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Preliminary Test Regression type Estimator of Finite Population Mean in Survey Sampling

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SUMMARY

A preliminary test regression type estimator (PTRE) of population mean in survey sampling has been developed when prior value of correlation coefficient between study variable and an auxiliary variable is available. It has been demonstrated that PTRE performs well as compared to the usual difference and regression estimators.

Keywords: Difference estimator, Regression estimator, Preliminary test of significance, Mellin transform, Legendre function.

1. INTRODUCTION

In finite population sampling for estimating the population parameters like population mean, total, etc. the use of information on auxiliary variable related to study variable is well known technique to find out more precise estimates of the population parameters. Well known methods of estimation which use the auxiliary information at estimation stage are ratio, regression and product method of estimators. In general, regression estimator is known to be more precise than ratio and product estimators. The usual regression estimator of population mean of the study variable y is given by

$$\bar{y}_{lr} = \bar{y} + \hat{\beta}\left(\bar{X} - \bar{x}\right) \tag{1.1}$$

where $\bar{y}$ and $\bar{x}$ are sample means of y and x (auxiliary variable related to y) based on random sample of size n drawn from the population by simple random sampling without replacement (SRSWOR). $\bar{X}$ is known population mean of x, and $\hat{\beta}$ is estimated value of regression coefficient of y on x from sample data. If $\bar{X}$ is not known, the double sampling regression estimator is used (Cochran, 1977). Han (1973) first time made use of preliminary test of significance to develop preliminary test estimator of finite population mean when some partial information on an auxiliary variable is available in sample surveys. Johnson *et al.* (1977) developed a procedure for pooling regression coefficients of similar regression equations subsequent to preliminary test of significance of equality of regression coefficients. When two prior values of $\bar{X}$ are known from two different sources, Shukla and Sisodia (2015) have developed preliminary test regression estimator of finite population mean. If $\bar{X}$ is known and some prior guess value of β, say, β_0, is available on the basis of past experience, it is better to use difference estimator (Cochran, 1977) replacing $\hat{\beta}$ by β_0 in (1.1). However, one may not be sure that how much β_0 is close to β. Grimes and Sukhatme (1980) suggested a regression type estimator after preliminary test of significance of the hypothesis H_0: $\beta=\beta_0$ against H_1: $\beta\neq\beta_0$. If the hypothesis H_0 is accepted, they suggested to use difference estimator, otherwise regression estimator (1.1) is recommended to use. The availability of a prior guess value of β is not easy in practice for a decision maker. But, a prior guess value of population correlation coefficient (ρ) between y and x is possibly easier to obtain than β on the basis of past experience or past data. Its value can also be ascertained by plotting scattered diagram of sample observations. For example, area under crop,

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crop production/productivity, and livestock products etc at two consecutive points of time, say t and t-1, are generally expected to be highly correlated and it can easily be guessed. Estimates of population parameters like mean or total of the aforesaid items over different occasions are required for policy formulation by public/private sectors in any country. Alam (1976) had suggested the regression estimator after preliminary test of significance of the hypothesis H_0: ρ=0 against H_1:ρ≠0. However, when we intend to use auxiliary information at estimation stage to find out more precise estimate of the population mean, it is generally desirable to choose an auxiliary variable x which is highly correlated and linearly related with the study variable. Therefore, testing the hypothesis H_0: ρ=0 is not worthwhile when one intends to use regression estimator. If a prior guess value of ρ, say, ρ_0 is available and variances y and x are approximately same, i.e. $\sigma_y^2 \cong \sigma_x^2$, one may perform a preliminary test of significance of hypothesis H_0 : $\rho=\rho_0$ against H_1 : $\rho\neq\rho_0$ and accordingly difference estimator or regression estimator may be recommended to use.

