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JOURNAL OF THE INDIAN SOCIETY OF AGRICULTURAL STATISTICS 74(1) 2020 63–68

An Alternative to Ratio and Product type Estimators of Finite Population Mean in Double Sampling for Stratification

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Received 11 April 2019; Revised 30 July 2019; Accepted 31 January 2020

SUMMARY

In this paper we have proposed an alternative to Ige and Tripathi (1987) estimators with their properties. The expressions for biases and mean squared errors have been obtained upto the first degree of approximation. The proposed estimators have been compared with usual unbiased estimator of population mean in double sampling for stratification and ratio and product type estimators given by Ige and Tripathi (1987). To judge the merits of the proposed estimators an empirical study have been carried out.

Keywords: Double sampling for stratification, Bias, Mean squared error.

1. INTRODUCTION

In survey sampling there can be the situations when strata weights are not available or if available, strata weights are outdated and can't be used. This type of situation occurs during the household survey, when investigator does not have information about newly added household in different colonies. This situation leads investigator to use double sampling for stratification. Neyman (1938) developed the theory of double sampling. The problem of estimating finite population mean in double sampling for stratification has been studied by few researchers including Ige and Tripathi (1987),Tripathi and Bahl (1991), Singh and Vishwakarma (2007), Chouhan (2012) ,Sharma (2012), Jatwa (2014), Tailor and Lone (2014a) and Tailor *et al.* (2014b).

Let us consider a finite population $U=\{U_1,U_2,U_3,...U_N\}$ of size N in which strata weight $\frac{N_h}{N},\{h=1,2,3,...L\}$ are unknown. In these conditions we use double sampling for stratification. The procedure for double sampling for stratification is given below

(a) a first phase sample S of size n' using simple random sampling without replacement is drawn and auxiliary variates x and z are observed.

(b) the sample is stratified into L strata on the basis of observed variables x and z. Let n'_h denotes the number of units in h^{th} stratum $(h=1,2,3,...,L)$ such that $n'=\sum_{h=1}^{L} n'_h$.

(c) from each n'_h unit, a sample of size $n_h=v_h n'_h$ is drawn where $0<v_h<1$, $\{h=1,2,3,...,L\}$, is the predetermined probability of selecting a sample of size n_h from each strata of size n'_h and it constitutes a sample S' of size $n=\sum_{h=1}^{L} n_h$. In S' both study variate y and auxiliary variates x and z are observed.

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Let y be the study variate and x and z are the two auxiliary variate respectively. Let us define

$\bar{x}_{ds} = \sum_{h=1}^{L} w_h \bar{x}_h$: Unbiased estimator of population mean $\bar{X}$ at second phase or double sampling mean of the auxiliary variate x

$\bar{y}_{ds} = \sum_{h=1}^{L} w_h \bar{y}_h$: Unbiased estimator of population mean $\bar{Y}$ at second phase or double sampling mean of the study variate y

$\bar{z}_{ds} = \sum_{h=1}^{L} w_h \bar{z}_h$: Unbiased estimator of population mean $\bar{Z}$ at second phase or double sampling mean of the auxiliary variate z

$\bar{x}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} \bar{x}_{hi}$: Mean of the second phase sample taken from h^{th} stratum for the auxiliary variate x

$\bar{y}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} \bar{y}_{hi}$: Mean of the second phase sample taken from h^{th} stratum for the study variate y

$\bar{z}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} \bar{z}_{hi}$: Mean of the second phase sample taken from h^{th} stratum for the auxiliary variate z

$\bar{X} = \frac{1}{N} \sum_{h=1}^{L} \sum_{i=1}^{N_h} x_{hi}$: Population mean of the auxiliary variate x

$\bar{Y} = \frac{1}{N} \sum_{h=1}^{L} \sum_{i=1}^{N_h} y_{hi}$: Population mean of the study variate y

$\bar{Z} = \frac{1}{N} \sum_{h=1}^{L} \sum_{i=1}^{N_h} z_{hi}$: Population mean of the auxiliary variate z

$\bar{X}_h = \frac{1}{N_h} \sum_{i=1}^{N_h} x_{hi}$: h^{th} stratum population mean for the auxiliary variate x

