J. Ind. Soc. Agril. Statist.
62(1), 2008 : 19-25

Possibility and Necessity Measures for Fuzzy Linear Regression Analysis: An Application

Rama Krishna Singh, Himadri Ghosh and Prajneshu
Indian Agricultural Statistics Research Institute, New Delhi
(Received : May 2007)

SUMMARY

For reliable policy planning at micro-level, estimates of crop yield at small area level, say block level, is required. Application of existing crop-cutting methodology would not be feasible in view of prohibitive cost involved. One possible alternative is to employ "Fuzzy regression methodology". Accordingly, in this paper Possibility and Necessity measures for obtaining reliable fuzzy estimates of crop yield have been thoroughly studied. Estimation of parameters is carried out using "Fuzzy least-squares" procedure. As an illustration, the methodology is applied to Pearl Millet crop yield data in order to build block level estimates for Bhiwani district, Haryana based on farmers' estimates at the same level. Performance evaluation criterion is used to compare results of Possibility and Necessity approaches at optimal value of fitness level.

Key words : Crop-cutting experiments, Possibility measures, Necessity measures, Fuzzy least-squares, LINGO, Pearl Millet.

1. INTRODUCTION

Estimation of crop production is an essential input in the process of planning and policy formulation for agricultural sector. In India, estimation of yield is carried out through carefully planned experiments known as Crop-cutting experiments (CCE) on sampled fields. At present, estimates of yield and crop production in country are available at somewhat higher level, such as State or district. With progress in agricultural sector, there is a growing demand for estimation of crop yield at lower level, such as Community development block and in some cases even Gram panchayat. One alternative for meeting requirement of building up precise estimators at small area level is to increase number of crop cutting experiments. However, this is usually not possible due to cost constraints. Therefore, a new cost effective technique needs to be developed, which may be adopted for implementation for estimation of yield of various crops at small area level (Rao, 2003).

Sud *et al*. (2006) carried out a detailed study for developing crop yield estimates at small area level using farmers' estimates with the objective to develop precise block level estimates. However, this would be meaningful only when inquiry based farmers' estimates can be used successfully for modelling actual yield based on CCE. Ghosh *et al*. (2007) applied "Possibilistic linear regression model" considering yield data as fuzzy while farmers' estimate as 'crisp' value. Linear programming (LP) approach of Tanaka *et al*. (1982) for modelling of fuzzy variable was used for its inherent simplicity in terms of computation.

The main shortcoming of Tanaka's LP approach is that it is not based on sound statistical concepts (Chang and Ayyub 2001). Therefore, another method for handling fuzzy data was developed by Diamond (1988), where Fuzzy least-squares (FLS) criterion was considered. Kandala and Prajneshu (2004) applied FLS method to some data. From a view point of risk, Modarres *et al*. (2004) developed fuzzy linear regression model by considering Necessity measures of inclusion of observed fuzzy numbers in estimated fuzzy numbers. Further, Modarres *et al*. (2005) also extended a FLS with high fitness level of estimated model by incorporating the concept of Possibility theory. In this paper, an attempt has been made to study FLS model using both the approaches of Possibility and Necessity measures. As an illustration, the above-mentioned methodology is applied for estimation of yield of Pearl Millet crop at block level of Bhiwani district in Haryana State using data given in Sud *et al*. (2006). Further, comparison has been done for performance evaluation of FLS with optimal fitness level of the estimated model expressed in terms of Possibility and Necessity measures.

2. POSSIBILITY AND NECESSITY OF EVENTS

2.1 Possibility measure

Let U be a universal set of elementary events. Any subset of U is called an event. An event $A \subseteq U$ is said to occur when some elementary event in A occurs. A possibility measure (Zadeh, 1965) on U is a set function Π from $\wp(U)$, the set of crisp subsets of U, to the unit interval [0, 1], such that

$$\Pi(\phi)=0,\ \Pi(U)=1 \text{ and } \forall\ A, B \in \wp(U)$$

$$\Pi(A \cup B) = \max(\Pi(A), \Pi(B)) \tag{2.1}$$

Let F be a normalized fuzzy set with membership function $\mu_F(u)$ such that $\mu_F(u)=1$ for some $u \in U$. Then, quantity $\Pi_F(A)$ derived from membership function $\mu_F(u)$ by