In the present paper a preliminary test regression type estimator of the population mean of y after preliminary test of H_0: $\rho=\rho_0$ against H_1: $\rho\neq\rho_0$ provided $\sigma_y^2 \cong \sigma_x^2$ is developed and its properties are discussed. The preliminary test regression type estimator is compared with difference and regression estimators to illustrate its relative efficiency theoretically and empirically both.

2. DEVELOPMENT OF PRELIMINARY TEST ESTIMATOR

In survey sampling, it is generally considered that the population under study is finite. If the population is, however, quite sufficiently large, we will assume without loss of generality in order to perform preliminary test of significance that the variable y and x follow a bi-variate normal distribution with mean (μ_y, μ_x) and variance covariance matrix Σ (say). It is assumed that σ_x^2 and σ_y^2, the variance of x and y is approximately equal, say σ^2. It is further assumed that a prior guess value of ρ, i.e. ρ_0 is available with the investigator. Consider that a random sample of size n is drawn from the population by simple random sampling without replacement and sampled units are observed for the values of y and x.

It is further assumed that the population mean of the auxiliary variable x, i.e. μ_x is known.

There could be two possible regression type estimators of μ_y, a difference estimator given by

$$\left.\begin{aligned} &T_1 = \bar{y} + \rho_0(\mu_x - \bar{x}) \\ &\text{with } V(T_1) = \frac{\sigma^2}{n}(1-\rho^2)(1+\delta^2),\ \delta = \frac{(\rho_0-\rho)}{\sqrt{(1-\rho^2)}} \end{aligned}\right\} \quad (2.1)$$

and the usual regression estimator given by

$$\left.\begin{aligned} &T_2 = \bar{y} + \hat{\beta}(\mu_x - \bar{x}) \\ &\text{with } V(T_2) = \frac{\sigma^2}{n}(1-\rho^2)\left(1+\frac{1}{n-3}\right) \end{aligned}\right\} \quad (2.2)$$

where $\bar{y}$ and $\bar{x}$ are sample means of y and x, respectively, and $\hat{\beta}$ is estimated value of β, the regression coefficient of y on x, based on sample values, $\hat{\beta}$ is explicitly expressed as

$$\hat{\beta} = r.\frac{s_y}{s_x},$$

where $s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$ and

$$s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

If an investigator has some doubt about the closeness of ρ_0 to ρ, he can perform a preliminary test of significance of the hypothesis

$$\left.\begin{aligned} &H_0 : \rho = \rho_0 \\ &\text{against } H_1 : \rho \neq \rho_0 \end{aligned}\right\} \quad (2.3)$$

at some α probability level of significance. On the basis of outcomes of the above test of the hypothesis, the investigator can use the estimator T_1 in (2.1) if H_0 is accepted or T_2 in (2.2) if H_1 is accepted. Hence, a preliminary test regression type estimator of μ_y is proposed as

$$\hat{\mu}_y = \begin{cases} T_1 = \bar{y} + \rho_0(\mu_x - \bar{x}) & \text{if } H_0 \text{ is accepted} \\ T_2 = \bar{y} + \hat{\beta}(\mu_x - \bar{x}) & \text{if } H_1 \text{ is accepted} \end{cases} \quad (2.4)$$

To derive the expectation and variance of the preliminary test estimator $\hat{\mu}_y$, we use a result which is stated in the following Lemma.

Lemma 2.1

Let x and y have a bivariate distribution with joint density function f(x,y), marginal density function h(y) for Y, and conditional density function $g(x|y)$ for $x|y$. Let ε>0 be a real number. Let β(x) and θ(y) be random variables which are continuous functions of the random variables x and y, then

$$E\left[\beta(x)\theta(y)\middle\||y|\le\varepsilon\right]=\frac{\int_{-\varepsilon}^{\varepsilon}\theta(y)E\left[\beta(x)|y\right]h(y)dy}{P\left[|y|\le\varepsilon\right]} \quad (2.5)$$

