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Stratified Randomized Response Model for Multiple Responses

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SUMMARY

Randomized response (RR) techniques are used to collect data on sensitive characteristics. Abdelfatah and Mazloum (2015) extended Odumade and Singh's (2009) RR techniques based on two decks of cards for stratified sampling and claim, on the basis of empirical studies, that their proposed estimators performs better that the Odumade and Singh (2009) RR technique in most situations. In this paper we have proposed alternative estimators for each of the Odumade and Singh (2009), Abdelfatah *et al.* (2011) and Abdelfatah and Mazloum (2015) RR techniques for stratified sampling. The proposed alternative estimators are found be more efficient than the existing estimators. Apart from increased efficiencies, the proposed estimators possess simpler expressions for the estimators of proportion, variances and unbiased estimators of variances.

Keywords: Optimum allocation, Randomized response; Relative efficiency, Sensitive characteristics.

1. INTRODUCTION

While collecting information, directly from respondents, relating to sensitive issues such as induced abortion, drug addiction, duration of suffering from Aids and so on, the respondents very often report untrue values or even refuse to respond (Arnab and Singh (2013)). Warner (1965) introduced an ingenious technique known as randomized response technique (RR) where a respondent supplies indirect response instead of direct response. Thus the RR technique protects privacy of the respondents while increase quality of data by reducing major sources of bias originated from evasive answers and non-responses. Warner's (1965) technique was modified by several researchers e.g. Horvitz et al. (1967), Greenberg *et al.* (1969), Kim (1978), Franklin (1989), Arcos *et al.* (2015) and Rueda *et al.* (2015) among others which increased cooperation of the respondents and improved efficiencies of the estimators. Applications of the RR techniques in real life surveys were reported by Greenberg *et al.* (1969): Illegitimacy of offspring; Abernathy *et al.* (1970): Incidence of induced abortions; Van der Heijden *et al.* (1998): Social security fraud, and Arnab and Mothupi (2015): Sexual habits of University students. Further details are given by Chaudhuri and Mukherjee (1988), Singh (2003) and Arnab (2017) among others.

Recently Abdelfatah and Mazloum (2015) extended Odumade and Singh (2009) and Abdelfatah *et al.* (2011)'s RR techniques to stratified sampling for estimating the proportion π of a certain sensitive characteristic of a population. On the basis of empirical studies Abdelfatah and Reda Mazloum (2015) showed that the Abdelfatah *et al.*'s (2011) RR technique performed better in about 22% of the cases than the Odumade and Singh (2009)'s RR technique when extended to stratified sampling. We will refer Odumade and Singh (2009), Abdelfatah *et al.* (2011) and Abdelfatah and Mazloum (2015) to this paper as OS, AF and AFM respectively.

In this paper, we have proposed alternative estimators for AFM and OS RR models for stratified sampling. The proposed alternative estimator for AFM model has been proven theoretically superior to the existing AF and AFM estimators. It is shown empirically that the proposed alternative estimator perform always better than the OS estimator. The proposed estimators, their variances and unbiased estimators of the variances

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are much simpler than the existing AF, AFM and OS estimators.

2. ALTERNATIVE ESTIMATORS FOR OS AND AF RR MODELS

2.1 OS RR technique

In OS RR technique, a sample of size n units is selected from a population by simple random sampling with replacement (SRSWR) method. Each of the selected respondents in the sample is asked to select one card at random from each of the decks: Deck 1 and Deck 2. Each of the decks consists of two types of cards written "I belong to the sensitive group A" and "I do not belong to the sensitive group A" with proportions P and T respectively. The respondent answers "Yes" or "No" if the statement matches his status with the statement written on the card (Arnab *et al.* (2017)).

Deck 1		Deck 2
I belong to the sensitive group A with proportion P		I belong to the sensitive group A with proportion T
I do not belong to the sensitive group A with proportion $1-P$		I do not belong to the sensitive group A with proportion $1-T$

For example if a respondent selects a card written "I belong to the sensitive group A" from the Deck-1 and selects the other card written "I do not belong to the sensitive group A" from the Deck-2, then he/she will supply with a response "Yes, No" if he/she belong to the sensitive group A. On the other hand if the respondent do not belongs to the group A, he/she will supply "No, Yes" as his/her response (Arnab and Shangodoyin (2015)).

Let $n_{11}(\theta_{11})$, $n_{10}(\theta_{10})$, $n_{01}(\theta_{01})$ and ${}_{00}\left({}_{00}\right)$ denote respectively the frequencies (probabilities) of responses (Yes, Yes), (Yes, No), (No, Yes) and (No, No).

Response from Deck 1	Response from Deck 2		Total
	Yes	No	
Yes	n_{11}	n_{10}	$n_{1\cdot}$
No	n_{01}	n_{00}	$n_{0\cdot}$
Total	$n_{\cdot 1}$	$n_{\cdot 0}$	n

2.1.1 Odumade and Singh's Estimator

Odumade and Singh (2009) proposed an unbiased estimator for the population proportion π by minimizing a distance function

$$D = \frac{1}{2}\sum_{i=0}^{1}\sum_{j=0}^{1}\left(\theta_{ij} - n_{ij}/n\right)^2$$

as

$$\hat{\pi}_{os} = \frac{1}{2} + \frac{(P+T-1)(n_{11}-n_{00})(P-T)(n_{10}-n_{00})}{2n[(P+T-1)^2+(P-T)^2]} \tag{2.1}$$