$$\Pi_F(A) = \operatorname*{Sup}_{u \in A} \mu_F(u) \quad \forall\ A \subseteq U \tag{2.2}$$

defines a possibility measure. Eq. (2.2) is interpreted as possibility of realizing event A when possibility of elementary events is expressed by fuzzy set F. Now, if Π_F is crisp (i.e., $\Pi_F(u) \in \{0, 1\}$), then $\Pi_F(A) = 1 \Leftrightarrow A \cap F \neq \phi$. When both A and F are fuzzy, (2.2) can be readily extended, using fuzzy set intersection, into

$$\Pi_F(A) = \operatorname*{Sup}_{u} \min(\mu_F(u), \mu_A(u)) \tag{2.3}$$

Eq. (2.2) is a special case of eq. (2.3) and such an extension can be interpreted in terms of the intersection of the level cuts of F and A.

2.2 Necessity measure

A necessity measure (Dubois and Prade, 1980) is a set function $N: \wp(U) \to [0, 1]$ such that

$$N(\phi) = 0,\ N(U) = 1, \text{ and}$$

$$N(A \cap B) = \min(N(A), N(B)), \quad \forall\ A, B \subseteq U \tag{2.4}$$

Let $\bar{A}$ be the complementary set of A, and Π be a possibility measure. Then it is easy to check that the set function N defined by

$$N(A) = 1 - \Pi(\bar{A}), \quad \forall\ A \subseteq U \tag{2.5}$$

is a necessity measure. If Π derives from a normalized membership function μ_F, then it is obvious that $\forall\ A$,

$$N_F(A) = 1 - \Pi_F(\bar{A}) = \inf_{u \in \bar{A}} (1 - \mu_F(u)) \tag{2.6}$$

When and are crisp, then

$$N_F(A) = \begin{cases} 1 & \text{if } F \subset A \\ 0 & \text{otherwise} \end{cases} \tag{2.7}$$

Hence, while possibility is related to intersection, necessity refers to set inclusion. Eq. (2.5) can be extended, consistently with eq. (2.3), by defining

$$\begin{aligned} N_F(A) &= 1 - \operatorname*{Sup}_{u} \min(\mu_F(u), 1 - \mu_A(u)) \\ &= \inf_{u} \max(1 - \mu_F(u), \mu_A(u)) \end{aligned} \tag{2.8}$$

This is a measure of fuzzy set F contained in fuzzy set A.

2.3 Possibilistic regression model

In the conventional regression model, deviations between observed and estimated values are supposed to be due to measurement errors. However, in many real world situations, the response/explanatory variables may not be taken as crisp values. Now, in modelling yield of Pearl Millet crop at block level, it is always meaningful to consider yield as a fuzzy variable because there are many representative values of yield of a particular block from several villages obtained by CCE. Also, significance of yield to be expressed as fuzzy variables is that it facilitates gradual transitions of actual yield and possesses a natural capability to express and deal with measurement uncertainties. In possibility theory, these deviations are characterized as fluctuation of system parameters, which can be represented by a fuzzy number. Accordingly, it has become important to deal with fuzzy data originated from a fuzzy phenomenon. The formulation of Possibilistic linear regression model has been introduced by Tanaka *et al*. (1982). There are m explanatory non-fuzzy variables, x_i, i = 1, 2, . . . , m, while the response variables are symmetric fuzzy number, $\tilde{Y}_i = (y_i, e_i)$. The objective is to estimate a fuzzy linear regression model, expressed as follows

$$\hat{\tilde{Y}}_i = \tilde{A}_0\, x_{i0} + \tilde{A}_1\, x_{i1} + ... + \tilde{A}_n\, x_{in} \qquad (2.9)$$