Corollary to lemma 2.5; If the hypothesis to lemma 2.1 holds, then

$$E\left[\beta(x)\theta(y)\middle\||y|>\varepsilon\right]=\frac{E\left[\beta(x)\theta(y)\right]-\int_{-\varepsilon}^{\varepsilon}\theta(y)E\left[\beta(x)|y\right]h(y)dy}{\Pr\left[|y|>\varepsilon\right]} \quad (2.6)$$

Test statistic

To perform a preliminary test significance of the hypothesis given in (2.3) at α probability level of significance, we have well known test statistic developed by Fisher which is given by

$$Z=\left(z-\xi_0\right)\sqrt{n-3} \quad (2.7)$$

Where $z=\frac{1}{2}\log\frac{1+r}{1-r}$, $\xi_0=\frac{1}{2}\log\frac{1+\rho_0}{1-\rho_0}$ and n is the sample size. r is the correlation coefficient between x and y calculated from the sample data.

Now if $|z|\le z_\alpha$, where z_α is $\left(1-\frac{\alpha}{2}\right)100$ per cent point of standard normal variate, then H_0 is accepted otherwise, rejected. Thus it follows that

$$\Pr H_0\left[\xi_0-\frac{z_\alpha}{\sqrt{n-3}}\le z\le\xi_0+\frac{z_\alpha}{\sqrt{n-3}}\right]=1-\alpha \quad (2.8)$$

The expression (2.8) implies that

$$\frac{1}{2}\log\frac{1+r}{1-r}\ge\frac{1}{2}\log\frac{1+\rho_0}{1-\rho_0}-\frac{z_\alpha}{\sqrt{n-3}}$$

$$\text{or } \frac{1+r}{1-r}\ge e^{\log\frac{1+\rho_0}{1-\rho_0}-\frac{2z_\alpha}{\sqrt{n-3}}}$$

$$\text{or } r\ge\frac{\left[\frac{1+\rho_0}{1-\rho_0}e^{-2z_\alpha/\sqrt{n-3}}-1\right]}{\left[\frac{1+\rho_0}{1-\rho_0}e^{-2z_\alpha/\sqrt{n-3}}+1\right]}=r_{\alpha_1}\text{ say} \quad (2.9)$$

$$\text{and } \frac{1}{2}\log\frac{1+r}{1-r}\le\frac{1}{2}\log\frac{1+\rho_0}{1-\rho_0}+\frac{z_\alpha}{\sqrt{n-3}}$$

$$\text{or } r\le\frac{\left[\frac{1+\rho_0}{1-\rho_0}e^{2z_\alpha/\sqrt{n-3}}-1\right]}{\left[\frac{1+\rho_0}{1-\rho_0}e^{2z_\alpha/\sqrt{n-3}}+1\right]}=r_{\alpha_2}\text{ say} \quad (2.10)$$

From the expression (2.8), (2.9) and (2.10), it follows that

$$\Pr H_0\left[r\ge r_{\alpha_1},r\le r_{\alpha_2}\right]=1-\alpha \quad (2.11)$$

Therefore, the hypothesis H_0 in (2.1) will be accepted if and only if $r\ge r_{\alpha1}$ and $r\le r_{\alpha2}$. This result is stated in Theorem 2.1.

Theorem 2.1

Necessary and sufficient conditions to accept the null hypothesis H_0: $\rho=\rho_0$ with a test of size α is given by

$$\Pr H_0\left[r\ge r_{\alpha_1},r\le r_{\alpha_2}\right]=1-\alpha$$

3. EXPECTED VALUE AND VARIANCE OF PRELIMINARY TEST REGRESSION TYPE ESTIMATOR $\left(\hat{\mu}_y\right)$

We use the lemma (2.1) and Mellin Transform to derive the expected value and variance of $\hat{\mu}_y$. The Mellin transform is stated below

Mellin Transform:

