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Efficient and Cost Effective Partial Three-Way Cross Designs for Breeding Experiments with Scarce Resources*

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SUMMARY

Three-way cross plans find a vital role in breeding experiments due to uniformity, stability and the relative simplicity of selecting and testing. Here, methodology has been developed for obtaining information matrices pertaining to general combining ability effects of full parents and half parents after eliminating specific combining ability effects. A new, efficient and cost effective series of designs involving three-way crosses for breeding experiments has been introduced and general expressions of information matrices, eigenvalues, variance factors, efficiency factor and degree of fractionation have been derived. The developed series has small degree of fractionation and high efficiency factor making them cost effective and suitable for scarce resource conditions.

Keywords: Degree of fractionation, Efficiency factor, General combining ability, Partial three-way cross, Triangular association scheme, Variance factor.

1. INTRODUCTION

The role of designing and analysis of experiments is indispensable for any scientific study and experimentation and so is for breeding experiments. Breeders are very much interested in the variability present in the experimental material as it can be used for creation of lines with desirable qualities. Hence, the objective of any breeder in a breeding programme is to create and utilize the variability or heterogeneity for the purpose of selecting better genotypes. These selected genotypes are grown further for breeding purpose. For this purpose breeders require to evaluate the combining abilities of individual parental lines or crosses. The information regarding the methods to evaluate the combining abilities is needed to make a correct choice of best parental lines and thus defining the success of a breeding programme. There are various ways of obtaining progenies and further studying the combining ability effects of the lines involved. Some of the most common methods are diallel or two-way cross, triallel or three-way cross, tetra-allele or four-way cross. Amongst these methods two-way is the most commonly practiced because of the simplicity in handling and lesser number of crosses are involved. In case of three-way and four-way crosses the number of crosses for a given number of line is higher but at the same time one can get extra information regarding the combining ability in terms of higher specific combining ability (sca). While comparing the three methods at a single stretch, three-way crosses are intermediate between two-way and four-way cross hybrids with respect to number of lines involved, complexity in handling and information regarding combining abilities. Hence three-way cross hybrids are economical in terms of uniformity, yield, stability and the relative simplicity of selecting and testing. These hybrids exhibit individual as well as population buffering mechanism because of the broad genetic base. There are many cases of plant (like maize) and animal (like swine and chicken) breeding where three-way and four-way crosses are the commonly used techniques of producing commercial hybrids. These techniques help the breeders to improve the quantitative traits which are of economical as well as nutritional importance in crops and animals (Shunmugathai and Srinivasan, 2012). Most of the common commercial

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hybrids in corn are either three-way or four-way cross hybrids. There are examples in the field of animal production where the technique is being utilized and performed better comparatively. For e.g., three-way crossbred chickens showed better egg traits than diallel crossbred chickens with lower mortality (Khawaja *et al.,* 2013). Three-way cross came out as the most practical and acceptable scheme for the production of slaughter pigs having fast growth rate, good feed efficiency, and carcass quality (http://www.pcaarrd.dost.gov.ph/home/momentum/swine/). Three-way is mostly used for exploitation of heterosis in case of commercial silkworm production. Before discussing the previous work done in this area complete three-way cross (CTC) and partial three-way Cross (PTC) are defined as follows for better understanding of the literature cited:

CTC: The three-way or the triallel crosses has been defined by Rawlings and Cockerham (1962) as a set of all possible three-way crosses among a group of lines. Given three lines i, j and k, there are three distinct three-way crosses, namely $(i \times j) \times k$, $(j \times k) \times i$ and $(i \times k) \times j$. The set of all possible three-way crosses based on v lines would be $N = \frac{v(v-1)(v-2)}{2}$ and it leads to a Complete Triallel Cross (CTC).

PTC: As the number of lines increases, the number of crosses in CTC increases manifold and becomes unmanageably large for the breeder to handle. This situation leads to take a sample of Complete Triallel Crosses, known as Partial Triallel Crosses (PTC). Hinkelmann (1965) defined PTC as a set of matings which satisfies the following conditions:

(i) Each line occurs exactly r_H times as half-parent and r_F times as full-parent and

(ii) Each cross $(i \times j) \times k$ occurs either once or not at all.

Condition (ii) does not exclude the simultaneous occurrence of $(i \times j) \times k$, $(j \times k) \times i$ and $(i \times k) \times j$. The total number of crosses are r_F. Since each line is equally often represented as half-parent it follows immediately that $r_H = 2r_F$. Let $r_F = r$, then $r_H = 2r$.

Hinkelmann (1965) for the first time introduced the concept of partial triallel crosses and gave a method of construction using generalized partially balanced incomplete block (GPBIB) designs and shown a concurrence between partial triallel crosses and incomplete block designs. Arora and Aggarwal (1984) discussed application of extended triangular designs as the confounded triallel experiments arising under a fixed effect model. The total degrees of freedom were partitioned into three orthogonal sets, said to belong to gca, first order sca and second order sca effects, respectively. Arora and Aggarwal (1989) extended their previous work for triallel experiments with reciprocal effects. A method of construction of PTC using a special class of Balanced Incomplete Block (BIB) designs and Partially Balanced Incomplete Block (PBIB) designs which preserves the property of triallel mating design has been developed by Ponnuswamy and Srinivasan (1991). Optimal block designs for triallel cross experiments are investigated and several series of nested block designs, leading to optimal designs for triallel crosses have been reported by Das and Gupta (1997).