In model (2.9), $\tilde{A} = (\tilde{A}_0, \tilde{A}_1, ..., \tilde{A}_n)$ is a vector of fuzzy parameters where $\tilde{A}_j = (\alpha_j, c_j)$ is a symmetric fuzzy number with α_j as center and c_j as spread. Fuzzy parameters of model are estimated for a certain fitness level h, $0 \le h \le 1$ such that h-level cut of the estimated fuzzy number contains the h-level cut of observed values. Problem is formulated as Minimization problem, which is given as follows

$$A_j = \underset{(\alpha_j,\ c_j)}{\text{Min}}\ J(c) = \sum c^t\ |x_i| \qquad (2.10)$$

where $c^t |x_i|$ is spread of estimated fuzzy output $\hat{\tilde{Y}}_i$, subject to following three constraints

$$y_i + e_i\ \left|L^{-1}(h)\right| \le \alpha^t x_i + c^t\ |x_i|\ \left|L^{-1}(h)\right|$$

$$y_i - e_i\ \left|L^{-1}(h)\right| \ge \alpha^t x_i - c^t\ |x_i|\ \left|L^{-1}(h)\right|$$

$$c \ge 0, \quad i = 1, \ldots, m \qquad (2.11)$$

The main shortcoming of Tanaka's linear programming approach as noticed by Chang and Ayyub (2001) is that, "As the number of data sets increase, the number of constraints (of the linear programming method) increases proportionally. This increase might result in computational difficulties". The other drawback of the above method is that the concept of least-squares is not considered; therefore a natural extension of fuzzy regression would be the integration of the least-square criterion into fuzzy regression as described in the next section.

3. FUZZY LEAST-SQUARES MODELS

In 1988, Diamond proposed the FLS method to determine fuzzy parameters by adopting concept of minimum fuzziness between observed and estimated values, minimization criteria similar to least squares method in Statistics was used. Working on the principle of Diamond's FLS criterion, a model is constructed in this paper based on following concepts:

(i) The objective function is to minimize total square of difference between estimated regression spread and observed spread of given data.

(ii) The degree of fitness of the FLS model, based on possibility measure, is greater than or equal to a threshold h, $0 \le h < 1$.

(iii) The degree of fitness of the FLS model, based on necessity measure, is greater than or equal to a threshold h, $0 \le h < 1$.

3.1 Fitness of FLS Model Based on Possibility Measures

In fuzzy linear regression model $\hat{\tilde{Y}}_i = \tilde{A}\ x_i$, let $\hat{\tilde{Y}}_i$ and $\tilde{Y}_i$ be estimated and observed data for a vector of independent variables x_i, respectively. For i = 1, 2, …, m, we define possibility of degree of fitness of estimated $\hat{\tilde{Y}}_i$ for given observed data $\tilde{Y}_i$ as

$$f_i = \text{Pos}\left(\tilde{Y}_i = \hat{\tilde{Y}}_i\right) \qquad (3.1)$$

The degree of fitness of estimated FLS model to all data X_1, X_2 ..., X_m is defined by

$$f = \min\{fi,\ i = 1, 2, ..., m\} \qquad (3.2)$$

A relation for possibility of equality of two fuzzy numbers as obtained by Modarres *et al*. (2005) is stated. If $\tilde{A} = (\alpha, c)$ and $\tilde{B} = (\beta, d)$, then

$$\text{Pos}\left(\tilde{A} = \tilde{B}\right) = L\left(\frac{\alpha - \beta}{c + d}\right) \qquad (3.3)$$

By applying extension Principle for fuzzy linear regression model $\hat{\tilde{Y}}_i = \tilde{A}\ x_i$ and for a vector of independent variables x_i, the center and spread of estimated symmetry fuzzy output is $\alpha\ x_i$ and $c^t\ |x_i|$, respectively. Therefore, following membership function of $\hat{\tilde{Y}}_i$ is derived

$$\hat{\tilde{Y}}_i(\mathbf{y_i}) = \begin{cases} L\left\{\dfrac{(y_i - \alpha^t\ x_i)}{c^t\ |x_i|}\right\} & \text{if } x_i \neq 0 \\ 1 & \text{if } x_i = 0,\ y_i = 0 \\ 0 & \text{otherwise} \end{cases} \tag{3.4}$$