The variance of $\hat{\pi}_{os}$ and an unbiased estimator of the variance of $\hat{\pi}_{os}$ were given respectively as

$$V\left(\hat{\pi}_{os}\right) = \frac{\begin{array}{c}(P+T-1)^2\{PT+(1-P)(1-T)\}+\\(P-T)^2\{T(1-P)+P(1-T)\}\end{array}}{4n[(P+T-1)^2+(P-T)^2]^2} - \frac{(2\pi-1)^2}{4n}$$

$$= \frac{\pi(1-\pi)}{n} + \frac{1}{4n}\left[\frac{\begin{array}{c}(P+T-1)^2\{PT+(1-P)(1-T)\}+\\(P-T)^2\{T(1-P)+P(1-T)\}\end{array}}{[(P+T-1)^2+(P-T)^2]^2} - 1\right] \tag{2.2}$$

and

$$\hat{V}\left(\hat{\pi}_{os}\right) = \frac{1}{4(n-1)}\left[\frac{\begin{array}{c}(P+T-1)^2\{PT+(1-P)(1-T)\}+\\(P-T)^2\{T(1-P)+P(1-T)\}\end{array}}{[(P+T-1)^2+(P-T)^2]^2} - (2\hat{\pi}-1)^2\right] \tag{2.3}$$

2.1.2 Jayraj *et al.* (2016) Estimator

Jayaraj *et al.* (2016) proposed an alternative estimator of π by minimizing a weighted distance function

$$D = \frac{1}{2}\sum_{i=0}^{1}\sum_{j=0}^{1} w_{ij}\left(\theta_{ij} - n_{ij}/n\right)^2$$

with $w_{00} = \dfrac{(1-P)(1-T)}{(1-P-T)}$, $w_{01} = \dfrac{(1-P)T}{(T-P)}$, $w_{10} = \dfrac{P(1-T)}{(P-T)}$ and $w_{11} = \dfrac{PT}{P+T-1}$

as

$$\hat{\pi}_J = \frac{\begin{array}{c}PTn_{11}+P(1-T)n_{10}+(1-P)Tn_{01}+(1-P)(1-T)n_{00}\\-4PT(1-P)(1-T)\end{array}}{n\left\{1-2P(1-P)-2T(1-T)\right\}} \tag{2.4}$$

They obtained the variance of $\hat{\pi}_J$ and an unbiased estimator of the variance of $\hat{\pi}_J$ as

$$V(\hat{\pi}_J) = \frac{\pi(1-\pi)}{n} + \frac{(2P-1)^2(2T-1)^2\{P(1-P)+T(1-T)\}\pi}{n\{1-2P(1-P)-2T(1-T)\}^2} + \frac{PT(1-P)(1-T)\{1-16PT(1-P)(1-T)\}}{n\{1-2P(1-P)-2T(1-T)\}^2} \tag{2.5}$$

and

$$\hat{V}(\hat{\pi}_J) = \frac{\pi_J(1-\pi_J)}{n-1} + \frac{(2P-1)^2(2T-1)^2\{P(1-P)+T(1-T)\}\hat{\pi}_J}{n\{1-2P(1-P)-2T(1-T)\}^2} + \frac{PT(1-P)(1-T)\{1-16PT(1-P)(1-T)\}}{n\{1-2P(1-P)-2T(1-T)\}^2} \tag{2.6}$$

3. ALTERNATIVE ESTIMATOR FOR OS MODEL

Let y_i be the value of the sensitive characteristic (say) of the study variable y for the ith respondent (unit) of the population U of size N. Let $y_i = 1$ if the respondent possesses the characteristic A and $y_i = 0$ otherwise. Then the proportion of the respondents possess the characteristic A in the population is

$$\pi = \frac{1}{N}\sum_{i \in U} y_i \tag{3.1}$$

Let

$$z_i(j) = \begin{cases} 1 \text{ if the ith respondent of the jth deck answers "Yes"} \\ 0 \text{ if the ith respondent of the jth deck answers "No"} \end{cases}$$

for $j = 1, 2$.

Then,

$$E_R\{z_i(1)\} = y_i P + (1-y_i)(1-P),\ V_R\{z_i(1)\} = P(1-P),$$
$$E_R\{z_i(2)\} = y_i T + (1-y_i)(1-T) \text{ and}$$
$$V_R\{z_i(2)\} = T(1-T) \tag{3.2}$$

where E_R and V_R denote respective the expectation and variance with respect to the RR technique.

From the above Eq. (3.2), we find that

$$r_i(1) = \frac{z_i(1) - P(1-P)}{2P-1} \text{ and } r_i(2) = \frac{z_i(2) - T(1-T)}{2T-1} \tag{3.3}$$

satisfy

$$E_R\{r_i(1)\} = E_R\{r_i(2)\} = y_i,\ V_R\{r_i(1)\} = \frac{P(1-P)}{(2P-1)^2} = \phi_1,$$
$$V_R\{r_i(2)\} = \frac{T(1-T)}{(2T-1)^2} = \phi_2 \text{ and the covariance}$$
$C_R\{r_i(1), r_i(2)\} = 0$ (as the cards are selected independently) (3.4)

From (3.3) and (3.4), we obtain unbiased estimators of π based on the answers from Deck1 and Deck 2 cards respectively as follows:

$$\hat{\pi}_1 = \frac{1}{n}\sum_{i=1}^{n} r_i(1) = \frac{\hat{\lambda}_1 - (1-P)}{(2P-1)} \text{ and}$$
$$\hat{\pi}_2 = \frac{1}{n}\sum_{i=1}^{n} r_i(1) = \frac{\hat{\lambda}_2 - (1-T)}{(2T-1)} \tag{3.5}$$

where $\hat{\lambda}_1$ and $\hat{\lambda}_2$ are the proportion of "Yes" answers from the sampled respondents based on the Deck 1 and Deck 2 cards respectively.