A systematic method of construction of PTC using Trojan Square design has been developed by Dharmalingam (2002) which requires only a fraction of the number of crosses to be made compared to triallel cross. Subsequently, some more plans have been obtained using generalized incomplete Trojan type designs by Jaggi *et al.* (2010) and Varghese and Jaggi (2011). A method of construction of mating designs for partial triallel cross is proposed by Sharma *et al.* (2012) by using Mutually Orthogonal Latin Squares (MOLS). Harun *et al.* (2016 a) developed some methods of constructing designs for breeding trials involving CTC/PTC based on MOLS and based on two-associate class PBIB designs. Further, Harun *et al.* (2016 b) developed a method for constructing a class of PTC designs for comparing a set of test lines with a control line.

In this paper methodology has been developed to obtain the information matrices related to gca effects of full parents and half parents after eliminating the sca effects. Further, the general form of information matrices, inverted matrices, eigen values along with multiplicities, variance factors, efficiency factor and degree of fractionation have been derived for a class of PTC plans based on two-associate triangular association scheme. The method is easy and gives a readymade layout of PTC arranged in blocks. The method of construction has been illustrated with an example. A catalogue of designs with canonical efficiency factor along with other parameters is also given.

2. MODEL AND EXPERIMENTAL SETUP

Consider an equi-replicated completely randomized setup with the known source of variability is the three-way crosses of the form $(i \times j) \times k$, $i, j, k\ (i \neq jk \neq) = 1,2, \ldots v$ each replicated N times. Let v be the number of inbred lines resulting in $N = \frac{v(v-1)(v-2)}{2}$ three-way crosses. The model for mating experiments can be expressed as:

$$y_{lr} = \mu + \tau_{(ijk)l} + e_{lr} \qquad (1)$$

where y_{lr} is the response from the r^{th} replication of the l^{th} cross, $(l = 1,2, \ldots N)$, μ is the grand mean, $\tau_{(ijk)l}$ the effect of the l^{th} cross and e_{lr} is $i.i.d\ \mathrm{N}(0, \sigma^2)$.

The model in eqn. (1) can be rewritten in matrix notation as:

$$\mathbf{y} = \mu\mathbf{1} + \Delta'\boldsymbol{\tau} + \mathbf{e}, \qquad (2)$$

where $\mathbf{y}$ is a $Nr \times 1$ vector of responses, $\mathbf{1}$ is a $Nr \times 1$ vector of ones, Δ' is a $Nr \times N$ incidence matrix of response versus crosses, $\boldsymbol{\tau}$ is a $N \times 1$ vector of cross effect and $\mathbf{e}$ is a $Nr \times 1$ vector of errors.

Now, the design matrix $\mathbf{X}_{Nr\times(N+1)}$ can be partitioned into parameters of interest ($\mathbf{X}_1$) and the nuisance parameters ($\mathbf{X}_2$). Thus, the design matrix corresponding to the eqn. (2) can be partitioned as:

$$\mathbf{X} = [\mathbf{X}_1 \quad \mathbf{X}_2] = [\Delta' \quad \mathbf{1}].$$

The information matrix can be obtained as $\mathbf{C}_{\boldsymbol{\tau}} = \mathbf{X}_1'\mathbf{X}_1 - \mathbf{X}_1'\mathbf{X}_2(\mathbf{X}_2'\mathbf{X}_2)^{-}\mathbf{X}_2'\mathbf{X}_1$, where $N \times N$ matrix $\mathbf{C}_{\boldsymbol{\tau}}$ is symmetric, non-negative definite and doubly centered matrix with zero row and column sums. Hence the information matrix for estimating the cross effects is obtained as:

$$\mathbf{C}_{\boldsymbol{\tau}} = r[\mathbf{I}_N - \mathbf{J}_{NN}/N],$$

where r is the number of replication of the crosses, $\mathbf{I}_N$ is an identity matrix of order N, $\mathbf{J}_{NN}$ is a matrix of ones of order $N \times N$ and N is the total number of three-way crosses obtained from v inbred lines. The matrix $\mathbf{C}_{\boldsymbol{\tau}}$ can be generalized in terms of number of lines as:

$$\mathbf{C}_{\boldsymbol{\tau}} = r[\mathbf{I}_N - \mathbf{J}_{NN}/v(v-1)(v-2)].$$

Now, the cross effect $\boldsymbol{\tau}$ is itself a mixture of various combining ability effects viz. gca effects of half as well as full parent and sca effects of first order and second order. In light of previous literature available, there are two models (full and reduced) which one can assume to proceed further.