On other hand, $\tilde{Y}_i = (y_i\ ,\ e_i)$. By substituting center and spread of $\hat{\tilde{Y}}_i = \tilde{A}\ x_i$ and $\tilde{Y}_i$ in eq. (3.4), degree of fitness of estimated FLR model, f_i is calculated as follows:

$$f_i = \text{Pos}\left(\hat{\tilde{Y}}_i = \tilde{Y}_i\right) = L\left(\frac{\alpha^t\ x_i - y_i}{c^t\ |x_i| + e_i}\right),\ x_i \neq 0 \tag{3.5}$$

where $\alpha^t = (\alpha_0, \alpha_1, ..., \alpha_n)$

$c^t = (c_0, c_1, ..., c_n)$

and $|x_i| = \left(|x_{i0}|,\ |x_{i1}|,\ ...,\ |x_{in}|\right)'$

The objective function of FLS model is to minimize square of total difference between observed spread, e_i, and estimated spread, $c^t\ |x_i|$. This can be achieved by minimizing following objective function

$$\text{Minimize } Z(h) = \sum_{i=1}^{n}\left(c^t\ |x_i| - e_i\right)^2 \tag{3.6}$$

The problem in FLS regression model is to determine fuzzy parameters $\tilde{A}$ such that $f_i \geq h,\ \forall\ i$. On substituting the value of f_i from eq. (3.2) and solving the above inequality, constraints of FLS regression model are as follows

$$\sum_{j=0}^{n}\alpha_j\ x_{ij} + \left|L^{-1}(h)\right|\sum_{j=0}^{n}c_j\ |x_{ij}| \geq y_i - \left|L^{-1}(h)\right|e_i$$

$$\sum_{j=0}^{n}\alpha_j\ x_{ij} - \left|L^{-1}(h)\right|\sum_{j=0}^{n}c_j\ |x_{ij}| \leq y_i + \left|L^{-1}(h)\right|e_i$$

$$c^t\ |x_i| \geq 0 \tag{3.7}$$

Decision maker selects a threshold $0 \leq h < 1$, as the least value for fitness of the FLS regression model. Therefore, optimal solution depends on the threshold value, h. The model is a quadratic programming model and can be solved by any nonlinear optimization solver.

3.2 Fitness of FLS Model Based on Necessity Measures

Let h-level set of two fuzzy numbers say $\tilde{A}$ and $\tilde{F}$ be $L_h(\tilde{A})$ or $L_h(\tilde{F})$ respectively for which degree of its membership function exceeds level h :

$$L_h(\tilde{A}) = \left\{u \in R^1 / \mu_{\tilde{A}}(u) \geq h\right\} = \left[A_h^L\ ,\ A_h^R\right]$$

$$L_h(\tilde{F}) = \left\{u \in R^1 / \mu_{\tilde{F}}(u) \geq h\right\} = \left[F_h^L\ ,\ F_h^R\right] \tag{3.8}$$

where A_h^L (F_h^L) and A_h^R (F_h^R) are left and right side extreme points of h-level set of $\tilde{A}(\tilde{F})$ respectively. It is already pointed out (in Sections 2.1 and 2.2) that possibility is related to intersection and necessity refers to set inclusion. Using notations for h-level sets $L_h(\tilde{A})$ and $L_h(\tilde{F})$, and eq. (2.8), the following results are obtained:

$$\text{Nes}\left(\tilde{F} \subset \tilde{A}\right) \geq h \text{ if and only if } A_h^L \leq F_{1-h}^L \text{ and } A_h^R \leq F_{1-h}^R \tag{3.9}$$

Considering, fuzzy linear regression model $\tilde{Y}_i = \tilde{A}X = \left(\sum_{j=0}^{n} a_j x_{ij}\ ,\ \sum_{j=0}^{n} c_j |x_{ij}|\right)$, and using the results obtained in eq. (3.9) for $\text{Nes}\left(\tilde{Y}_i \subset \hat{\tilde{Y}}_i\right) \geq h$, the following inequality is obtained

$$\sum_{j=0}^{n}\alpha_j\ x_{ij} - \left|L^{-1}(h)\right|\sum_{j=0}^{n}c_j |x_{ij}| \leq y_i - \left|L^{-1}(1-h)\right|e_i$$

$$\sum_{j=0}^{n}\alpha_j\ x_{ij} + \left|L^{-1}(h)\right|\sum_{j=0}^{n}c_j |x_{ij}| \geq y_i + \left|L^{-1}(1-h)\right|e_i \tag{3.10}$$

Now, based on concepts of fuzzy least-squares the objective function (3.6) is minimized with respect to necessity conditions of (3.10) to yield fuzzy least-squares results under necessity measure.