$\bar{Y}_h = \frac{1}{N_h} \sum_{i=1}^{N_h} y_{hi}$: h^{th} stratum population mean for the study variate y

$\bar{Z}_h = \frac{1}{N_h} \sum_{i=1}^{N_h} x_{hi}$: h^{th} stratum population mean for the auxiliary variate z

$S_x^2 = \frac{1}{N-1} \sum_{h=1}^{L} \sum_{i=1}^{N_h} \left(x_{hi} - \bar{X}_h \right)^2$: Population mean square of the auxiliary variate x

$S_y^2 = \frac{1}{N-1} \sum_{h=1}^{L} \sum_{i=1}^{N_h} \left(y_{hi} - \bar{Y}_h \right)^2$: Population mean square of the study variate y

$S_z^2 = \frac{1}{N-1} \sum_{h=1}^{L} \sum_{i=1}^{N_h} \left(z_{hi} - \bar{Z}_h \right)^2$: Population mean square of the auxiliary variate z

$S_{xh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} \left(x_{hi} - \bar{X}_h \right)^2$: h^{th} stratum population mean square of the auxiliary variate x

$S_{yh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} \left(y_{hi} - \bar{Y}_h \right)^2$: h^{th} stratum population mean of the study variate y

$S_{zh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} \left(z_{hi} - \bar{Z}_h \right)^2$: h^{th} stratum population mean square of the auxiliary variate z

$\rho_{yxh} = \frac{S_{yxh}}{S_{yh} S_{xh}}$: Correlation coefficient between y and x in the stratum h,

$\bar{x}'_h = \frac{1}{n'_h} \sum_{h=1}^{n_h} \bar{x}_{hi}$: First phase sample mean of the h^{th} stratum for the auxiliary variate x

$\bar{z}'_h = \frac{1}{n'_h} \sum_{h=1}^{n_h} \bar{z}_{hi}$: First phase sample mean of the h^{th} stratum for the auxiliary variate z

$f = \frac{n'}{N}$: First phase sampling fraction.

$n = \sum_{h=1}^{L} n_h$: size of the sample S'

$w'_h = \frac{n'_h}{n'}$: h^{th} stratum weight in the first phase sample

$\bar{x}' = \frac{1}{n'_h} \sum_{h=1}^{n_h} w_h \bar{x}'_h$: Unbiased estimator of population mean $\bar{X}$ for the first phase

$\bar{z}' = \frac{1}{n'_h} \sum_{h=1}^{n_h} w_h \bar{z}'_h$: Unbiased estimator of population mean $\bar{Z}$ for the first phase

Ige and Tripathi (1987) defined classical ratio and product estimators in double sampling for stratification as

$$\bar{y}_{Rd} = \bar{y}_{ds}\left(\frac{\bar{x}'}{\bar{x}_{ds}}\right). \tag{1.1}$$

and

$$\bar{y}_{Pd} = \bar{y}_{ds}\left(\frac{\bar{z}_{ds}}{\bar{z}'}\right). \tag{1.2}$$

where z is an auxiliary variate which is negatively correlated with the study variate y and notations $\bar{z}_{ds}$ and $\bar{z}'$ have their usual meanings.

The biases and mean squared errors of estimators $\bar{y}_{Rd}$ and $\bar{y}_{Pd}$ up to the first degree of approximation are given by

$$B(\bar{y}_{Rd}) = \frac{1}{\bar{X}}\left[\sum_{h=1}^{L}\frac{W_h}{n'}\left(\frac{1}{v_h}-1\right)\left\{R_1 S_{xh}^2 - S_{yxh}\right\}\right], \tag{1.3}$$

$$B(\bar{y}_{Pd}) = \frac{1}{\bar{Z}}\left[\sum_{h=1}^{L}\frac{W_h}{n'}\left(\frac{1}{v_h}-1\right)S_{yzh}\right], \tag{1.4}$$

$$MSE(\bar{y}_{Rd}) = S_y^2\left(\frac{1-f}{n'}\right) + \frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2 + R_1^2 S_{xh}^2 - 2R_1 S_{yxh}\right], \tag{1.5}$$

and

$$MSE(\bar{y}_{Pd}) = S_y^2\left(\frac{1-f}{n'}\right) + \frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2 + R_2^2 S_{zh}^2 + 2R_2 S_{yzh}\right] \tag{1.6}$$