2.1 Full model approach

Taking all the effects into consideration the full model for expressing the three-way cross effect is written as:

$$\tau_{(ij)k} = \bar{\tau} + h_i + h_j + g_k + d_{(ij)} + s_{(i)k} + s_{(j)k} + t_{(ij)k} + e_{(ij)k}, \qquad (3)$$

where $i, j, k\ (i \neq j \neq k) = 1,2, \ldots v$, $\tau_{(ij)k}$ is the effect of three-way cross of the type $(i \times j) \times k$, $\bar{\tau}$ is the mean effect of crosses, h_i is the gca effect of i_{th} half parent involved in the three-way cross, h_j is the gca effect of j_{th} half parent involved in the three-way cross, g_k is the gca effect of k_{th} full parent involved in the three-way cross, $d_{(ij)}$ is two-line specific combining ability effects involving two half-parents $s_{(i)k}$, $s_{(j)k}$ are two-line specific combining ability effects involving a half-parent and a full-parent, $t_{(ij)k}$ is three-line specific combining ability effect and $e_{(ij)k}$ is $i.i.d\ \mathrm{N}(0, \sigma^2)$.

The model (3) can be rewritten by considering the sca effects as one component in the following manner:

$$\tau_{(ij)k} = \bar{\tau} + h_i + h_j + g_k + s_{(ij)k} + e_{(ij)k},$$
$$i, j, k\ (i \neq j \neq k) = 1,2, \ldots v$$

where $s_{(ij)k}$ is the total sca effect associated with a cross.

In order to estimate the gca effect free from the sca effect, sca effect is given a place in the model and then the gca effect are estimated by making the model orthogonal. For the purpose of orthogonal estimation of gca effects the model (Harun *et al., 2016 a* and *b*) is taken as:

$$\boldsymbol{\tau}_{(ij)k} = \bar{\boldsymbol{\tau}} + \mathbf{Q}'\begin{bmatrix}\mathbf{h}\\ \mathbf{g}\end{bmatrix} + \mathbf{S} + \mathbf{e}, \qquad (4)$$

where $\boldsymbol{\tau}_{(ij)k}$ is the $N \times 1$ vector of response due to cross $(i \times j) \times k$, $\bar{\boldsymbol{\tau}}$ is the mean effect of crosses, $\mathbf{h}$ is the vector of gca effect due to half parent, $\mathbf{g}$ is the vector of gca effect due to full parent, $\mathbf{S}$ is the vector of sca effects and $\mathbf{e}$ is the error vector. $\mathbf{Q}$ is a $2v \times N$ matrix with rows indexed by $1,2, \ldots v$

and columns by the three-way cross $(i \times j) \times k$, $i, j, k\ (i \neq j \neq k) = 1,2, \dots v$ such that the $\{u, (i \times j) \times k\}^{th}$ entry of $\mathbf{Q}$ is 0.5 if $u \in (ij)$, is 1 if $u \in k$ and zero otherwise. The normal equations for the model are as:

$$\boldsymbol{\tau}_{(ij)k} = \bar{\boldsymbol{\tau}} + \mathbf{Q}' \begin{bmatrix} \mathbf{h} \\ \mathbf{g} \end{bmatrix} + \mathbf{S}$$

$$\mathbf{Q}\boldsymbol{\tau}_{(ij)k} = \mathbf{Q}\bar{\boldsymbol{\tau}} + \mathbf{QQ}' \begin{bmatrix} \mathbf{h} \\ \mathbf{g} \end{bmatrix} + \mathbf{QS}$$

Solving the two normal equations estimates of the combining effects can be obtained. The estimate of combined gca effects of half as well as full parent is given as:

$$\begin{bmatrix} \hat{h} \\ \hat{g} \end{bmatrix} = (\mathbf{QQ}')^{-}(\mathbf{Q}\boldsymbol{\tau}_{(ij)k} - \mathbf{Q}\bar{\boldsymbol{\tau}}\mathbf{1}_N)$$

$$= [(\mathbf{QQ}')^{-}\mathbf{Q} - (\mathbf{QQ}')^{-}\mathbf{QJ}_{NN}/N)]\ \boldsymbol{\tau}_{(ij)k}$$

$$= \mathbf{H}_1\boldsymbol{\tau}_{(ij)k}.$$

The estimate of combined sca effect is given as:

$$\hat{\mathbf{S}} = \boldsymbol{\tau}_{(ij)k} - \bar{\boldsymbol{\tau}}\mathbf{1} + \mathbf{Q}'g$$

$$= (\mathbf{I}_N - \mathbf{J}_{NN} - \mathbf{Q}'\mathbf{H}_1)\boldsymbol{\tau}_{(ij)k}$$

$$= \mathbf{H}_2\boldsymbol{\tau}_{(ij)k}.$$

The restrictions being imposed in order to estimate the gca effects free from sca effects are as:

$$\mathbf{1}' \begin{bmatrix} \hat{h} \\ \hat{g} \end{bmatrix} = \mathbf{H}_1\mathbf{1} = \mathbf{H}_2\mathbf{1} = \mathbf{H}_1'\mathbf{H}_2 = \mathbf{QS} = \mathbf{0},$$

rank $(\mathbf{H}_1) = v - 2$ and rank $(\mathbf{H}_2) = N - v.$

Now the joint information matrix regarding $\begin{pmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \end{pmatrix}\boldsymbol{\tau}$ is given by:

$$\mathbf{C} = \begin{bmatrix} \mathbf{H}_1\mathbf{C}_\tau\mathbf{H}_1' & \mathbf{H}_1\mathbf{C}_\tau\mathbf{H}_2' \\ \mathbf{H}_2\mathbf{C}_\tau\mathbf{H}_1' & \mathbf{H}_2\mathbf{C}_\tau\mathbf{H}_2' \end{bmatrix}.$$