Let E_p, V_p and C_p be operators for expectation, variance and covariance with respect to the sampling design p, we have the following estimators:

$$E(\hat{\pi}_1) = E_p\left[\frac{1}{n}\sum_{i=1}^{n} E_R\{r_i(1)\}\right] = E_p\left(\frac{1}{n}\sum_{i=1}^{n} y_i\right) = \pi, \tag{3.6}$$

$$V(\hat{\pi}_1) = E_p\left[\frac{1}{n^2}\sum_{i=1}^{n} V_R\{r_i(1)\}\right] + V_p\left[\frac{1}{n}\sum_{i=1}^{n} E_R\{r_i(1)\}\right] = E_p[\phi_1/n] + V_p\left(\frac{1}{n}\sum_{i=1}^{n} y_i\right)$$
$$= \frac{\pi(\quad \pi)}{n} + \frac{\ }{n} \tag{3.7}$$

Similarly,

$$E(\hat{\pi}_2) = \pi \text{ and } V(\hat{\pi}_2) = \frac{\pi(1-\pi)}{n} + \frac{\phi_2}{n} \tag{3.8}$$

Further,

$$\mathrm{Cov}(\hat{\pi}_1, \hat{\pi}_2) = C_p[E_R(\hat{\pi}_1), E_R(\hat{\pi}_2)] + E_p[C_R(\hat{\pi}_1, \hat{\pi}_2)] = C_p\left[\left(\frac{1}{n}\sum_{i=1}^{n} y_i\right), \left(\frac{1}{n}\sum_{i=1}^{n} y_i\right)\right] = \frac{\pi(1-\pi)}{n} \tag{3.9}$$

This study propose the following theorem:

Theorem 3.1.

(i) The optimum estimator π based on the linear combination of $\hat{\pi}_1$ and $\hat{\pi}_2$ is

$$\hat{\pi}_{w0} = w_0\hat{\pi}_1 + (1-w_0)\hat{\pi}_2$$

where $w_0 = \dfrac{\phi_2}{\phi_1 + \phi_2}$

(ii) The variance of $\hat{\pi}_{w0}$ is

$$V(\hat{\pi}_{w0}) = \frac{\pi(1-\pi)}{n} + \frac{\overline{\phi}}{n}$$

where

$$\overline{\phi} = \left(\frac{1}{\phi_1} + \frac{1}{\phi_2}\right)^{-1} = \frac{P(1-P)T(1-T)}{P(1-P)(2T-1)^2 + T(1-T)(2P-1)^2}$$

(iii) An unbiased estimator of $V(\hat{\pi}_{w0})$ is

$$\hat{V}(\hat{\pi}_{w0}) = \frac{\hat{\pi}_{w0}(1-\hat{\pi}_{w0}) + \overline{\phi}}{n-1}$$

Proof:

Consider an unbiased estimator of π based on the linear combination of $\hat{\pi}_1$ and $\hat{\pi}_2$ as

$$\hat{\pi}_w = w\hat{\pi}_1 + (1-w)\hat{\pi}_2 \tag{3.10}$$

The variance of $\hat{\pi}_w$ is

$$V(\hat{\pi}_w) = V_p\left[E_R(\hat{\pi}_w)\right] + E_p\left[V_R(\hat{\pi}_w)\right]$$

$$= V_p\left[\frac{1}{n}\sum_{i=1}^{n} y_i\right] + E_p\left[w^2\phi_1 + (1-w)^2\phi_2\right]/n$$

$$= \frac{\pi(1-\pi)}{n} + \frac{w^2\phi_1 + (1-w)^2\phi_2}{n} \tag{3.11}$$

Differentiating $V(\hat{\pi}_w)$ with respect to w and then equating it to zero, the optimum value of w comes out as $w_0 = \dfrac{\phi_2}{\phi_1 + \phi_2}$.

(ii) Substituting $w = w_0 = \dfrac{\phi_2}{\phi_1 + \phi_2}$ in (3.11), we find

$$V(\hat{\pi}_{w0}) = \frac{\pi(1-\pi)}{n} + \frac{\overline{\phi}}{n}$$

(iii) $$E\left[\hat{V}(\hat{\pi}_{w0})\right] = \frac{E\left[\hat{\pi}_{w0}(1-\hat{\pi}_{w0})\right] + \overline{\phi}}{n-1}$$

$$= \frac{E(\hat{\pi}_{w0}) - E(\hat{\pi}_{w0})^2 + \overline{\phi}}{n-1}$$

$$= \frac{\pi(1-\pi) + \overline{\phi} - V(\hat{\pi}_{w0})}{n-1}$$

$$= V(\hat{\pi}_{w0})$$

4. EFFICIENCY OF THE PROPOSED ESTIMATOR

The percentage relative efficiency of Jayraj *et al.* (2016) estimator $\hat{\pi}_J$ and the proposed estimator $\hat{\pi}_{w0}$ compared with Odumade and Singh (2009) estimator $\hat{\pi}_{os}$ are given by

$$E(1) = \frac{V(\hat{\pi}_{os})}{V(\hat{\pi}_J)} \times 100 \tag{4.1}$$

and

$$E(2) = \frac{V(\hat{\pi}_{os})}{V(\hat{\pi}_{w0})} \times 100 \tag{4.2}$$

It is important to note that the differences $V(\hat{\pi}_J) - V(\hat{\pi}_{os})$ and $V(\hat{\pi}_J) - V(\hat{\pi}_{wo})$ increase with π. Hence Jayraj *et al.* (2016) estimator performs worse than $V(\hat{\pi}_{os})$ and $V(\hat{\pi}_{wo})$ for higher values of π. The relative percentage efficiencies $E(1)$ and $E(2)$ are given in the following Table 4.1 for different values of P, T and π. The Table 4.1 shows that the proposed estimator $\hat{\pi}_{wo}$ perform better than $\hat{\pi}_{os}$ in all situations while $\hat{\pi}_{wo}$ performs better than $\hat{\pi}_J$ in most of situations. Both the estimators $\hat{\pi}_{wo}$ and $\hat{\pi}_{os}$ perform better than $\hat{\pi}_J$ for higher values of π in general. However for (P, T)=(0.1, 0.4), (0.2, 0.4) and (0.3, 0.4), $\hat{\pi}_J$ perform better than the other two while for (P, T)=(0.2, 0.1), (0.3, 0.1), (0.3, 0.2), (0.4, 01) and (0.4, 0.2), $\hat{\pi}_J$ performs very poor.

