement error in y.

2.2 Estimation of finite population total under Model-II:

An estimator of T for a given sample s of size n units from the population consisting of N units can be written as

$$\hat{T}_2 = \sum_{i\in s} Y_i + \hat{\beta}\sum_{i\in\bar{s}} x_i \quad (2.2.1)$$

where $\hat{\beta} = \dfrac{\sum_{i\in s} Y_i}{\sum_{i\in s} x_i}$ is blue of β, which is obtained after the fitting of the Model-II with data contained in s by least square technique. We state and prove the following theorem.

Theorem (2.2.1): The estimator $\hat{T}_2$ is the unbiased estimator of T with model variance

$$V\left(\hat{T}_2\right) = \sigma_e^2\left[\frac{X\sum_{i\in\bar{s}} x_i}{\sum_{i\in s} x_i} + n\delta\left(\frac{X}{\sum_{i\in s} x_i}\right)^2\right]$$

Proof: We take the model expectation of the estimator $\hat{T}_2$ as follows

$$E\left(\hat{T}_2 - T\right) = E\left[\sum_{i\in s} Y_i + \frac{\sum_{i\in s} Y_i}{\sum_{i\in s} x_i}.\sum_{i\in\bar{s}} x_i - \sum_{i=1}^{N} y_i\right]$$

$$= E_\xi\left[\sum_{i\in s}\left(\beta x_i + e_i x_i^{1/2} + v_i\right) + \frac{\sum_{i\in s}\left(\beta x_i + e_i x_i^{1/2} + v_i\right)}{\sum_{i\in s} x_i}.\sum_{i\in \bar{s}} x_i - \sum_{i=1}^{N}\left(\beta x_i + e_i x_i^{1/2}\right)\right]$$

$$= \left[\beta\left(\sum_{i\in s} x_i + \sum_{i\in \bar{s}} x_i\right) - \beta\sum_{i=1}^{N} x_i\right] = 0$$, as per assumptions of the model. (2.2.2)

This prove that the estimator $\hat{T}_2$ is model unbiased estimator of T.

The model variance of $\hat{T}_2$ is derived as fallows

$$V\left(\hat{T}_2\right) = E\left(\hat{T}_2 - T\right)^2 = E\left[\sum_{i\varepsilon s} Y_i + \frac{\sum_{i\varepsilon s} Y_i}{\sum_{i\varepsilon s} x_i}\sum_{i\varepsilon s} x_i - \sum_{i=1}^{N} x_i\right]^2$$

$$= E_\xi\left[\sum_{i\in s}\left(\beta x_i + e_i x_i^{1/2} + v_i\right) + \frac{\sum_{i\in s}\left(\beta x_i + e_i x_i^{1/2} + v_i\right)}{\sum_{i\in s} x_i}.\sum_{i\in \bar{s}} x_i - \sum_{i=1}^{N}\left(\beta x_i + e_i x_i^{1/2}\right)\right]$$

$$= E\left[\sum_{i\in s}\left(e_i x_i^{1/2} + v_i\right) + B\sum_{i\in s}\left(e_i x_i^{1/2} + v_i\right). - \sum_{i=1}^{N} e_i x_i^{1/2}\right], \quad \mathbf{B} = \sum_{\mathbf{i}\varepsilon\bar{\mathbf{s}}} \mathbf{x_i} \Big/ \sum_{\mathbf{i}\varepsilon \mathbf{s}} \mathbf{x_i}$$

Squaring the above expression and taking model expectation as per assumptions of model-II, we obtain the required model variance of $\hat{T}_2$ after little algebraic simplification as fallows.

$$v\left(\hat{T}_2\right) = \frac{\sigma_e^2 X \sum_{i\in \bar{s}} x_i}{\sum_{i\in s} x_i} + n\left(\frac{X}{\sum_{i\in s} x_i}\right)^2 \sigma_v^2, \text{ where } X = \sum_{i=1}^{N} x_i \quad (2.2.3)$$

It may also be noted that the first term of (2.2.3) is the model variance of $\hat{T}_2$ when there is no measurement error in y under model-II. The expression (2.2.3) can further be written as

$$V\left(\hat{T}_2\right) = \sigma_e^2\left[\frac{X\sum_{i\in \bar{s}} x_i}{\sum_{i\in s} x_i} + n\delta\left(\frac{X}{\sum_{i\in s} x_i}\right)^2\right] \quad (2.2.4)$$

This proves the theorem.

Remarks-1: The variance of $\hat{T}_2$ has obviously increased by the second term in equation (2.2.3) when the study variable is subject measurement error.