Let f(w) be a function of w given by $f(w)=(1+2w\,\text{Cos}\theta+w^2)^{-\lambda}$, $-\pi\le\theta\le\pi$ and $\lambda\ge0$. The Mellin transform of f(w) is reproduced from Tables of Integral Transform (1954) as given below

$$g(s)=\int_0^\infty f(w)w^{s-1}dw$$

$$=2^{\lambda-\frac{1}{2}}\left(Sin\theta\right)^{\lambda-\frac{1}{2}}\Gamma(\lambda+\frac{1}{2})B\left(s,2\lambda-s\right)P_{s-\lambda-\frac{1}{2}}^{\frac{1}{2}-\lambda}\left(Cos\theta\right),$$

$$0<s<2\lambda$$

where $B(s, 2\lambda\text{-}s)=\frac{\Gamma s\Gamma(2\lambda-s)}{\Gamma(2\lambda)}$ and $P_{s-\lambda-\frac{1}{2}}^{\frac{1}{2}-\lambda}\left(Cos\theta\right)$ is a Legendre function defined in general as

$$P_\lambda^\mu(x)=\frac{1}{\Gamma(1-\mu)}\left(\frac{1+x}{1-x}\right)^{\mu/2}\ {}_2F_1\left(-\lambda,\ \lambda+1;\ 1-\mu,\frac{1}{2}-\frac{1}{2}x\right)$$

, -1<x<1

where ${}_2F_1$ is hyper-geometric function of single variable defined, in general, as

$$ {}_2F_1(a_1,a_2,a_3;x)=\sum_{k=0}^{\infty}\frac{(a_1)_k(a_2)_k}{(a_3)_k}\ \frac{x^k}{k!} $$

where $(a_i)_k = a_i(a_i+1)(a_i+2).....(a_i+k-1)$ for k= 1, 2, … and $(a_i)_0 = 1$

Now, we state and prove the following theorem.

Theorem 3.1.

The preliminary test regression type estimator $\hat{\mu}_y$ of μ_y is unbiased with variance

$$ V(\hat{\mu}_y)=\frac{\sigma^2}{n}(1-\rho^2)\left(1+\frac{1}{n-3}\right)+\rho_0\frac{\sigma^2}{n}(\rho_0-2\rho)(1-\alpha) $$

$$ -\frac{\sigma^2}{n}D_2\sum_{k=0}^{\infty}\sum_{i=0}^{\frac{2n+2k-3}{2}}\frac{\left(-\frac{3}{2}\right)_k\left(\frac{5}{2}\right)_k}{\left(n-\frac{1}{2}\right)_k 2^k k!}\binom{\frac{2n+2k-3}{2}}{i}\rho^i B\left(\frac{i+3}{2},\frac{n-2}{2}\right) $$

$$ \left[\mathrm{I}_{r^2_{\alpha 2}}\left(\frac{i+3}{2},\frac{n-2}{2}\right)\mathrm{I}_{r^2_{\alpha 1}}\left(\frac{i+2}{2},\frac{n-2}{2}\right)\right]+2\rho\frac{\sigma^2}{n} $$

$$ D_1\sum_{k=0}^{\infty}\sum_{i=0}^{\frac{2n+2k-3}{2}}\frac{\left(-\frac{1}{2}\right)_k\left(\frac{3}{2}\right)_k}{\left(n-\frac{1}{2}\right)_k 2^k k!}\binom{\frac{2n+2k-3}{2}}{i}\rho^i B\left(\frac{i+3}{2},\frac{n-2}{2}\right) $$

$$ \left\{I_{r^2_{\alpha_2}}\left(\frac{i+2}{2},\frac{n-2}{2}\right)-I_{r^2_{\alpha_1}}\left(\frac{i+2}{2},\frac{n-2}{2}\right)\right\}\Bigg]\Bigg] $$