Remark 4.1

Jayaraj *et al.* (2017) proposed an alternative estimator for the proportion π and found empirically that their proposed estimator is superior to the Odumade and Singh (2009) estimator. The proposed Jayaraj *et al.*'s (2017) will be subject of our future investigation.

5. AF RR MODELS

Under AF model, each respondent is asked to draw two cards; one from the "Deck 1" and another from "Deck 2". Deck 1 comprises two types of cards as in Warner (1965) model viz. "I belong to the sensitive group A" with proportion P and "I do not belong to the sensitive group A" with proportion $1-P$. The respondent should answer truthfully "Yes" if the statement matches his/her status otherwise, answers "No". The Deck 2 comprises also two types of cards written "Yes" with proportion Q and "No" with proportion 1-Q. Regardless of his/her actual status the

Table 4.1: Relative efficiencies E(1) and (E2)

π	P = 0.1							
	T= 0.1		T= 0.2		T= 0.3		T= 0.4	
	E(1)	E(2)	E(1)	E(2)	E(1)	E(2)	E(1)	E(2)
0.1	128.0	100	156.5	104.3	179.3	107.1	185.9	103.5
0.2	108.0	100	127.9	103.1	146.0	105.4	153.7	102.7
0.3	99.7	100	116.5	102.6	133.2	104.6	141.7	102.3
0.4	94.3	100	109.9	102.4	126.6	104.2	136.3	102.1
0.5	89.7	100	105.1	102.3	122.8	104.1	134.1	102.0
0.6	85.0	100	101.1	102.4	120.6	104.2	134.3	102.1
0.7	79.4	100	96.9	102.6	119.6	104.6	137.2	102.3
0.8	71.7	100	91.9	103.1	119.9	105.4	144.7	102.7
0.9	59.5	100	84.2	104.3	122.6	107.1	164.5	103.5
π	P = 0.2							
	T= 0.1		T= 0.2		T= 0.3		T= 0.4	
	E(1)	E(2)	E(1)	E(2)	E(1)	E(2)	E(1)	E(2)
0.1	93.3	104	130.8	100	188.8	101.2	244.4	101.2
0.2	82.3	103	112.2	100	159.0	101.1	204.8	101.1
0.3	75.8	103	102.3	100	144.3	101.0	186.6	101.0
0.4	70.6	102	95.4	100	135.6	100.9	177.4	100.9
0.5	65.5	102	89.7	100	129.9	100.9	173.4	100.9
0.6	60.0	102	84.3	100	125.9	100.9	173.3	100.9
0.7	53.6	103	78.5	100	123.0	101.0	177.3	101.0
0.8	45.8	103	71.8	100	120.9	101.1	187.2	101.1
0.9	35.7	104	63.1	100	119.4	101.2	208.8	101.2
π	P = 0 .3							
	T= 0.1		T= 0.2		T= 0.3		T= 0.4	
	E(1)	E(2)	E(1)	E(2)	E(1)	E(2)	E(1)	E(2)
0.1	32.1	107	54.8	101.2	116.8	100	242.5	100.3
0.2	33.1	105	53.3	101.1	108.7	100	220.0	100.2
0.3	32.5	105	51.3	101.0	102.7	100	206.4	100.2
0.4	31.0	104	48.9	100.9	97.9	100	198.2	100.2
0.5	28.8	104	46.1	100.9	93.7	100	193.5	100.2
0.6	25.9	104	42.9	100.9	89.7	100	191.7	100.2
0.7	22.5	105	39.1	101.0	85.7	100	192.6	100.2
0.8	18.5	105	34.7	101.1	81.4	100	196.5	100.2
0.9	13.8	107	29.5	101.2	76.5	100	204.4	100.3
π	P=0.4							
	T= 0.1		T= 0.2		T= 0.3		T= 0.4	
	E(2)	E(1)	E(2)	E(1)	E(2)	E(1)	E(2)	E(1)
0.1	2.5	104	5.3	101.2	18.4	100.3	104.4	100
0.2	3.0	103	5.7	101.1	18.7	100.2	102.7	100
0.3	3.3	102	6.0	101.0	18.8	100.2	101.1	100
0.4	3.4	102	6.0	100.9	18.7	100.2	99.6	100
0.5	3.4	102	5.9	100.9	18.3	100.2	98.1	100
0.6	3.1	102	5.7	100.9	17.8	100.2	96.7	100
0.7	2.8	102	5.3	101.0	17.0	100.2	95.3	100
0.8	2.3	103	4.7	101.1	16.1	100.2	93.9	100
0.9	1.7	104	4.1	101.2	15.0	100.3	92.4	100

respondent has to answer "Yes" if he /she receives card written "Yes". Alternatively, if the respondent receives the card written "No" the respondent should answer "No" as his or her response.

Deck 1		Deck 2
I $\in A$ with proportion P		"Yes" with proportion Q
I $\in A^c$ with proportion $1-P$		"No" with proportion $1-Q$

Let the responses of the selected sample of n units by SRSWR method be classified as follows:

Response from Deck 1	Response from Deck 2		Total
	Yes	No	
Yes	n_{11}	n_{10}	$n_{1\bullet}$
No	n_{01}	n_{00}	$n_{0\bullet}$
Total	$n_{\bullet 1}$	$n_{\bullet 0}$	n

By using the above scenario, AF (2011) derived the following results:

(i) An unbiased estimator of the population π is

$$\hat{\pi}_f = \frac{1}{2} + \frac{Q(n_{11}/n - n_{01}/n) + (1-Q)(n_{10}/n - n_{00}/n)}{2(2P-1)[Q^2 + (1-Q)^2]},$$

$$P \neq 0.5 \tag{5.1}$$

(ii) The variance of $\hat{\pi}_f$ is

$$V(\hat{\pi}_f) = \frac{Q^3 + (1-Q)^3}{4n(2P-1)^2[Q^2 + (1-Q)^2]^2} - \frac{(2\pi - 1)^2}{4n},$$