Remarks-2: Although there is no sense of comparison of $\hat{T}_1$ and $\hat{T}_2$ as these have been developed under different models, but it may be worth to see that under which model the loss in precision of the estimator is more. Assuming σ_v^2 is same for both the models, the increased variance in $\hat{T}_1$ due to measurement error is the second term in the expression (2.1.4) and let it be denoted as

$$V_1 = \sigma_v^2\left[\frac{\left(\sum_{i\in \bar{s}} x_i\right)^2}{\left(\sum_{i\in s} x_i^2\right)} + 2\sum_{i\in s} x_i \frac{\sum_{i\in \bar{s}} x_i}{\sum_{i\in s} x_i^2} + n\right] \quad (2.2.5)$$

Similarly, let V_2 be the increased variance of the estimator $\hat{T}_2$ due to measurement error, which is second term in the expression (2.2.3), i.e.

$$V_2 = n\sigma_v^2\left(\frac{(X)}{\left(\sum_{i\in s} x_i\right)}\right)^2 \quad (2.2.6)$$

Comparing V_1 and V_2 by taking their difference we get that $V_1 > V_2$. This implies that if the population under study satisfies the model-I, the loss in the efficiencies of estimator under this model is expected to be more when there is measurement error in y.

3. A LIMITED SIMULATION STUDY

A limited simulation study has been conducted to examine the effect of measurement error on the precision of the estimators under both the model-I and II as given below

$$y_i = \beta x_i + e_i, \ \mathbf{i = 1,2,.....,N} \quad (3.1)$$

with $V\left(y_i\right) = \sigma_e^2$

and model-II

$$y_i = \beta x_i + e_i x_i^{1/2}, \ \mathbf{i = 1,2,.....,N} \quad (3.2)$$

with $V\left(y_i\right) = \sigma_e^2 x_i$

It is assumed that x_i follows chi-square with 5 degree of freedom in both the models. Therefore, the values of x_i were generated from chi-square with 5 degree of freedom. It is also assumed that e_i's follow independently normal distribution with mean zero and variance $\sigma_e^2 = 2$. Therefore, random effect e_i's were generated from normal distribution with mean zero and variance 2. The value of β is assumed to be 0.5. Using these models along with their defined parameters, the populations of y of size N=1500 were generated using the both models. Fifty thousands samples of size n=150 were drawn by simple random sampling without replacement from both populations generated from the model-I&II. The simulation study was done using R- software. The values of $\delta = \sigma_v^2/\sigma_e^2$ have been considered as 0.75, 1.00 and 1.25. The variances of the estimators with and without measurement error in y for each simulation run (i=1,2,…,L) have been computed under both the models. Let V_i be the variance of the estimator without measurement error in y for ith run of simulation (i=1,2,…,L). Similarly, let V_i' be the variance of the estimator with measurement error in y for ith run of simulation. Here L=50,000.

Let us define

$\bar{V} = \frac{1}{L}\sum_{i=1}^{L} V_i$, the average variance of the estimator without measurement error

$\bar{V}' = \frac{1}{L}\sum_{i=1}^{L} V_i'$, the average variance of the estimator with measurement error

The effect of measurement error on the precision of the estimator has been computed as percent relative increase (%R.I.) in the standard error of the estimator with measurement error over that of the estimator without measurement error,

$$\text{i.e. } \%R.I. = \frac{\sqrt{\bar{V}'} - \sqrt{\bar{V}}}{\sqrt{\bar{V}}} \times 100 \qquad (3.3)$$

The results of simulation in terms of average variances of the estimators under model-I &II with and without measurement error are presented in the table 4.1

It is obvious from the results of the Table 3.1, that variances of the estimators of finite population total have increased when there is measurement error in y. The increase in variance depends on the value of $\delta = \sigma_v^2/\sigma_e^2$. It clearly shows that variances increases with increase in δ. However, if variability in measurement error is smaller as compared to variability in the model error (e_i), then increase in variances would be marginal one. It is also obvious that $\bar{V} = \bar{V}'$ for $\delta = 0$ i.e. $\sigma_v^2 = 0$. This is also obvious from the variance expression (2.1.5) and (2.2.4) that they reduces to the variance of the estimators, when there is no measurement error in y in other words when $\delta = 0$, i.e. if $\sigma_v^2 = 0$.

Table 3.1. Average variances of the estimators

S. No.	Model	$\bar{V}$	$\bar{V}'$ for different values of δ		
			0.75	1.00	1.25
1.	Model-I	**20365.21 (142.71)**	**36767.44 (191.75)**	**42234.85 (205.51)**	**47702.26 (218.41)**
2	Model-II	**140364.89 (374.65)**	**163021.77 (403.76)**	**170574.06 (413.01)**	**178126.35 (422.05)**

NB: figures in parentheses denote the average standard error of the estimators.