3.3 Performance Evaluation

To determine fuzzy parameters such that estimation error is minimized, the following bisection algorithm is suggested : (i) Set h = 0, $h_L = 0$ and $h_U = 1$, where h_L and h_U are upper and lower bounds for h, respectively.(ii) Solve quadratic problem (3.6) and denote value of optimal objective function by z^0. (iii) Set $h = (h_L + h_U)/2$ and solve the problem (3.6), again. Denote the value of optimal objective function by z*. Update values of h_L and h_U as $h_L = h$, if $z^* = z^0$ and $h_U = h$, otherwise. (iv) If difference between two consecutive values of h is less than ε, then algorithm is finished and fuzzy parameters are determined where ε is an acceptable tolerance; otherwise go to (iii).

In a fuzzy linear regression model, values of response variable are represented as fuzzy numbers with membership functions characterized by explanatory variable. In order to evaluate the closeness of observed and estimated fuzzy numbers, support of both fuzzy numbers should be close to each other, where support of a fuzzy set A is defined by $S_A = \{u : \mu_A(u) > 0\}$. Therefore, for performance evaluation of a fuzzy regression model, Kim and Bishu (1998) used ratio of difference between membership values to observed membership values as follows :

$$E_i = \frac{\int_{S_{\hat{\tilde{Y}}_i} \cup S_{\tilde{Y}_i}} \left| \hat{\tilde{Y}}_i(y) - \tilde{Y}_i(y) \right| dy}{\int_{S_{\tilde{Y}_i}} \tilde{Y}_i \, dy} \quad (3.11)$$

where $S_{\hat{\tilde{Y}}_i}$ and $S_{\tilde{Y}_i}$ are the support of $\hat{\tilde{Y}}_i$ and $\tilde{Y}_i$, respectively.

4. RESULTS AND DISCUSSION

As an illustration, a part of data given in Sud *et al*. (2006) concerned with yield of Pearl Millet crop at block levels of Bhiwani district of Haryana State is considered here to develop a fuzzy estimate of Pearl Millet yield. Nine blocks in the district are: B. Khera, Bhiwani, Kairu, Tosham, Siwani, Loharu, Badhra, Dadri-I and Dadri-II. The explanatory variable at block level is farmers' estimate while response variable at the same level is actual Pearl Millet crop yield based on Crop-cutting experiments, and are fuzzy numbers. Entire data analysis is carried out using LINGO, Version 8, software package (LINDO, 2002) available at I.A.S.R.I., New Delhi. The data for present investigation, culled from Sud *et al*. (2006), is reproduced in Table 1 for ready reference.

Table 1. Pearl Millet yield (based on CCE) as triangular fuzzy numbers with farmers' estimates for Bhiwani district

Block Number	Blocks	Farmers' Estimate (quintals/ hectare)	Lower limit of yield (quintals / hectare)	Upper limit of yield (quintals/ hectare)
1	B. Khera	13.36	10.00	15.00
2	Bhiwani	19.69	12.50	20.00
3	Kairu	10.01	6.00	12.42
4	Tosham	10.66	5.00	10.80
5	Siwan	9.98	6.25	12.01
6	Loharu	11.93	9.09	14.51
7	Badhra	11.96	7.33	15.01
8	Dadri-I	10.08	8.75	13.75
9	Dadri-II	9.75	11.43	15.01

Yield as function of farmers' estimates can be expressed as

$$\hat{\tilde{Y}} = (\alpha_1 \, , \, c_1) + (\alpha_2 \, , \, c_2) x_i \, , \; i = 1, \ldots, 9 \quad (4.1)$$