$P \neq 0.5$

$$= \frac{\pi(1-\pi)}{n} + \frac{1}{4n}\left[\frac{Q^3 + (1-Q)^3}{(2P-1)^2[Q^2 + (1-Q)^2]^2} - 1\right] \tag{5.2}$$

(iii) An unbiased estimator of the variance of $\hat{\pi}_f$ is

$$\hat{V}(\hat{\pi}_f) = \frac{1}{4(n-1)}\left[\frac{Q^3 + (1-Q)^3}{(2P-1)^2[Q^2 + (1-Q)^2]^2} - (2\hat{\pi}_f - 1)^2\right] \tag{5.3}$$

5.1 Improved estimator for AF model

AF argued that the force responses “Yes” or “No” will increase respondents’ confidentiality and cooperation as the respondents need not answer sensitive question twice as in OS model. Since, the response of the Deck 2 has no relation with the sensitive characteristic of the respondents (variable under study), Arnab and Singh (2013) recommended that one should ignore response of the Deck 2 for the analysis of the data. So, they proposed a modified estimator based on the responses from the Deck 1 only. The proposed alternative estimator is

$$\hat{\pi}_1 = \frac{\hat{\lambda}_1 - (1-P)}{2P-1} \tag{5.4}$$

where $\hat{\lambda}_1$ is the proportion of “Yes” answers from the Deck 1.

The properties of the estimator $\hat{\pi}_1$ are obtained from Chaudhuri and Mukherjee (1988) as follows:

(i) $\left(\hat{\pi}\ \right)\ \pi$

(ii) $V\left(\hat{\pi}_1\right) = \dfrac{\pi(1-\pi)}{n} + \dfrac{P(1-P)}{n(2P-1)^2}$

(iii) An unbiased estimator of $V\left(\hat{\pi}_1\right)$ is

$$\hat{V}\left(\hat{\pi}_1\right) = \frac{\hat{\lambda}_1(1-\hat{\lambda}_1)}{n(2P-1)^2}$$

6. STRATIFIED SAMPLING

Consider a finite population stratified into H strata. Let N_h be the total number of units and π_h be the proportion of individuals possess the sensitive characteristic A in the stratum h. Then, $\pi = \sum_{h=1}^{h} N_h \pi_h / N$ be the proportion of individuals possessing the characteristic A in the entire population of size $N = \sum_{h=1}^{H} N_h$. From each of the stratum samples are selected by SRSWR method independently. Let n_h be the number of respondents selected from the hth stratum.

In this section we will compare performances of the alternative estimators for OS and AF methods of RR techniques when extended to the stratified sampling. The RR techniques for the stratified samplings is described as follows:

6.1 OS model

Each of the respondents of the selected sample of the hth stratum is asked to draw one card from each of the two decks independently with proportions $P_h\left(\neq 1/2\right)$ and $T_h\left(\neq 1/2\right)$. The details of the cards and data obtained from the stratum h are as follows:

Stratum h

Deck 1		Deck 2
I $\in A$ with proportion P_h		I $\in A$ with proportion T_h
I $\in A^c$ with proportion $1-P_h$		I $\in A^c$ with proportion $1-T_h$

Responses obtained from stratum h

Response from Deck 1	Response from Deck 2		Total
	Yes	No	
Yes	$n_{11}(h)$	$n_{10}(h)$	$n_{1\cdot}(h)$
No	$n_{01}(h)$	$n_{00}(h)$	$n_{0\cdot}(h)$
Total	$n_{\cdot 1}(h)$	$n_{\cdot 0}(h)$	n_h

OS estimator of π for the stratified sampling was proposed by AFM as follows:

$$T_{os} = \frac{1}{N}\sum_{h=1}^{H} N_h \hat{\pi}_{os}^h \tag{6.1}$$

where

$$\hat{\pi}_{os}^h = \frac{1}{2} + \frac{(P_h+T_h-1)(n_{11}(h)-n_{00}(h))(P_h-T_h)(n_{10}(h)-n_{00}(h))}{2n_h[(P_h+T_h-1)^2+(P_h-T_h)^2]}.$$

Proposed alternative estimator for stratified sampling based on OS RR technique is

$$T_{ws} = \frac{1}{N}\sum_{h=1}^{H} N_h \hat{\pi}_{wo}^h \tag{6.2}$$

where

$\hat{\pi}_{wo}^h = w_o^h \hat{\pi}_1^h + \left(1-w_o^h\right)\hat{\pi}_2^h$, $\hat{\pi}_1^h = \dfrac{\hat{\lambda}_1^h - (1-P_h)}{(2P_h-1)}$, $\hat{\pi}_2^h = \dfrac{\hat{\lambda}_2^h - (1-T_h)}{(2T_h-1)}$, $\hat{\lambda}_1^h\left(\hat{\lambda}_2^h\right)$ = proportion of “Yes” answers from the Deck 1 (Deck 2), $w_0^h = \dfrac{\phi_2^h}{\phi_1^h+\phi_2^h}$, $\phi_1^h = \dfrac{P_h(1-P_h)}{\left(2P_h-1\right)^2}$, $\phi_2^h = \dfrac{T_h(1-T_h)}{\left(2T_h-1\right)^2}$.