Proof:- Taking the expectation of $\hat{\mu}_y$, we have

$$ E(\hat{\mu}_y)=E\left[\{\bar{y}+\rho_0(\mu_x-\bar{x})\}\middle|Accept\ \mathrm{H}_0\right]\Pr(Accept\ \mathrm{H}_0)+ $$
$$ E\left[\{\bar{y}+\hat{\beta}(\mu_x-\bar{x})\}\middle|Accept\ \mathrm{H}_1\right]\Pr(Accept\ \mathrm{H}_1) $$

$$ =E\left(\bar{y}\middle|r\ge r_{\alpha_1},r\le r_{\alpha_2}\right).\Pr\left(r\ge r_{\alpha_1},r\le r_{\alpha_2}\right)+E\left\{\rho_0(\mu_x-\bar{x})\middle|r\ge r_{\alpha_1},r\le r_{\alpha_2}\right\}.\ \Pr\left(\mathrm{r}\ge \mathrm{r}_{\alpha_1},\mathrm{r}\le \mathrm{r}_{\alpha_2}\right) $$
$$ +E\left(\bar{y}\middle|\mathrm{r}\le \mathrm{r}_{\alpha_1},\mathrm{r}\ge \mathrm{r}_{\alpha_2}\right)\Pr\left(\mathrm{r}\le \mathrm{r}_{\alpha_1},\mathrm{r}\ge \mathrm{r}_{\alpha_2}\right)+E\left\{\hat{\beta}(\mu_x-\bar{x})\middle|r\le r_{\alpha_1},r\ge r_{\alpha_2}\right\}.\Pr\left(\mathrm{r}\le \mathrm{r}_{\alpha_1},\mathrm{r}\ge \mathrm{r}_{\alpha_2}\right) $$

$$ =E(\bar{y})+\rho_0E\left\{(\mu_x-\bar{x})\middle|r\ge r_{\alpha_1},r\le r_{\alpha_2}\right\}.\ \Pr\left(\mathrm{r}\ge \mathrm{r}_{\alpha_1},\mathrm{r}\le \mathrm{r}_{\alpha_2}\right) $$
$$ +E\left\{\hat{\beta}(\mu_x-\bar{x})\middle|r\le r_{\alpha_1},r\ge r_{\alpha_2}\right\}.\Pr\left(\mathrm{r}\le \mathrm{r}_{\alpha_1},\mathrm{r}\ge \mathrm{r}_{\alpha_2}\right) \quad (3.1) $$

Since sample means are independently distributed with $\hat{\beta}$ and r in case of bivariate normal population then $E(\hat{\mu}_y)$ follows as

$$ E(\hat{\mu}_y)=\mu_y+\rho_0E(\mu_x-\bar{x})\int_{r_{\alpha_1}}^{r_{\alpha_2}}f(r)dr+E(\mu_x-\bar{x}) $$
$$ E\left(\hat{\beta}\middle|r\le r_{\alpha_1},r\ge r_{\alpha_2}\right).\Pr\left(r\le r_{\alpha_1},r\ge r_{\alpha_2}\right) \quad (3.2) $$

$=\mu_y$, as $E(\mu_x-\bar{x})=\mu_x-\mu_x=0$

Hence, the estimator $\hat{\mu}_y$ is unbiased.

The variance of $\hat{\mu}_y$ is defined as

$$ V(\hat{\mu}_y)=E(\hat{\mu}_y^2)-\left[E(\hat{\mu}_y)\right]^2 \quad (3.3) $$

The $E(\hat{\mu}_y^2)$ will be evaluated in similar way using lemma (2.1) as follows

$$ E(\hat{\mu}_y^2)=E\left[\{\bar{y}+\rho_0(\mu_x-\bar{x})\}^2\middle|r\ge r_{\alpha_1},r\le r_{\alpha_2}\right].\Pr\left(r\ge r_{\alpha_1},r\le r_{\alpha_2}\right) $$
$$ +\ E\left[\{\bar{y}+\hat{\beta}(\mu_x-\bar{x})\}^2\middle|r\le r_{\alpha_1},r\ge r_{\alpha_2}\right].\Pr\left(r\le r_{\alpha_1},r\ge r_{\alpha_2}\right) $$