In order to derive the gca effects and sca effects independent from each other the off-diaogonal terms should vanish, the information matrices for gca and sca effects are as:

$\mathbf{C}_{gca} = \mathbf{H}_1\mathbf{C}_\tau\mathbf{H}_1'$ and $\mathbf{C}_{sca} = \mathbf{H}_2\mathbf{C}_\tau\mathbf{H}_2'$ provided that $\mathbf{H}_2\mathbf{C}_\tau\mathbf{H}_1' = \mathbf{H}_1\mathbf{C}_\tau\mathbf{H}_2' = \mathbf{0}$

In order to generalize the theory for the class of three-way designs, the various results are recalculated in terms of number of lines as:

$$\mathbf{QQ}' = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}' & \mathbf{D} \end{bmatrix} = \begin{bmatrix} \frac{(v-2)}{4}\{(v-2)\mathbf{I}_v + \mathbf{J}_{vv}\} & -\frac{(v-2)}{2}(\mathbf{I}_v - \mathbf{J}_{vv}) \\ -\frac{(v-2)}{2}(\mathbf{I}_v - \mathbf{J}_{vv}) & \frac{(v-1)(v-2)}{2}\mathbf{I}_v \end{bmatrix}$$

$(\mathbf{QQ}')^{-}$ can be calculated as follows:

$(\mathbf{QQ}')^{-} = \begin{bmatrix} \mathbf{A}^{-} + \mathbf{FE}^{-}\mathbf{F}' & \mathbf{FE}^{-} \\ \mathbf{E}^{-}\mathbf{F}' & \mathbf{E}^{-} \end{bmatrix}$, where $\mathbf{F} = \mathbf{A}^{-}\mathbf{B}$ and $\mathbf{D} = \mathbf{B}'\mathbf{A}^{-}\mathbf{B}$.

$$\mathbf{A}^{-} = \frac{4}{(v-2)^2}\left[\mathbf{I}_v - \frac{\mathbf{J}_{vv}}{2(v-1)}\right],\ \mathbf{E} = \frac{(v-3)}{2}[v\mathbf{I}_v - \mathbf{J}_{vv}],$$

$$\mathbf{E}^{-} = \frac{2}{v^2(v-3)}[v\mathbf{I}_v - \mathbf{J}_{vv}],\qquad \mathbf{F} = \frac{-1}{(v-2)}[2\mathbf{I}_v - \mathbf{J}_{vv}],$$

$$-\mathbf{FE}^{-} = \frac{4}{v^2(v-2)^2(v-3)}[v\mathbf{I}_v - \mathbf{J}_{vv}],$$

$$\mathbf{A}^{-} + \mathbf{FE}^{-}\mathbf{F}' = \frac{4}{v(v-2)(v-3)}\left[(v-1)\mathbf{I}_v - \frac{(v^2-v+2)}{2v(v-1)}\mathbf{J}_{vv}\right]$$

These generalized form can be used to calculate the combined gca effects free from sca effect and the total sca effect.

2.2 Reduced model approach

In case of reduced model approach, it is assumed that the sca effects are contributing much less to the total combining ability effects of a three-way cross as compared to gca effects. The reduced model for expressing the cross effects can be written as

$$\tau_{(ij)k} = \bar{\tau} + h_i + h_j + g_k + e_{(ij)k},$$

where $i, j, k\ (i \neq j \neq k) = 1,2, \dots v,\ \tau_{(ij)k}$ is the effect of three-way cross of the type $(i \times j) \times k$, $\bar{\tau}$ is the mean effect of crosses, h_i is the gca effect of i_{th} half parent involved in the three-way cross, h_j is the gca effect of j_{th} half parent involved in the three-way cross, g_k is the gca effect of k_{th} full parent involved in the three-way cross and $e_{(ij)k}$ is $i.i.d$ $\mathrm{N}(0, \sigma^2)$.

In order to estimate the gca effect of half parents free from the gca effect due to full parents, both effects are given a place in the model and then they are estimated by making the model orthogonal. For the purpose of orthogonal estimation of both gca effects the model is taken as:

$$\boldsymbol{\tau}_{(ij)k} = \bar{\boldsymbol{\tau}} + \mathbf{Q}_1'\mathbf{h} + \mathbf{Q}_2'\mathbf{g} + \mathbf{e} \qquad (5)$$