The variances of T_{os} and T_{ws} are given by

$$V\left(T_{os}\right) = \frac{1}{N^2}\sum_{h=1}^{H} N_h^2 \frac{\sigma_{hos}^2}{n_h} \text{ and}$$

$$V\left(T_{ws}\right)=\frac{1}{N^2}\sum_{h=1}^{H}N_h^2\frac{\sigma_{hws}^2}{n_h} \tag{6.3}$$

where

$$\sigma_{hos}^2=\pi_h\left(1-\pi_h\right)+\frac{1}{4}\left[\frac{\begin{array}{l}(P_h+T_h-1)^2\{P_hT_h+(1-P_h)(1-T_h)\}+\\(P_h-T_h)^2\{T_h(1-P_h)+P_h(1-T_h)\}\end{array}}{[(P_h+T_h-1)^2+(P_h-T_h)^2]^2}-1\right],$$

$$\sigma_{hws}^2=\pi_h(1-\pi_h)+\bar{\phi}_h \text{ and } \bar{\phi}_h=\left(\frac{1}{\phi_1^h}+\frac{1}{\phi_2^h}\right)^{-1}.$$

6.2 AF model

In this model also each respondents of the stratum h is asked to draw one card at random from each of two decks independently with proportions $W_h\left(\neq 1/2\right)$ and $Q_h\left(\neq 1/2\right)$ respectively. Here the respondent matches his/her status with the statement written on the card drawn from the Deck-1 and answers “Yes” or “No”. For the card drawn from the Deck-2, respondents answer “Yes” or “No” on the basis of “Yes” or “No” written in the card.

Stratum h

Deck 1		Deck 2
I $\in A$ with proportion W_h		“Yes” with proportion Q_h
I $\in A^c$ with proportion $1-W_h$		“No” with proportion $1-Q_h$

Responses obtained from stratum h

Response from Deck 1	Response from Deck 2		Total
	Yes	No	
Yes	$n_{11}(h)$	$n_{10}(h)$	$n_{1\cdot}(h)$
No	$n_{01}(h)$	$n_{00}(h)$	$n_{0\cdot}(h)$
Total	$n_{\cdot 1}(h)$	$n_{\cdot 0}(h)$	n_h

Using this scenario, Abdelfatah and Mazloum (2015) proposed the following estimator for the population proportion π.

$$T_f=\frac{1}{N}\sum_{h=1}^{H}N_h\hat{\pi}_f^h \tag{6.4}$$

where

$$\hat{\pi}_f^h=\frac{1}{2}+\frac{Q_h\left(n_{11}(h)-n_{01}(h)\right)+(1-Q_h)\left(n_{10}(h)-n_{00}(h)\right)}{2(2W_h-1)[Q_h^2+(1-Q_h)^2]n_h}$$

The proposed alternative estimator for AFM is

$$T_{1f}=\frac{1}{N}\sum_{h=1}^{H}N_h\hat{\pi}_{1f}^h \tag{6.5}$$

where $\hat{\pi}_{1f}^h=\dfrac{\hat{\lambda}_{1h}-W_h(1-W_h)}{2W_h-1}$ and $\hat{\lambda}_{h1}$ = Proportion of “Yes” answers from Deck 1 of hth stratum for AFM RR technique.

The variances of T_f and T_{1f} are as follows:

$$V\left(T_f\right)=\frac{1}{N^2}\sum_{h=1}^{H}N_h^2\frac{\sigma_{hf}^2}{n_h} \tag{6.6}$$

$$V\left(T_{1f}\right)=\frac{1}{N^2}\sum_{h=1}^{H}N_h^2\frac{\sigma_{1hf}^2}{n_h} \tag{6.7}$$

where

$$\sigma_{hf}^2=\pi_h(1-\pi_h)+\frac{1}{4}\left[\frac{Q_h^3+(1-Q_h)^3}{(2W_h-1)^2[Q_h^2+(1-Q_h)^2]^2}-1\right]$$

and $\sigma_{1hf}^2=\pi_h\left(1-\pi_h\right)+\dfrac{W_h(1-W_h)}{\left(2W_h-1\right)^2}.$

6.3 Optimum allocation

Consider the simple cost function for stratified sampling suggest by Cochran (1977) as

$$C=c_o+\sum_{h=1}^{H}c_hn_h \tag{6.8}$$

where c_0 is the overhead fixed cost and c_h is the cost per unit for the hth stratum.

The optimum sample sizes n_h that minimizes the variance of the form

$$\Psi=\frac{1}{N^2}\sum_{h=1}^{H}N_h^2\frac{\sigma_h^2}{n_h} \tag{6.9}$$

keeping the total cost of the survey fixed as C^* is given by

$$n_{h0}=\frac{C^*-c_0}{\sum_{h=1}^{H}N_h\sigma_h\sqrt{c_h}}\frac{N_h\sigma_h}{\sqrt{c_h}} \tag{6.10}$$

The optimum value of Ψ with $n_h=n_{ho}$ is

$$\Psi_0=\frac{1}{N^2\left(C^*-c_0\right)}\left(\sum_{h=1}^{H}N_h\sigma_h\sqrt{c_h}\right)^2 \tag{6.11}$$

For the Neyman allocation $c_h=c$ and the total sample size $n=\sum_h n_h=\left(C^*-c_0\right)/c$ is fixed. In this case Ψ_0 in (6.11) reduces to

$$\Psi_{ney}=\frac{1}{N^2}\left(\sum_{h=1}^{H}N_h\sigma_h\right)^2/n \tag{6.12}$$

The expressions of the variances under Neyman allocation for the estimators T_{0s}, T_{ws}, T_f and T_{1f} are respectively given by

$$\mathrm{V}_{os}=\frac{1}{n}\left(\sum_{h=1}^{H} Z_h\sigma_{hos}\right)^2,\quad \mathrm{V}_{ws}=\frac{1}{n}\left(\sum_{h=1}^{H} Z_h\sigma_{hws}\right)^2,$$

$$\mathrm{V}_{f}=\frac{1}{n}\left(\sum_{h=1}^{H} Z_h\sigma_{hf}\right)^2 \text{ and } \mathrm{V}_{1f}=\frac{1}{n}\left(\sum_{h=1}^{H} Z_h\sigma_{1hf}\right)^2 \quad (6.13)$$

where $Z_h = N_h / N$.