4.1 Fuzzy least-squares regression model

4.1.1 Possibility approach

When $\mathrm{Pos}(\hat{\tilde{Y}}_i = \tilde{Y}_i)$, the fuzzy linear regression model with least-squares error can be formulated with following objective function to be minimized

$$\text{Min } Z(h) = \{(\alpha_1 + \alpha_2) - 2.50\}^2 + \{(\alpha_1 + \alpha_2) - 3.75\}^2 + \ldots + \{(\alpha_1 + \alpha_2) - 1.79\}^2 \quad (4.2)$$

subject to

$$12.50 + 2.50 \left| L^{-1}(h) \right| \geq (\alpha_1 * 1 + \alpha_2 * 13.36) - (c_1 * 1 + c_2 * 13.36) \left| L^{-1}(h) \right|$$

$$12.50 - 2.50 \left| L^{-1}(h) \right| \leq (\alpha_1 * 1 + \alpha_2 * 13.36) + (c_1 * 1 + c_2 * 13.36) \left| L^{-1}(h) \right|$$

$$\vdots$$

$$13.22 \; + 1.79 \left| L^{-1}(h) \right| \geq (\alpha_1 * 1 + \alpha_2 * 9.75) - (c_1 * 1 + c_2 * 9.75) \left| L^{-1}(h) \right|$$

$$13.22 - 1.79 \left|L^{-1}(h)\right| \le (\alpha_1 * 1 + \alpha_2 * 9.75) + (c_1 * 1 + c_2 * 9.75)\left|L^{-1}(h)\right|$$

$$c_1 * 1 + c_2 * 13.36 \ge 0, \ldots, \; c_1 * 1 + c_2 * 9.75 \ge 0 \qquad (4.3)$$

The above nonlinear quadratic optimization problem is solved to obtain FLS regression model. The optimal value of fitness level is obtained using bisection algorithm as discussed in Section 3.3. A program is written in LINGO and objective function value z^0 is obtained by taking $h = 0$. Then, the value of h is updated according to bisection algorithm to obtain subsequent values of objective function. At last iteration, the optimal value of fitness level h is determined. The optimum value of fitness level at tolerance level, $\varepsilon = 0.001$, is $h = 0.451$. Using the above fitness level and solving quadratic optimization problem, the model constructed is as follows

$$\hat{\tilde{Y}}_i = (9.66,\ 1.60) + (0.13,\ 0.11)\ x_i,\ i = 1,2,\ldots 9 \qquad (4.4)$$

4.1.2 Necessity approach

For $Nes\left(\tilde{Y}_i \subset \hat{\tilde{Y}}_i\right)$, the fuzzy linear regression model with least-squares error can be formulated with following objective function to be minimized

$$\text{Min } Z(h) = \{(\alpha_1 + \alpha_2) - 2.50\}^2 + \{(\alpha_1 + \alpha_2) - 3.75\}^2 + \ldots + \{(\alpha_1 + \alpha_2) - 1.79\}^2 \qquad (4.5)$$

subject to

$$12.50 - 2.50 \left|L^{-1}(1-h)\right| \ge (\alpha_1 * 1 + \alpha_2 * 13.36) - (c_1 * 1 + c_2 * 13.36)\left|L^{-1}(h)\right|$$

$$12.50 + 2.50 \left|L^{-1}(1-h)\right| \le (\alpha_1 * 1 + \alpha_2 * 13.36) + (c_1 * 1 + c_2 * 13.36)\left|L^{-1}(h)\right|$$

$$\vdots$$

$$13.22 - 1.79\left|L^{-1}(1-h)\right| \ge (\alpha_1 * 1 + \alpha_2 * 9.75) - (c_1 * 1 + c_2 * 9.75)\left|L^{-1}(h)\right|$$

$$13.22 + 1.79\left|L^{-1}(1-h)\right| \le (\alpha_1 * 1 + \alpha_2 * 9.75) + (c_1 * 1 + c_2 * 9.75)\left|L^{-1}(h)\right|$$

$$c_1 * 1 + c_2 * 13.36 \ge 0, \ldots, \; c_1 * 1 + c_2 * 9.75 \ge 0 \qquad (4.6)$$