where $\boldsymbol{\tau}_{(ij)k}$ is the $N \times 1$ vector of response due to cross $(i \times j) \times k$, $\bar{\boldsymbol{\tau}}$ is the mean effect of crosses, $\mathbf{h}$ is the vector of gca effect due to half parent, $\mathbf{g}$ is the vector of gca effect due to full parent and $\mathbf{e}$ is the error vector. Q_1 is a $v \times N$ matrix with rows indexed by $1,2, \ldots v$ and columns by the three-way cross $(i \times j) \times k, i, j, k\ (i \neq j \neq k) = 1,2, \ldots v$ such that the $\{u, (i \times j) \times k\}^{th}$ entry of Q_1 is 0.5 if $u \in (ij)$ and zero otherwise and Q_2 is a $v \times N$ matrix with rows indexed by $1,2, \ldots v$ and columns by the three-way cross $(i \times j) \times k,\ \ i, j, k\ (i \neq j \neq k) = 1,2, \ldots v$ such that the $\{u, (i \times j) \times k\}^{th}$ entry of Q_2 is 1 if $u \in k$ and zero otherwise. The normal equations for the model are as

$$\boldsymbol{\tau}_{(ij)k} = \bar{\boldsymbol{\tau}} + \mathbf{Q}_1'\mathbf{h} + \mathbf{Q}_2'\mathbf{g}$$

$$\mathbf{Q}_1\boldsymbol{\tau}_{(ij)k} = \mathbf{Q}_1\bar{\boldsymbol{\tau}} + \mathbf{Q}_1\mathbf{Q}_1'h + \mathbf{Q}_1\mathbf{Q}_2'g$$

$$\mathbf{Q}_2\boldsymbol{\tau}_{(ij)k} = \mathbf{Q}_2\bar{\boldsymbol{\tau}} + \mathbf{Q}_2 Q_1'h + \mathbf{Q}_2\mathbf{Q}_2'g$$

On solving the three normal equations the estimates of the combining effects, the estimate of gca effects of half parent is given as:

$$\hat{\mathbf{h}} = (\mathbf{Q}_1\mathbf{Q}_1')^-(\mathbf{Q}_1\boldsymbol{\tau}_{(ij)k} - \mathbf{Q}_1\bar{\boldsymbol{\tau}}\mathbf{1}_N)$$

$$= [(\mathbf{Q}_1\mathbf{Q}_1')^-\mathbf{Q}_1 - (\mathbf{Q}_1\mathbf{Q}_1')^-\mathbf{Q}_1\mathbf{J}_{NN}/N)]\ \boldsymbol{\tau}_{(ij)k}$$

$$= \mathbf{G}_1\boldsymbol{\tau}_{(ij)k} \text{ (say)},$$

and the estimate of gca effects of full parent is given as:

$$\hat{\mathbf{g}} = (\mathbf{Q}_2\mathbf{Q}_2')^-(\mathbf{Q}_2\boldsymbol{\tau}_{(ij)k} - \mathbf{Q}_2\bar{\boldsymbol{\tau}}\mathbf{1}_N)$$

$$= [(\mathbf{Q}_2\mathbf{Q}_2')^-\mathbf{Q}_2 - (\mathbf{Q}_2\mathbf{Q}_2')^-\mathbf{Q}_2\mathbf{J}_{NN}/N)]\ \boldsymbol{\tau}_{(ij)k}$$

$$= \mathbf{G}_2\boldsymbol{\tau}_{(ij)k} \text{ (say)}.$$

The restrictions being imposed in order to estimate the gca effect of half parents free from gca effect of full parents are as:

$\mathbf{1}'\hat{\mathbf{h}} = \mathbf{1}'\hat{\mathbf{g}} = \mathbf{G}_1\mathbf{1} = \mathbf{G}_2\mathbf{1} = \mathbf{G}_1'\mathbf{G}_2 = \mathbf{0},$ rank $(\mathbf{G}_1)$ = rank $(\mathbf{G}_2) = v - 2.$

Now the joint information matrix regarding $\begin{pmatrix}\mathbf{G}_1\\ \mathbf{G}_2\end{pmatrix}\boldsymbol{\tau}$ is given by:

$$\mathbf{C}_{\text{gca}} = \begin{bmatrix}\mathbf{G}_1\mathbf{C}_\tau\mathbf{G}_1' & \mathbf{G}_1\mathbf{C}_\tau\mathbf{G}_2'\\ \mathbf{G}_2\mathbf{C}_\tau\mathbf{G}_1' & \mathbf{G}_2\mathbf{C}_\tau\mathbf{G}_2'\end{bmatrix}$$

The gca effect of half parent can be derived independent of full parent if the off-diagonal terms become null. The information matrices for gca and sca effects are as:

$$\mathbf{C}_{\text{gca\_half}} = \mathbf{G}_1\mathbf{C}_\tau\mathbf{G}_1' \text{ and } \mathbf{C}_{\text{gca\_full}} = \mathbf{G}_2\mathbf{C}_\tau\mathbf{G}_2'$$

provided that $\mathbf{G}_2\mathbf{C}_\tau\mathbf{G}_1' = \mathbf{G}_1\mathbf{C}_\tau\mathbf{G}_2' = \mathbf{0}$.

3. A CLASS OF PTC PLANS

A general method of constructing a series of PTC plans is described below.

Let there be $v = \frac{n(n-1)}{2}$ lines, where $n > 4$. Arrange these v lines in a two-associate triangular association scheme, i.e. allot v lines to the off diagonal positions above the principal diagonal in a natural order and repeat the same below the diagonal such that the final arrangement is symmetrical about the diagonal. Diagonal positions are left empty.