Srivenkataramana (1980) and Bandhyopadhyaya (1980) used the transformation $x_i^* = \frac{N\bar{X} - nx_i}{N-n}$ and $z_i^* = \frac{N\bar{Z} - nz_i}{N-n}$ on auxiliary variate x and z and obtained dual to classical ratio and product estimator as

, (1.7)

and

$$\hat{\bar{Y}}_P^* = \bar{y}\left(\frac{\bar{Z}}{\bar{z}^*}\right). \tag{1.8}$$

where $\bar{x}^* = \frac{N\bar{X} - n\bar{x}}{N-n}$ and $\bar{z}^* = \frac{N\bar{Z} - n\bar{z}}{N-n}$ are unbiased estimators of population mean $\bar{X}$ and $\bar{Z}$ respectively.

2. PROPOSED ESTIMATORS

Following Srivenkataramana (1980) and Bondyopadhyayh (1980) transformation, we proposed an alternative to Ige and Tripathi (1987) estimators in double sampling for stratification as

$$\bar{y}_{Rd}^* = \bar{y}_{ds}\left(\frac{\bar{x}_{ds}^*}{\bar{x}'}\right)$$

$$\text{or } \bar{y}_{Rd}^* = \frac{\bar{y}_{ds}}{\bar{x}'}\left[\frac{N\bar{x}' - n\bar{x}_{ds}}{N-n}\right] \tag{2.1}$$

and

$$\bar{y}_{Pd}^* = \bar{y}_{ds}\left(\frac{\bar{\mathbf{z}}'}{\bar{z}_{ds}^*}\right)$$

$$\text{or } \bar{y}_{Pd}^* = \frac{\bar{y}_{ds}}{\bar{\mathbf{z}}'^{-1}}\left[\frac{N-n}{N\bar{z}' - n\bar{z}_{ds}}\right] \tag{2.2}$$

Where $\bar{x}_{ds}^* = \frac{N\bar{x}' - n\bar{x}_{ds}}{N-n}$ and $\bar{z}_{ds}^* = \frac{N\bar{z}' - n\bar{z}_{ds}}{N-n}$

To obtain the biases and mean squared errors of the proposed estimators $\bar{y}_{Rd}^*$ and $\bar{y}_{Pd}^*$ we write

$\bar{y}_{ds} = \bar{Y}(1+e_o)$, $\bar{x}_{ds} = \bar{X}(1+e_1)$, $\bar{x}' = \bar{X}(1+e_1')$, $\bar{z}_{ds} = \bar{Z}(1+e_2)$ and $\bar{z}' = \bar{Z}(1+e_2')$

such that $E(e_o) = E(e_1) = E(e_1') = E(e_2) = E(e_2') = 0$ and

$$E(e_0^2) = \frac{1}{\bar{Y}^2}\left[S_y^2\left(\frac{1-f}{n'}\right) + \frac{1}{n'}\sum_{h=1}^{L}W_h S_{yh}^2\left(\frac{1}{v_h}-1\right)\right],$$

$$E(e_1^2) = \frac{1}{\bar{X}^2}\left[S_x^2\left(\frac{1-f}{n'}\right) + \frac{1}{n'}\sum_{h=1}^{L}W_h S_{xh}^2\left(\frac{1}{v_h}-1\right)\right],$$

$$E(e_2^2) = \frac{1}{\bar{Z}^2}\left[S_z^2\left(\frac{1-f}{n'}\right) + \frac{1}{n'}\sum_{h=1}^{L}W_h S_{zh}^2\left(\frac{1}{v_h}-1\right)\right],$$

$$E(e_0e_1) = \frac{1}{\bar{Y}\bar{X}}\left[\left(\frac{1-f}{n'}\right)S_{yx} + \frac{1}{n'}\sum_{h=1}^{L}W_h S_{yxh}\left(\frac{1}{v_h}-1\right)\right],$$