6.4 Efficiency Comparison

For the AFM RR model, the proposed alternative estimator T_{1f} is more efficient than T_f as $\sigma_{1hf} \le \sigma_{hf}$. The modified estimator T_{ws} for OS strategy with $P_h = W_h$ is more efficient than T_{1f} as $\sigma_{1hf} \ge \sigma_{hws}$. Following Abdelfatah and Mazloum (2015), we compare relative percentage efficiencies of the estimators T_{0s}, T_{ws}, T_{1f} with respect to T_f numerically and these are given in Table 6.1 for $h = 2$, $P_1 = P_2 = P$; $T_1 = T_2 = T$; $W_1 = W_2 = W$; $Q_1 = Q_2 = Q$ and different combination of Z_h, π_{1h} and π_{2h} as follows: $P(=W) = 0.1, 0.2, 0.3, 0.4$, $T(=Q) = 0.1, 0.2, 0.3, 0.4$, $Z_1(=1-Z_2) = 0.1, 0.3, 0.5, \; 0.7, 0.9$ and $(\pi_1, \pi_2) =$ (0.08, 0.13), (0.38, 0.53), (0.78, 0.83), (0.85, 0.95). The relative percentage efficiencies of T_{0s}, T_{ws}, T_{1f} with respect to T_f are given by

$$EOS = \frac{V_f}{V_{os}} \times 100,\; E1 = \frac{V_f}{V_{1f}} \times 100 \text{ and } EW = \frac{V_f}{V_{ws}} \times 100$$

The empirical studies reveal that the estimator T_{ws} performs the best in all the situations. The next place is occupied by T_{0s}. The improved estimator T_{1f} is more efficient than T_f but less efficient than T_{0s}. However, the comparison between T_{1f} and T_{0s} is not fair as the estimator T_{1f} is based on the responses of Deck 1 cards only while T_{0s} is based on the responses of both Deck 1 and Deck 2 cards.

Table 6.1. Relative efficiencies of the estimators T_{0s}, T_{1f} and T_{ws} with respect to T_f

P(=W)	T(=Q)	$Z_1(=1-Z_2)=0.1$											
		$\pi_1=.08,\ \pi_2=0.13$			$\pi_1=0.38,\ \pi_2=0.53$			$\pi_1=0.78,\ \pi_2=0.83$			$\pi_1=0.85,\ \pi_2=0.95$		
		EOS	E1	EW	EOS	E1	EW	EOS	E1	EW	EOS	E1	EW
0.1	0.1	133	119	138	133	109	133	148	112	148	182	117	182
	0.2	115	116	122	120	113	123	129	117	133	143	125	151
	0.3	104	106	107	110	110	114	113	114	119	119	120	129
	0.4	273	111	284	102	104	104	103	105	106	105	107	109
0.2	0.1	193	116	193	207	109	212	250	110	258	328	112	345
	0.2	140	113	141	166	112	166	184	115	184	211	117	211
	0.3	109	104	111	130	110	131	137	112	138	145	114	147
	0.4	618	109	659	107	104	108	109	104	110	110	105	112
0.3	0.1	362	114	366	434	109	452	555	109	587	759	110	823
	0.2	206	111	206	300	112	303	343	113	347	398	114	403
	0.3	124	104	125	190	110	190	201	111	201	214	111	214
	0.4	2611	109	2695	122	104	122	124	104	124	125	104	126
0.4	0.1	1302	113	1317	1726	109	1761	2307	109	2372	3282	109	3417
	0.2	566	110	568	1052	112	1062	1227	113	1241	1437	113	1456
	0.3	204	104	204	518	110	519	553	110	554	588	110	590
	0.4	133	119	138	199	104	199	202	104	202	205	104	205

P(=W)	T(=Q)	$Z_1(=1-Z_2)=0.3$											
		$\pi_1=.08,\ \pi_2=0.13$			$\pi_1=0.38,\ \pi_2=0.53$			$\pi_1=0.78,\ \pi_2=0.83$			$\pi_1=0.85,\ \pi_2=0.95$		
		EOS	E1	EW	EOS	E1	EW	EOS	E1	EW	EOS	E1	EW
0.1	0.1	160	114	160	133	109	133	147	112	147	173	116	173
	0.2	134	120	140	121	113	123	128	117	132	140	123	146
	0.3	115	116	123	110	110	114	113	113	119	117	118	127
	0.4	104	106	107	102	104	104	103	105	106	104	107	108
0.2	0.1	280	111	291	208	109	213	246	110	254	308	112	323
	0.2	195	116	195	166	113	166	183	115	183	205	117	205
	0.3	140	113	142	130	110	132	136	112	138	143	113	145
	0.4	109	105	111	107	104	108	109	104	110	110	105	111

0.3	0.1	635	109	678	436	109	455	545	109	576	709	110	765
	0.2	367	114	371	301	112	304	340	113	344	386	114	391
	0.3	207	111	207	190	110	190	201	111	201	212	111	212
	0.4	124	104	125	122	104	122	123	104	124	125	104	125
0.4	0.1	2691	109	2781	1737	109	1773	2262	109	2324	3047	109	3163
	0.2	1320	113	1336	1056	112	1066	1215	113	1228	1394	113	1412
	0.3	569	110	571	518	110	520	551	110	552	581	110	583
	0.4	204	104	204	199	104	199	202	104	202	205	104	205