The above problem is solved similarly and the optimal value for h is 0.003 at tolerance level of $\varepsilon = 0.001$. The model constructed is as follows

$$\hat{\tilde{Y}}_i = (8.53,1.73) + (0.20,0.10)\ x_i,\ i = 1, 2, \ldots, 9 \qquad (4.7)$$

4.2 Minimization Approach

The problem is formulated as follows

$$\text{Min } J(h) = \sum_{i=1}^{9} (c_1 * 1 + c_2 * x_i),\ i = 1, \ldots, 9 \qquad (4.8)$$

subject to

$$12.50 + 2.50\left|L^{-1}(h)\right| \le (\alpha_1 * 1 + \alpha_2 * 13.36) + (c_1 * 1 + c_2 * 13.36)\left|L^{-1}(h)\right|$$

$$12.50 - 2.50\left|L^{-1}(h)\right| \ge (\alpha_1 * 1 + \alpha_2 * 13.36) - (c_1 * 1 + c_2 * 13.36)\left|L^{-1}(h)\right|$$

$$\vdots$$

$$13.22 + 1.79\left|L^{-1}(h)\right| \le (\alpha_1 * 1 + \alpha_2 * 9.75) + (c_1 * 1 + c_2 * 9.75)\left|L^{-1}(h)\right|$$

$$13.22 - 1.79\left|L^{-1}(h)\right| \ge (\alpha_1 * 1 + \alpha_2 * 9.75) - (c_1 * 1 + c_2 * 9.75)\left|L^{-1}(h)\right|$$

$$c_1,\ c_2 \ \ge\ 0 \qquad (4.9)$$

Now, solving the above linear programming problem for $h = 0.451$, fuzzy linear regression model constructed is as follows :

$$\hat{\tilde{Y}}_i = (6.04\ ,7.53\ ) + (0.41\ ,\ \ 0\ )x_i,\ i = 1, 2, \ldots, 9 \qquad (4.10)$$

Substituting values of farmers' estimates as given in Table 1 to (4.4), (4.7) and (4.10), the estimated fuzzy Pearl Millet yield corresponding to Possibility, Necessity, and Minimization methods are computed and reported in Table 2.

Table 2. Estimated fuzzy yield corresponding to Possibility, Necessity and Minmization methods

Blocks	Estimated Yields								
	Possibility			Necessity			Minimization		
	$\hat{Y}_{il}$	$\hat{Y}_{im}$	$\hat{Y}_{iu}$	$\hat{Y}_{il}$	$\hat{Y}_{im}$	$\hat{Y}_{iu}$	$\hat{Y}_{il}$	$\hat{Y}_{im}$	$\hat{Y}_{iu}$
B. Khera	8.29	11.34	14.40	8.14	11.20	14.27	4.02	11.56	19.09
Bhiwani	8.40	12.14	15.88	8.77	12.47	16.17	6.64	14.17	21.71
Kairu	8.23	10.92	13.61	7.80	10.53	13.26	2.64	10.17	17.71
Tosham	8.25	11.00	13.76	7.87	10.66	13.46	2.91	10.44	17.98
Siwan	8.23	10.92	13.60	7.80	10.53	13.26	2.63	10.16	17.70
Loharu	8.27	11.16	14.06	7.99	10.92	13.84	3.43	10.97	18.50
Badhra	8.27	11.17	14.07	8.00	10.92	13.85	3.44	10.98	18.52
Dadri-I	8.24	10.93	13.63	7.81	10.55	13.28	2.67	10.20	17.74
Dadri-II	8.23	10.89	13.55	7.78	10.48	13.19	2.53	10.07	17.60

Further, using eq. (3.12), errors in estimation for optimal fitness level are computed for all approaches and are reported in Table 3.