$$E(e_0e_2) = \frac{1}{\bar{Y}\bar{Z}}\left[\left(\frac{1-f}{n'}\right)S_{yz} + \frac{1}{n'}\sum_{h=1}^{L}W_h S_{yzh}\left(\frac{1}{v_h}-1\right)\right],$$

$$E(e_1e_2) = \frac{1}{\bar{X}\bar{Z}}\left[\left(\frac{1-f}{n'}\right)S_{xz} + \frac{1}{n'}\sum_{h=1}^{L}W_h S_{xzh}\left(\frac{1}{v_h}-1\right)\right],$$

$$E(e_0e_1') = \frac{1}{\bar{Y}\bar{X}}\left(\frac{1-f}{n'}\right)S_{yx},\quad E(e_1'^2) = \frac{1}{\bar{X}^2}S_x^2\left(\frac{1-f}{n'}\right),$$

$$E\left(e_2'^2\right)=\frac{1}{\bar{Z}^2}S_z^2\left(\frac{1-f}{n'}\right),\ E\left(e_1e_1'\right)=\frac{1}{\bar{X}^2}\left(\frac{1-f}{n'}\right)S_x^2,$$

$$E\left(e_2e_2'\right)=\frac{1}{\bar{Z}^2}S_z^2\left(\frac{1-f}{n'}\right),\ E\left(e_1'e_2'\right)=\frac{1}{\bar{X}\bar{Z}}\left(\frac{1-f}{n'}\right)S_{xz},$$

$$E\left(e_0e_2'\right)=\frac{1}{\bar{Y}\bar{Z}}\left(\frac{1-f}{n'}\right)S_{yz}\ \text{and}\ E\left(e_1e_2'\right)=\frac{1}{\bar{X}\bar{Z}}\left(\frac{1-f}{n'}\right)S_{xz}.$$

The biases and mean squared errors of the proposed estimators $\bar{y}_{Rd}^*$ and $\bar{y}_{Pd}^*$ upto the first degree of approximation are obtained as

$$B\left(\bar{y}_{Rd}^*\right)=-\frac{g}{\bar{X}}\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)S_{yxh}, \tag{2.3}$$

$$B\left(\bar{y}_{Pd}^*\right)=\frac{1}{\bar{Z}}\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[g^2R_2\,S_{zh}^2+g\,S_{yzh}\right], \tag{2.4}$$

$$MSE\left(\bar{y}_{Rd}^*\right)=S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2+g^2R_1^2S_{xh}^2-2gR_1S_{yxh}\right], \tag{2.5}$$

and

$$MSE\left(\bar{y}_{Pd}^*\right)=S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2+g^2R_2^2S_{zh}^2+2gR_2S_{yzh}\right]. \tag{2.6}$$

3. EFFICIENCY COMPARISONS

The variance of usual unbiased estimator $\bar{y}_{ds}$ in double sampling for stratification is given as

$$V\left(\bar{y}_{ds}\right)=S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\,S_{yh}^2\left(\frac{1}{v_h}-1\right). \tag{3.1}$$

Efficiency comparisons of proposed dual to ratio estimator $\bar{y}_{Rd}^*$

Comparisons of (2.5) with equation (1.5) and (3.1) shows that

(i) $MSE\left(\bar{y}_{Rd}^*\right)<V\left(\bar{y}_{ds}\right)$ if

$$S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2+g^2R_1^2S_{xh}^2-2gR_1S_{yxh}\right]<S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\,S_{yh}^2\left(\frac{1}{v_h}-1\right)$$

$$\Rightarrow\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2+g^2R_1^2S_{xh}^2-2gR_1S_{yxh}\right]<\sum_{h=1}^{L}W_h\,S_{yh}^2\left(\frac{1}{v_h}-1\right)$$

$$\Rightarrow R_1g\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)S_{xh}^2<2\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)S_{yxh} \tag{3.2}$$