Consider all possible pairs that can be made from each row of the array. Add a third element which appears at the row-column intersection of the pair considered, to each pair to form triplets. Make three-way cross from these triplets considering lines in the pairs as half parents and third added line in the triplet as full parent. This will result in a partial three-way cross design with following parameters $v = \frac{n(n-1)}{2}$, $N = \frac{n(n-1)(n-2)}{2}$, $b = n$, $k = \frac{(n-1)(n-2)}{2}$, $r_H = 2(n-2)$ and $r_F = (n-2)$.

Example 3: The method can be well understood by an example for $n = 6$ giving rise to v = 15.

*	1	2	3	4	5
1	*	6	7	8	9
2	6	*	10	11	12
3	7	10	*	13	14
4	8	11	13	*	15
5	9	12	14	15	*

The crosses for first block can be obtained from the first row. The first cross of first block is obtained by considering the first pair of lines (i.e.1 &2) as half parents and then crossing it with the line present at the row-column intersection of lines 1 and 2 (i.e.6), treating 6 as the full parent in the cross. Proceeding in the same manner, for all possible pairs of first row one can obtain the three-way crosses to be placed in first block. In a similar manner, from other rows, remaining blocks can be obtained. The design so obtained is:

The parameters of this designs are $n = 6, v = 15$, $N = 60, b = 6, k = 10, r_H = 8$ and $r_F = 4$.

Block 1	Block 2	Block 3	Block 4	Block 5	Block 6
(1×2)×6	(1×6)×2	(2×6)×1	(3×7)×1	(4×8)×1	(5×9)×1
(1×3)×7	(1×7)×3	(2×10)×3	(3×10)×2	(4×11)×2	(5×12)×2
(1×4)×8	(1×8)×4	(2×11)×4	(3×13)×4	(4×13)×3	(5×)14×3
(1×5)×9	(1×9)×5	(2×12)×5	(3×14)×5	(4×15)×5	(5×15)×4
(2×3)×10	(6×7)×10	(6×10)×7	(7×10)×6	(8×11)×6	(9×12)×6
(2×4)×11	(6×8)×11	(6×11)×8	(7×13)×8	(8×13)×7	(9×14)×7
(2×5)×12	(6×9)×12	(6×12)×9	(7×14)×9	(8×15)×9	(9×15)×8
(3×4)×13	(7×8)×13	(10×11)×13	(10×13)×11	(11×13)×10	(12×14)×10
(3×5)×14	(7×9)×14	(10×12)×14	(10×14)×12	(11×15)×12	(12×15)×11
(4×5)×15	(8×9)×15	(11×12)×15	(13×14)×15	(13×15)×14	(14×15)×13

From this example, it can be seen that the above method of construction gives a layout in which the crosses are arranged in six groups. This excludes the necessity for an environmental design for laying out the crosses as these groups can be treated as blocks of a design. Hence, through this method one can get mating as well as environmental design at one go. The Model (2.2.1) needs to be modified by including block effects ($\boldsymbol{\beta}$) as given below:

$$\boldsymbol{\tau}_{(ij)k} = \bar{\boldsymbol{\tau}} + \mathbf{Q}_1'\mathbf{h} + \mathbf{Q}_2'\mathbf{g} + \boldsymbol{\beta} + \mathbf{e} \qquad (6)$$

Therefore, the results obtained in the Section (2.2) are derived again as per Model (6) for the proposed class of designs as discussed in next section.

4. GENERAL FORMS OF INFORMATION MATRICES, EIGENVALUES AND ESTIMATED VARIANCE FACTORS

4.1 Information Matrices

The general form of information matrix related to half parents is derived as:

$$\mathbf{C}_{\text{half}} = a_0\,\mathbf{I}_v + a_1\,\mathbf{A}_v + a_2\,\mathbf{B}_v,$$

where $a_0 = \frac{2(n-3)(n-4)}{(n-2)}$, $a_1 = -\frac{2(n-3)(n-4)}{(n-2)^2}$, $a_2 = \frac{4(n-4)}{(n-2)^2}$, $\mathbf{I}_v$ is an identity matrix of order v, $\mathbf{A}_v$ is a matrix of order v whose elements, $\{a_{ij}\}$ takes value 1 if i and j are first associates otherwise 0, and $\mathbf{B}_v$ is a matrix of order v whose elements, $\{b_{ij}\}$ takes value 1 if i and j are second associates, otherwise 0. For the example illustrated in Section (2.3) $\mathbf{C}_{\text{half}} = 3\mathbf{I}_{15} - 0.75\mathbf{A}_{15} + 0.5\mathbf{B}_{15}$.