Table 3. Error in estimation for Least-squares (Possibility and Necessity) and Minimization methods

Blocks	Errors in estimation of yield for different methods		
	Possibility	Necessity	Minimization
B. Khera	0.83	1.76	2.01
Bhiwani	1.59	1.81	1.01
Kairu	0.91	0.73	1.35
Tosham	1.56	1.45	1.60
Siwan	1.05	0.85	1.62
Loharu	0.44	1.51	1.79
Badhra	0.24	0.25	0.97
Dadri-I	0.25	1.51	2.01
Dadri-II	1.92	2.41	3.22
Total	**8.79**	**12.28**	**15.84**

A perusal shows that values of sums of errors for Possibility and Necessity methods are lower than those for Minimization method for all the blocks. In other words, least-squares approach gives more reliable estimates for crop yield vis-à-vis linear programming approach. To get a visual idea, observed and estimated Pearl Millet yields obtained from using Possibility approach with h = 0.451 are depicted in Fig. 3. Evidently, the observed yields (in solid lines) and estimated yields (in dotted lines) are found to be quite close to each other, thereby indicating that farmers' estimates are able to explain actual crop yield with high fitness levels.

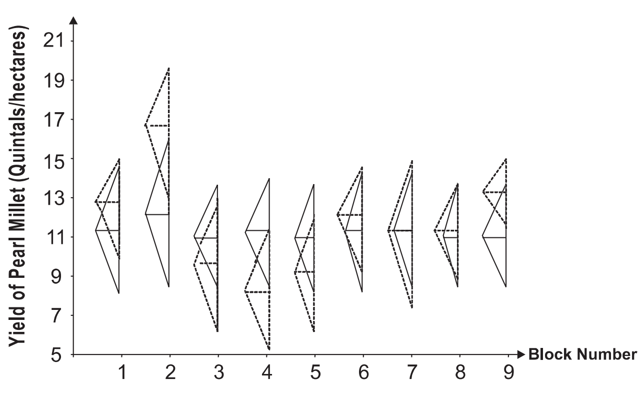

Fig. 3: Observed (in solid lines) and estimated (in doted lines) Pearl Millet crop yield based on Possibility approach at optimal fitness level

5. CONCLUDING REMARK

The extension of above work when explanatory variable is also fuzzy is in progress and shall be reported separately in due course of time.

REFERENCES

Chang, Y.O. and Ayyub, B.M. (2001). Fuzzy regression methods — a comparative assessment. *Fuzz. Sets Sys.,* **119,** 187-203.

Diamond, P. (1988). Fuzzy least squares. *Inform. Sci.,* **46**, 141-157.

Dubois, D. and Prade, H. (1980). *Fuzzy Sets and Systems: Theory and Applications.* Academic Press, New York.

Ghosh, H., Singh, R.K. and Prajneshu (2007). Possibilistic linear regression analysis with fuzzy response variable for crop yield estimation. (Communicated).

Kandala, V. M. and Prajneshu (2004). Fitting allometric model using fuzzy least squares. *J. Mar. Biol. Assoc.,* **46**, 120-123.

Kim, B. and Bishu, R.R. (1998). Evaluation of fuzzy linear regression models by comparing membership functions. *Fuzz. Set Sys.*, **100,** 343-352.

LINDO Systems Inc. (2002). *LINGO User's Guide*. LINDO Systems Inc., Chicago, U.S.A.

Modarres, M., Nasrabadi, E. and Nasrabadi, M.M. (2004). Fuzzy, linear regression analysis from the point of view risk. *Int. J. Uncert. Fuzzi. Knowl. Bas. Syst*., **12**, 635-649.

Modarres, M., Nasrabadi, E. and Nasrabadi, M.M. (2005). Fuzzy linear regression models with least square errors. *Appl. Math. Compu.*, **163**, 977-989.

Rao, J.N.K. (2003). *Small Area Estimation*. Wiley Interscience, U.S.A.

Sud, U.C., Mathur, D.C., Sethi, S.C., Jha, G.K. and Bhasin, R.M. (2006). Crop Yield Estimation at Smaller Area Level Using Farmers' Estimates. I.A.S.R.I. Project Report, New Delhi, 1-72.

Tanaka, H., Uegima, S. and Asia, K. (1982). Linear regression analysis with fuzzy model. *IEEE Trans. Sys. Ma. Cyb.,* **12**, 903-907.

Zadeh, L.A. (1965). Fuzzy sets. *Inform. Cont*., **8**, 338-353.