Utilizing the fact that sample means are independently distributed with $\hat{\beta}$ and r in case of bivariate normal population, then it follows that

$$ E(\hat{\mu}_y^2)=E(\bar{y}^2)+\rho_0^2E(\mu_x-\bar{x})^2\int_{r_{\alpha_1}}^{r_{\alpha_2}}f(r)dr+2\rho_0\mu_xE(\bar{y})\int_{r_{\alpha_1}}^{r_{\alpha_2}}f(r)dr $$
$$ -2\rho_0E(\bar{x}\,\bar{y})\int_{r_{\alpha_1}}^{r_{\alpha_2}}f(r)dr $$
$$ +E(\mu_x-\bar{x})^2E\left(\hat{\beta}^2\middle|r\le r_{\alpha_1},r\ge r_{\alpha_2}\right).\Pr\left(r\le r_{\alpha_1},r\ge r_{\alpha_2}\right) $$
$$ +2\mu_xE(\bar{y})E\left(\hat{\beta}\middle|r\le r_{\alpha_1},r\ge r_{\alpha_2}\right).\Pr\left(r\le r_{\alpha_1},r\ge r_{\alpha_2}\right) $$
$$ -2E(\bar{x}\,\bar{y})E\left(\hat{\beta}\middle|r\le r_{\alpha_1},r\ge r_{\alpha_2}\right).\Pr\left(r\le r_{\alpha_1},r\ge r_{\alpha_2}\right) \quad (3.4) $$

We have $E(\mu_x-\bar{x})^2=\frac{\sigma_x^2}{n}=\frac{\sigma^2}{n}$, $E(\bar{x}\,\bar{y})=\rho\frac{\sigma^2}{n}+\mu_x\mu_y$ and $E(\bar{y}^2)=\frac{\sigma^2}{n}+\mu_y^2$ Substituting these values in (3.4) and after little simplification, we get

$$ E(\hat{\mu}_y^2)=\frac{\sigma^2}{n}+\mu_y^2+\rho_0^2\frac{\sigma^2}{n}\int_{r_{\alpha_1}}^{r_{\alpha_2}}f(r)dr-2\rho_0\rho\frac{\sigma^2}{n}\int_{r_{\alpha_1}}^{r_{\alpha_2}}f(r)dr+\frac{\sigma^2}{n}E\left(\hat{\beta}^2\middle|r\le r_{\alpha_1},r\ge r_{\alpha_2}\right). $$
$$ \Pr\left(r\le r_{\alpha_1},r\ge r_{\alpha_2}\right)-2\rho\frac{\sigma^2}{n}E\left(\hat{\beta}\middle|r\le r_{\alpha_1},r\ge r_{\alpha_2}\right).\ \Pr\left(r\le r_{\alpha_1},r\ge r_{\alpha_2}\right) $$

Since $\int_{r_{\alpha_1}}^{r_{\alpha_2}}f(r)dr=1-\alpha$, the above expression reduces to

$$ E(\hat{\mu}_y^2)=\frac{\sigma^2}{n}+\mu_y^2+\frac{\sigma^2}{n}(\rho_0^2-2\rho\rho_0)(1-\alpha)+\frac{\sigma^2}{n}E\left(\hat{\beta}^2\middle|r\le r_{\alpha_1},r\ge r_{\alpha_2}\right).\Pr\left(r\le r_{\alpha_1},r\ge r_{\alpha_2}\right) $$
$$ -2\rho\frac{\sigma^2}{n}E\left(\hat{\beta}\middle|r\le r_{\alpha_1},r\ge r_{\alpha_2}\right).\ \Pr\left(r\le r_{\alpha_1},r\ge r_{\alpha_2}\right) \quad .....(3.5) $$