The simulation results in terms of % R.I. under model-I&II are presented in the Table 3.2.

Table 3.2. Percent relative increase (%R.I.) in standard error of the estimators with measurement error over the estimators without measurement error

S. No.	Model	Percent relative Increase (%R.I.) for different value of δ		
		0.75	1.00	1.25
1	Model-I	**34.36**	**44.01**	**53.05**
2	Model-II	**7.76**	**10.23**	**12.65**

The results of the Table 3.2 show that the percent relative increase (%RI) due to measurement error in y is smaller in model-II (up to 12%) than in the model-I (up to 53%). It is also very evident that %RI depends upon the value of $\delta = \sigma_v^2/\sigma_e^2$. That means smaller the value of δ, smaller is the %RI in the variances.

4. AN EMPIRICAL ILLUSTRATION ABOUT RELATIVE INCREASE IN VARIANCE DUE TO MEASUREMENT ERROR WITH REAL DATA

Let V_1 be the variance of the estimator of $\hat{T}_1$ without measurement error under model-I, i.e. first part of the expression (2.1.4). let V_1'' be the variance of $\hat{T}_1$ with measurement error under this model, i.e. expression (2.1.4). Similarly, let V_2 and V_2'' be the variance of $\hat{T}_2$ without and with measurement error, respectively, under model-II. V_2 is the first part of the expression (2.2.3) where V_2'' is the entire expression of (2.2.3).

The percent relative increase in standard error of the estimators due to measurement error in y under model-I and II are, respectively, given by

$$\%R.I.(I) = \frac{\sqrt{V_1'} - \sqrt{V_1}}{\sqrt{V_1}} \times 100 \quad (4.1)$$

$$\%R.I.(II) = \frac{\sqrt{V_2'} - \sqrt{V_2}}{\sqrt{V_2}} \times 100 \quad (4.2)$$

Two real populations have been considered..First population relates to model-I while second population relates to model-II. Description of the populations are given below.

Population-I: The Table 4.8 on page 185 of Sukhatme and Sukhatme(1970). The variable under study variate(y) is area under wheat during the year 1936, and the auxiliary variable (x) is total cultivated area during the year1931 After fitting the regression equation of y on x, we find that fitted model satisfies the model-I. The population size N= 34. A sample of size n=8 has been taken from the population. Various statistics have been computed which are as follows:

$$\bar{x}_s = 722.125$$
$$\bar{x}_{\bar{s}} = 778.6538$$
$$\bar{x}_s^{(2)} = \frac{1}{n}\sum x_i^2 \quad = 617159.375$$
$$\bar{x} = 765.3529$$

Population-II: The example 4.1 of the Sukhatme and Sukhatme (1970), page: 150 and the Table 4.1 and 5.1 on pages 152 and 205, respectively. The variable under study is livestock numbers (y) and auxiliary variable is agricultural area (x) in the villages of district Eawah, U.P., India. Here the data of the Table 4.1 up to column 6 have been considered as the data satisfies the model-II as pointed out by Sukhatme and Sukhatme(1970). Therefore, the population size N=319, n=64 as per Table 5.1. The value of $\bar{x} = 367.5348$, $\bar{x}_{\bar{s}} = 362.1242$ and $\bar{x}_s$ =389.0926.

The percent relative increase in variance due to measurement error have been computed using the formula given in (4.1) and (4.2) for different values of δ = 0.75, 1.00 and 1.25. The results are presented in the Table 4.1.

The perusal of the results of the Table 3.2 and the Table 4.1 reveals that the percent relative increase in standard error due to measurement error in y are of the almost same order in both the cases of simulation with hypothetical data and real data set. Hence, results with real data confirm the results of simulation.

Table 4.1. % R.I. in the variance due to measurement error for different value of δ

Model	% R.I. in the variance		
	δ		
	0.75	1.00	1.25
Model-I	**37.60**	**45.03**	**52.11**
Model-II	**5.17**	**8.13**	**10.31**

5. DISCUSSION AND CONCLUDING REMARKS

The overall results of the Tables 3.1 and 3.2 indicate that there would be considerable loss in the precision of the estimator developed under both the models if there is measurement error or response error in y. The results of the Table 4.1 with real data also confirm the results of simulation However, these losses would be marginal if the variability in measurement error (v_i) is small as compared to the variability in the model error (e_i). Therefore, it is imperative to survey statisticians that the efforts must be made to collect relible data free of measurement or response error in the surveys by ensuring proper training and guidance to the field investigators to minimize the loss in the precision of the estimates of the population parameters

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