(ii) $MSE\left(\bar{y}_{Rd}^*\right)<MSE\left(\bar{y}_{Rd}\right)$ if

$$S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2+g^2R_1^2S_{xh}^2-2gR_1S_{yxh}\right]<S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2+R_1^2S_{xh}^2-2R_1S_{yxh}\right]$$

$$\Rightarrow\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2+g^2R_1^2S_{xh}^2-2gR_1S_{yxh}\right]<\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2+R_1^2S_{xh}^2-2R_1S_{yxh}\right]$$

$$\Rightarrow\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[g^2R_1^2S_{xh}^2-2gR_1S_{yxh}\right]<\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[R_1^2S_{xh}^2-2R_1S_{yxh}\right]$$

$$\Rightarrow R_1(g^2-1)\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)S_{xh}^2<2(g-1)\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)S_{yxh} \tag{3.3}$$

Efficiency comparison of proposed dual to product estimator $\bar{y}_{Pd}^*$

Comparisons of equations (2.6) with equations (1.6) and (3.1) shows that

(i) $MSE\left(\bar{y}_{Pd}^*\right)<V\left(\bar{y}_{ds}\right)$ if

$$S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2+g^2R_2^2S_{zh}^2+2gR_2S_{yzh}\right]<S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\,S_{yh}^2\left(\frac{1}{v_h}-1\right)$$

$$\Rightarrow\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2+g^2R_2^2S_{zh}^2+2gR_2S_{yzh}\right]<\sum_{h=1}^{L}W_h\,S_{yh}^2\left(\frac{1}{v_h}-1\right)$$

$$\Rightarrow\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[g^2R_2^2S_{zh}^2+2gR_2S_{yzh}\right]<0$$

$$\Rightarrow R_2g\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)S_{zh}^2<-2\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)S_{yzh} \tag{3.4}$$

(ii) $MSE\left(\overline{y}_{Pd}^{*}\right) < MSE\left(\overline{y}_{Pd}\right)$ if

$$S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2+g^2R_2^2S_{zh}^2+2gR_2S_{yzh}\right]<$$

$$S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2+R_2^2S_{zh}^2+2R_2S_{yzh}\right]$$

$$\Rightarrow\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2+g^2R_2^2S_{zh}^2+2gR_2S_{yzh}\right]<$$

$$\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2+R_2^2S_{zh}^2+2R_2S_{yzh}\right]$$

$$\Rightarrow\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[g^2R_2^2S_{zh}^2+2gR_2S_{yzh}\right]<$$

$$\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[R_2^2S_{zh}^2+2R_2S_{yzh}\right]$$

$$\Rightarrow R_2(g^2-1)\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)S_{zh}^2<-2(g-1)\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)S_{yzh} \tag{3.5}$$

where $R_1=\frac{\overline{Y}}{\overline{X}}$, $R_2=\frac{\overline{Y}}{\overline{Z}}$ and $g=\frac{n}{N-n}$.

4. EMPIRICAL STUDY

To exhibit the performance of the proposed estimators in comparison to other considered estimators, two population data sets are being used. The descriptions of population are given below.

Population I- [Source: Tailor *et al.* (2014b)]

y: Production (MT/hectare), x: Production in ‘000Tons and z: Area in ‘000hectare

Constant	Stratum I	Stratum II
n_h	4	4
n'_h	7	7
N_h	10	10
$\overline{Y}_h$	1925.8	3115.6
$\overline{X}_h$	214.4	333.8
$\overline{Z}_h$	51.80	60.60
S_{yh}	615.92	340.38
S_{xh}	74.87	66.35
S_{zh}	0.75	4.84
S_{yxh}	39360.68	22356.50
S_{yzh}	411.16	1536.24
S_{xzh}	38.08	287.92
ρ_{yxh}	0.85	0.98
ρ_{yzh}	0.89	0.93
S_y^2	668351.00	

Population- II [Chouhan, S. (2012)

y: Snowy days,

x: rainy days and

z: Total annual sunshine hours

Constant	Stratum I	Stratum II
n_h	4	4
n'_h	7	7
N_h	10	10
$\overline{Y}_h$	142.80	102.60
$\overline{X}_h$	149.70	91.00
$\overline{Z}_h$	1630.00	2036.00
S_{yh}	6.09	12.60
S_{xh}	13.46	6.57
S_{zh}	102.17	103.46
S_{yxh}	18.44	23.30
S_{yzh}	-239.30	-655.30
S_{xzh}	-1073.00	-240.30
ρ_{yxh}	0.22	0.28
ρ_{yzh}	-0.38	-0.50
S_y^2	528.43	