P(=W)	T(=Q)	$Z_1(=1-Z_2)=$ **0.5**											
		$\pi_1=$ **.08**, $\pi_2=$ **0.13**			$\pi_1=$ **0.38**, $\pi_2=$ **0.53**			$\pi_1=$ **0.78**, $\pi_2=$ **0.83**			$\pi_1=$ **0.85**, $\pi_2=$ **0.95**		
		EOS	**E1**	**EW**	**EOS**	**E1**	**EW**	**EOS**	**E1**	**EW**	**EOS**	**E1**	**EW**
0.1	0.1	163	114	163	133	109	133	146	111	146	166	115	166
	0.2	136	121	141	121	113	124	127	116	131	137	121	142
	0.3	116	117	124	110	110	114	112	113	119	116	117	125
	0.4	104	106	107	102	104	104	103	105	106	104	106	108
0.2	0.1	286	111	298	209	109	214	243	110	251	292	111	304
	0.2	198	116	198	166	113	166	182	114	182	200	116	200
	0.3	141	113	143	130	110	132	136	112	137	142	113	143
	0.4	110	105	111	107	104	108	109	104	110	110	105	111
0.3	0.1	653	110	699	439	109	457	536	109	566	667	110	715
	0.2	372	114	376	302	113	305	337	113	341	375	114	380
	0.3	208	111	208	190	110	190	200	111	200	209	111	209
	.4	125	104	125	122	104	122	123	104	124	125	104	125
0.4	0.1	2777	109	2873	1748	109	1785	2218	109	2278	2845	109	2945
	0.2	1339	113	1355	1060	112	1070	1203	113	1216	1353	113	1370
	0.3	573	110	574	519	110	520	548	110	550	575	110	576
	0.4	204	104	204	199	104	199	202	104	202	204	104	204

P(=W)	T(=Q)	$Z_1(=1-Z_2)=$ **0.7**											
		$\pi_1=$ **.08**, $\pi_2=$ **0.13**			$\pi_1=$ **0.38**, $\pi_2=$ **0.53**			$\pi_1=$ **0.78**, $\pi_2=$ **0.83**			$\pi_1=$ **0.85**, $\pi_2=$ **0.95**		
		EOS	**E1**	**EW**	**EOS**	**E1**	**EW**	**EOS**	**E1**	**EW**	**EOS**	**E1**	**EW**
0.1	0.1	167	115	167	133	109	133	145	111	145	160	114	160
	0.2	137	122	143	121	113	124	127	116	131	134	120	139
	0.3	116	117	125	110	110	114	112	113	118	115	116	123
	0.4	104	106	108	102	104	105	103	105	106	104	106	107
0.2	0.1	294	111	307	210	109	215	240	110	247	278	111	289
	0.2	200	116	200	167	113	167	180	114	180	195	116	195
	0.3	142	113	144	131	110	132	135	111	137	140	113	142
	0.4	110	105	111	107	104	108	108	104	110	109	104	111
0.3	0.1	672	110	721	441	109	460	528	109	556	630	109	672
	0.2	377	114	381	303	113	306	334	113	338	365	114	370
	0.3	210	111	210	191	110	191	199	111	199	207	111	207
	0.4	125	104	125	122	104	122	123	104	123	124	104	125
0.4	0.1	2869	109	2971	1760	109	1797	2176	109	2234	2668	109	2756
	0.2	1358	113	1375	1064	112	1074	1192	113	1205	1315	113	1330
	0.3	576	110	577	520	110	521	546	110	548	568	110	570
	0.4	204	104	204	199	104	199	202	104	202	204	104	204

P(=W)	T(=Q)	$Z_1(=1-Z_2)=$ **0.9**											
		$\pi_1=$ **.08**, $\pi_2=$ **0.13**			$\pi_1=$ **0.38**, $\pi_2=$ **0.53**			$\pi_1=$ **0.78**, $\pi_2=$ **0.83**			$\pi_1=$ **0.85**, $\pi_2=$ **0.95**		
		EOS	**E1**	**EW**	**EOS**	**E1**	**EW**	**EOS**	**E1**	**EW**	**EOS**	**E1**	**EW**
0.1	0.1	170	115	170	134	109	134	143	111	143	155	113	155
	0.2	138	122	145	121	113	124	126	116	130	132	119	136
	0.3	117	118	126	110	110	114	112	113	118	114	115	121
	0.4	104	106	108	102	104	105	103	104	106	103	105	107
0.2	0.1	302	111	315	210	109	216	237	110	244	266	111	275
	0.2	203	117	203	167	113	167	179	114	179	190	115	190
	0.3	143	113	144	131	110	132	135	111	136	139	112	140
	0.4	110	105	111	107	104	108	108	104	109	109	104	110

0.3	0.1	693	110	745	444	109	462	519	109	546	598	109	635
	0.2	382	114	387	304	113	307	331	113	335	356	114	360
	0.3	211	111	211	191	110	191	198	111	198	205	111	205
	0.4	125	104	125	122	104	122	123	104	123	124	104	124
0.4	0.1	2967	109	3076	1772	109	1809	2136	109	2191	2513	109	2590
	0.2	1378	113	1395	1068	112	1078	1180	113	1193	1279	113	1293
	0.3	579	110	581	521	110	522	544	110	545	562	110	564
	0.4	205	104	205	200	104	200	202	104	202	203	104	203

7. CONCLUSION

An alternative estimator $\hat{\pi}_{wo}$ for OS RR model has been proposed. The proposed estimator perform better than OS estimator $\hat{\pi}_{os}$ always while it performs better than the estimator $\hat{\pi}_J$ most of the situations. Abdelfatah *et al.* (2011) and Abdelfatah and Mazloum (2015) used RR techniques based two decks of cards where the Deck 1 relates to sensitive questions and Deck 2 relates to non-sensitive questions. They anticipated that their proposed RR techniques increase level of confidentiality and hence co-operation from the respondents. They showed empirically that their proposed RR strategy for stratified sampling performs better than Odumade and Singh (2009) RR model. In this paper, we have shown that Abdelfatah *et al.* (2011) and Abdelfatah and Mazloum (2015) estimators can always be improved by using information of sensitive question on Deck 1 card and ignoring responses for the unrelated questions based on the card 2. Table 6.1 shows that the alternative estimator T_{ws} always perform better than Odumade and Singh (2009) estimator. It is also worth noting that the both the proposed estimator possess very simple expression for the estimator of the proportions, variances and unbiased estimator of variances.

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