The general form of information matrix related to full parents is:

$$\mathbf{C}_{\text{full}} = b_0\,\mathbf{I}_v + b_1\,\mathbf{A}_v + b_2\,\mathbf{B}_v,$$

where $b_0 = (n-4)$, $b_1 = -\frac{(n-4)}{(n-2)}$, $b_2 = \frac{2(n-4)}{(n-2)(n-3)}$, $\mathbf{I}_v$ is an identity matrix of order v, $\mathbf{A}_v$ is a matrix of order v whose elements, $\{a_{ij}\}$ takes value 1 if i and j are first associates otherwise 0, and $\mathbf{B}_v$ is a matrix of order v whose elements, $\{b_{ij}\}$ takes value 1 if i and j are second associates, otherwise 0. For the example illustrated in Section (3) $\mathbf{C}_{full} = 2\mathbf{I}_{15} - 0.5\mathbf{A}_{15} + 0.33\mathbf{B}_{15}$.

4.2 Inverted Information matrices

The general form of inverse of information matrix related to half parents is given as:

$$\mathbf{C}_{\text{half}}^{-} = c_0\,\mathbf{I}_v + c_1\,\mathbf{A}_v + c_2\,\mathbf{B}_v,$$

where $c_0 = \frac{(n-2)(n-3)}{2(n-1)^2(n-4)}$, $c_1 = -\frac{(n-3)}{2(n-1)^2(n-4)}$, $c_2 = \frac{1}{(n-1)^2(n-4)}$, $\mathbf{I}_v$ is an identity matrix of order v, $\mathbf{A}_v$ is a matrix of order v whose elements, $\{a_{ij}\}$ takes value 1 if i and j are first associates otherwise 0, and $\mathbf{B}_v$ is a matrix of order v whose elements, $\{b_{ij}\}$ takes value 1 if i and j are second associates, otherwise 0. For the example illustrated in Section (3) $\mathbf{C}_{\text{half}}^{-} = 0.12\mathbf{I}_{15} - 0.03\mathbf{A}_{15} + 0.02\mathbf{B}_{15}$.

The general form of inverted information matrix related to full parents is given as:

$$\mathbf{C}_{\text{full}}^{-} = d_0\,\mathbf{I}_v + d_1\,\mathbf{A}_v + d_2\,\mathbf{B}_v,$$

where $d_0 = \frac{(n-3)^2}{(n-1)^2(n-4)}$, $d_1 = -\frac{(n-3)^2}{(n-1)^2(n-2)(n-4)}$, $d_2 = \frac{2(n-3)}{(n-1)^2(n-2)(n-4)}$, $\mathbf{I}_v$ is an identity matrix of order v, $\mathbf{A}_v$ is a matrix of order v whose elements, $\{a_{ij}\}$ takes value 1 if i and j are first associates otherwise 0, and $\mathbf{B}_v$ is a matrix of order v whose elements, $\{b_{ij}\}$ takes value 1 if i and j are second associates, otherwise 0. For the example illustrated in Section (3) $\mathbf{C}_{\text{full}}^{-} = 0.18\mathbf{I}_{15} - 0.045\mathbf{A}_{15} + 0.03\mathbf{B}_{15}$.

4.3 Eigenvalues

The eigenvalues of the information matrix $\mathbf{C}_{\text{half}} = a_0\,\mathbf{I}_v + a_1\,\mathbf{A}_v + a_2\,\mathbf{B}_v$ are given as:

$a_0 + (n-2)a_2$ with multiplicity $\frac{n(n-3)}{2}$ and 0 with multiplicity n.

The eigenvalues of the information matrix for the example illustrated in Section (3) are 5 with multiplicity 9 and $\mathbf{0}$ with multiplicity 6.

The eigenvalues of the information matrix $\mathbf{C}_{\text{full}} = b_0\,\mathbf{I}_v + b_1\,\mathbf{A}_v + b_2\,\mathbf{B}_v$ are given as:

$b_0 + (n-2)b_2$ with multiplicity $\frac{n(n-3)}{2}$ and $\mathbf{0}$ with multiplicity n.

The eigenvalues of the information matrix for the example illustrated in Section (3) are 3.33 with multiplicity 9 and $\mathbf{0}$ with multiplicity 6.

4.4 Variance factors

The general expressions for variance factors of estimated contrasts for half parents is given as:

$$\mathrm{V}_{\text{half}}(\widehat{h_i - h_j}) = 2(c_0 - c_1) = \frac{(n-3)}{(n-1)(n-4)},$$

when i and j $(i \neq j)$ are first associates to each other, and

$$\mathrm{V}_{\text{half}}(\widehat{h_i - h_j}) = 2(c_0 - c_2) = \frac{1}{(n-1)},$$

when i and j $(i \neq j)$ are second associates to each other.

The general expressions for average variance factor of estimated contrasts for half parents is:

$$\overline{\mathrm{V}}_{\text{half}}(\widehat{h_i - h_j}) = \frac{n(n-3)}{(n^2-1)(n-4)}.$$

For the example illustrated in Section (3), the variance factor of estimated contrasts for half parents is 0.30 , when i and j $(i \neq j)$ are first associates to each other and 0.20 when i and j $(i \neq j)$ are second associates to each other. The general expressions for average variance of estimated contrasts for half parents is 0.257.