The variance of $(\hat{\mu}_y)$ is therefore obtained as

$$ V(\hat{\mu}_y)=\frac{\sigma^2}{n}+\frac{\sigma^2}{n}(\rho_0^2-2\rho\rho_0)(1-\alpha)+\frac{\sigma^2}{n}E\left(\hat{\beta}^2\middle|r\le r_{\alpha_1},r\ge r_{\alpha_2}\right). $$
$$ \Pr\left(r\le r_{\alpha_1},r\ge r_{\alpha_2}\right)-2\rho\frac{\sigma^2}{n}E\left(\hat{\beta}\middle|r\le r_{\alpha_1},r\ge r_{\alpha_2}\right).\ \Pr\left(r\le r_{\alpha_1},r\ge r_{\alpha_2}\right) \quad (3.6) $$

Let us define

$$E\left(\hat{\beta}^2|A\right)\Pr(A)+E\left(\hat{\beta}^2|A^c\right)\Pr\left(A^c\right)=E\left(\hat{\beta}^2\right)$$

$$=\frac{1-\rho^2}{n-3}+\rho^2 \tag{3.7}$$

and

$$E\left(\hat{\beta}\;|A\right)\Pr(A)+E\left(\hat{\beta}\;|A^c\right)\Pr\left(A^c\right)=E\left(\hat{\beta}\;\right)=\beta=\rho \tag{3.8}$$

where $A=\left(r\le r_{\alpha_1}, r\ge r_{\alpha_2}\right)$ and $A^c=\left(r\ge r_{\alpha_1}, r\le r_{\alpha_2}\right)$ and $\sigma_x^2=\sigma_y^2$

Incorporating the above results (3.7) and (3.8) into expression (3.6) we get

$$V\left(\hat{\mu}_y\right)=\frac{\sigma^2}{n}+\frac{\sigma^2}{n}\left(\rho_0^2-2\rho\rho_0\right)(1-\alpha)+\frac{\sigma^2}{n}.\left(\frac{1-\rho^2}{n-3}\right).-\frac{\rho^2\sigma^2}{n}$$
$$-\frac{\sigma^2}{n}E\left(\hat{\beta}^2|A^c\right)\Pr\left(A^c\right)+2\rho\frac{\sigma^2}{n}E\left(\hat{\beta}|A^c\right)\Pr\left(A^c\right) \tag{3.9}$$

To have partial check, suppose α =0 then $r_{\alpha_1}=-1$, $r_{\alpha_1}=1$ and hence

$$A^c=\left(r\ge r_{\alpha_1},\ r\le r_{\alpha_2}\right)=\left(r\ge -1,\ \mathrm{r}\le 1\right)$$

Thus, $E\left(\hat{\beta}^2|A^c\right)\Pr\left(A^c\right)=E\left(\hat{\beta}^2\right)=\frac{1-\rho^2}{n-3}+\rho^2$ (3.10)

and $E\left(\hat{\beta}^2|A^c\right)\Pr\left(A^c\right)=E\left(\hat{\beta}\right)=\rho$ (3.11)

Therefore, with the above results, the expression (3.9) reduces to

$$V\left(\hat{\mu}_y\right)=\frac{\sigma^2}{n}\left(1-\rho^2\right)\left(1+\delta^2\right)=V\left(T_1\right) \tag{3.12}$$

The expression (3.12) is the variance of T_1 when H_0 is always accepted. At α=1 it may be seen from the expressions (2.9) and (2.10) that $r_{\alpha_1}=r_{\alpha_2}=\rho_0$. Thus, the range of A^c reduced to r=ρ_0. Therefore

$$E\left(\hat{\beta}^2|A^c\right)\Pr\left(A^c\right)=E\left(\hat{\beta}|r\ge r_{\alpha_1},r\le r_{\alpha_2}\right).\ \Pr\left(r\ge r_{\alpha_1},r\le r_{\alpha_2}\right)=0 \tag{3.13}$$

and similarly

$$E\left(\hat{\beta}|A^c\right)\Pr\left(A^c\right)=E\left(\hat{\beta}|r\ge r_{\alpha_1},r\le r_{\alpha_2}\right).\ \Pr\left(r\ge r_{\alpha_1},r\le r_{\alpha_2}\right)=0 \tag{3.14}$$