Table 1 reveals that the proposed ratio estimator $\overline{y}_{Rd}^{*}$ has maximum percent relative efficiency in comparison to usual unbiased estimator $\overline{y}_{ds}$ and Ige and Tripathi (1987) ratio estimator $\overline{y}_{Rd}$ for populations 1. Proposed product type estimator $\overline{y}_{Pd}^{*}$ also has highest percent relative efficiency in comparison to usual unbiased estimator $\overline{y}_{ds}$ and Ige and Tripathi (1987) product estimator $\overline{y}_{Pd}$.

Table 1. Percent relative Efficiencies of $\bar{y}_{ds}$, $\bar{y}_{Rd}$, $\bar{y}_{Pd}$, $\bar{y}^*_{Rd}$ and $\bar{y}^*_{Pd}$ with respect to $\bar{y}_{ds}$

Estimators	$\bar{y}_{ds}$	$\bar{y}_{Rd}$	$\bar{y}_{Pd}$	$\bar{y}^*_{Rd}$	$\bar{y}^*_{Pd}$
Population I	100.00	138.99	82.20	**158.12**	*
Population II	100.00	80.66	104.24	*	**106.66**

*** Not applicable**

Table 2. Empirical exhibition of theoretical conditions given in Section 3

Conditions for proposed dual to ratio estimator $\bar{y}^*_{Rd}$	**Population- I**
$MSE\left(\bar{y}^*_{Rd}\right) < V\left(\bar{y}_{ds}\right)$ if $R_1 g\sum_{h=1}^{L} W_h\left(\frac{1}{v_h}-1\right)S^2_{xh} < 2\sum_{h=1}^{L} W_h\left(\frac{1}{v_h}-1\right)S_{yxh}$	**23002.4<46287.9** **-19169.7<-15429.3**
$MSE\left(\bar{y}^*_{Rd}\right) < MSE\left(\bar{y}_{Rd}\right)$ if $R_1(g^2-1)\sum_{h=1}^{L} W_h\left(\frac{1}{v_h}-1\right)S^2_{xh} < 2(g-1)\sum_{h=1}^{L} W_h\left(\frac{1}{v_h}-1\right)S_{yxh}$	
Conditions for proposed dual to product estimator $\bar{y}^*_{Pd}$	**Population- II**
$MSE\left(\bar{y}^*_{Pd}\right) < V\left(\bar{y}_{ds}\right)$ if $R_2 g\sum_{h=1}^{L} W_h\left(\frac{1}{v_h}-1\right)S^2_{zh} < -2\sum_{h=1}^{L} W_h\left(\frac{1}{v_h}-1\right)S_{yzh}$	**359.94<670.92** **-294.3<-223.69**
$MSE\left(\bar{y}^*_{Pd}\right) < MSE\left(\bar{y}_{Pd}\right)$ if $R_2(g^2-1)\sum_{h=1}^{L} W_h\left(\frac{1}{v_h}-1\right)S^2_{zh} < -2(g-1)\sum_{h=1}^{L} W_h\left(\frac{1}{v_h}-1\right)S_{yzh}$	

5. CONCLUSION

We have proposed an alternative to Ige and Tripathi (1987) estimators with their properties. In Section 3 the theoretical efficiency comparisons of the proposed estimators with other considered estimators have been given. The conditions under which the proposed estimators have less mean squared errors in comparison to usual unbiased estimator and Ige and Tripathi (1987) ratio and product type estimators are calculated empirically and tabulated in Table 2. The proposed product type estimator $\bar{y}^*_{Pd}$ also has highest percent relative efficiency in comparison to usual unbiased estimator $\bar{y}_{ds}$ and Ige and Tripathi (1987) product estimator $\bar{y}_{Pd}$. Thus the proposed estimators are recommended for use in practice for estimating the finite population mean provided the conditions given in section 3 are satisfied.

ACKNOWLEDGMENT

The authors are highly thankful to the learned referees and the Editor for their valuable on the original version of the paper.

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