The general expressions for variance factor of estimated contrasts for full parents is given as:

$$\mathrm{V}_{\text{full}}(\widehat{g_i - g_j}) = 2(d_0 - d_1) = \frac{2(n-3)^2}{(n-1)(n-2)(n-4)},$$

when i and j $(i \neq j)$ are first associates to each other, and

$$\mathrm{V}_{\text{full}}(\widehat{g_i - g_j}) = 2(d_0 - d_2) = \frac{2(n-3)}{(n-1)(n-2)}$$

when i and j $(i \neq j)$ are second associates to each other.

The general expressions for average variance factor of estimated contrasts for full parents is:

$$\overline{\mathrm{V}}_{\text{full}}(\widehat{g_i - g_j}) = \frac{2n(n-3)^2}{(n^2-1)(n-2)(n-4)}.$$

For the example illustrated in Section (3), the variance factor of estimated treatment contrasts for full parents is 0.45, when i and j $(i \neq j)$ are first associates to each other and 0.30 when i and j $(i \neq j)$ are second associates to each other. The general expressions for average variance of estimated treatment contrasts for full parents is 0.386.

5. RESULTS AND DISCUSSION

5.1 Degree of fractionation

The value of degree of fractionation (f) can be obtained for any PTC design for any given number of lines. In order to obtain the value of f, one has to find out the ratio of total number of crosses involved in the PTC design ($N_{\mathbf{PTC}}$) and the total number of crosses involved in the CTC ($N_{\mathbf{CTC}}$). The value of f plays an important role in selecting a design involving three-way cross as it is directly proportional to the resources consumption. A lower value of f is always desirable when the resources are scarce. The degree of fractionation for this series of designs involving three-way crosses is $f = \frac{2(n-2)}{(v-1)(v-2)}$. For the example illustrated in Section (3), $f = \frac{4}{91}$.

5.2 Efficiency factor

The selection of appropriate design cannot be done alone on the basis of degree of fractionation. There must be some theoretical basis to decide the goodness of designs. For this purpose, criterion of calculating the efficiency factor of designs has been introduced. Considering the model for three-way cross under blocked setup, the canonical efficiency factor of these designs so obtained as compared to an orthogonal design with same number of lines can be used for this purpose. The canonical efficiency is calculated relative to an orthogonal design with the same number of lines by working out the harmonic mean of (1/ r) times the non-zero eigen values of the information matrix. Here, r is the replication of lines and it is assumed that the error variance is same for both situations. The canonical efficiency factor pertaining to gca effects of half parents for the developed class of designs is $E_h = \frac{(n-1)(n-4)}{(n-2)^2}$ and the canonical efficiency factor pertaining to gca effects full parents for the developed class of designs is $E_f = \frac{(n-1)(n-4)}{(n-2)(n-3)}$. Here the point must be noted that for the developed series of design, $r = 2(n-2)$ in case of half parents and $r = (n-2)$ in case of full parents. For the example illustrated in Section (3), $E_h = 0.625$ and $E_f = 0.833.$

5.3 Catalogue

Considering Model (3.1) under blocked setup, the canonical efficiency factor of the designs as compared to an orthogonal design with same number of replications has been calculated and listed along with other parameters in Table 4.1.

Table 4.1. List of designs for three-way crosses under blocked setup

S. No.	n	v	b	k	N	f	$\bar{V}_{half}$ $(\widehat{h_i - h_j})$	$\bar{V}_{full}$ $(\widehat{g_i - g_j})$	r_H	r_F	E_h	E_f
1.	5	10	5	6	30	0.083	0.417	0.556	6	3	0.44	0.67
2.	6	15	6	10	60	0.044	0.257	0.386	8	4	0.63	0.83
3.	7	21	7	15	105	0.026	0.194	0.311	10	5	0.72	0.90
4.	8	28	8	21	168	0.017	0.159	0.265	12	6	0.78	0.93
5.	9	36	9	28	252	0.012	0.135	0.231	14	7	0.82	0.95
6.	10	45	10	36	360	0.008	0.118	0.206	16	8	0.84	0.96
7.	11	55	11	45	495	0.006	0.105	0.186	18	9	0.86	0.97
8.	12	66	12	55	660	0.005	0.094	0.170	20	10	0.88	0.98
9.	13	78	13	66	858	0.004	0.086	0.156	22	11	0.89	0.98
10.	14	91	14	78	1092	0.003	0.079	0.145	24	12	0.90	0.98

It can be seen from the table that degree of fractionation is very small and efficiency factor is quite high making these series of PTC designs much desirable.

6. CONCLUSIONS

Methodology has been developed for obtaining information matrices related to gca effects of full parents and half parents after eliminating the sca effects in a three-way cross plan. General expressions of information matrices, eigen values, variance factors, efficiency factors and degree of fractionation have been derived for a class of PTC plans obtained using triangular association scheme. The method gives readymade layouts of crosses which can be used directly, after randomization of blocks and crosses, as environmental designs. Hence, no further designing is required for carrying out environmental trials for such plans. Another advantage of the developed series is that the degree of fractionation is very low which further decreases with increase in number of lines and hence can be used when there is scarcity of resources. The efficiency factor is high in initial cases and it and reaches to near optimal value for higher number of lines. Breeders can easily select appropriate PTC plans based on efficiency factor and degree of fractionation.

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