Thus, the variance expression (3.9) at α=1 reduces to

$$V\left(\hat{\mu}_y\right)=\frac{\sigma^2}{n}\left(1-\rho^2\right)\left(1+\frac{1}{n-3}\right)=\mathrm{V}(\mathrm{T}_2) \tag{3.15}$$

The expression (3.15) is the variance of T_2 when alternative hypothesis H_1 is always accepted.

To evaluate the conditional expectations in (3.9), it requires the joint distribution function of correlation coefficient r and $v=\frac{S_y^2}{S_x^2}$ where $S_y^2=\frac{1}{n-1}\sum_{i=1}^{n}\left(y_i-\overline{y}\right)^2$ and $S_x^2=\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i-\overline{x}\right)^2$.

Joint density function of r and v has been obtained by Alam (1976) as follows

$$f(r,v)=\frac{2^{n-3}(n-2)}{\pi}\frac{\left[\mathrm{R}\left(1\text{-}\rho^2\right)\right]^{\frac{n-1}{2}}\left(1-r^2\right)^{\frac{n-4}{2}}v^{\frac{n-3}{2}}}{\left(R+v-2\rho r\sqrt{vR}\right)^{n-1}} \tag{3.16}$$

where $R=\frac{\sigma_y}{\sigma_x}$. Since R=1 in the present situation, we have

$$f(r,v)=\frac{Q\left(1-r^2\right)^{\frac{n-4}{2}}v^{\frac{n-3}{2}}}{\left(1+v-2\rho r\sqrt{v}\right)^{n-1}} \tag{3.17}$$

where $Q=\frac{2^{n-3}(n-2)\left(1-\rho^2\right)^{\frac{n-1}{2}}}{\pi}$

Using lemma (2.1), the conditional expectations will be obtained as follows

$$E\left(\hat{\beta}|A^c\right)\Pr\left(A^c\right)=E\left(\hat{\beta}|r\ge r_{\alpha_1},r\le r_{\alpha_2}\right)\Pr\left(r\ge r_{\alpha_1},r\le r_{\alpha_2}\right)$$
$$=E\left(r\sqrt{v}|r\ge r_{\alpha_1},r\le r_{\alpha_2}\right)\Pr\left(r\ge r_{\alpha_1},r\le r_{\alpha_2}\right)$$
$$=\int_{r_{\alpha_1}}^{r_{\alpha_2}}rE\left(\sqrt{v}|r\right)f(r)dr \tag{3.18}$$

$$E\left(\sqrt{v}|r\right)=\int_0^\infty\sqrt{v}\frac{f(r,v)}{f(r)}dv \tag{3.19}$$

Substituting (3.19) into (3.18), we have

$$E\left(\hat{\beta}|A^c\right)\Pr\left(A^c\right)=\int_{r_{\alpha_1}}^{r_{\alpha_2}}\int_0^\infty r\sqrt{v}\ \mathrm{f(r},v)\mathrm{d}v\ \mathrm{dr}$$
$$=Q\left[\int_{r_{\alpha_1}}^{r_{\alpha_2}}r\left(1-r^2\right)^{\frac{n-4}{2}}\int_0^\infty\frac{v^{\frac{n-2}{2}}}{\left(1+v-2\rho r\sqrt{v}\right)^{n-1}}\,\mathrm{d}v\right]\mathrm{dr} \tag{3.20}$$

Let $\mathrm{I}_1=\int_0^\infty\frac{v^{\frac{n-2}{2}}}{\left(1+v-2\rho r\sqrt{v}\right)^{n-1}}dv$

On putting $v=w^2$ we have

$$I_1 = 2\int_0^\infty \frac{w^{n-1}}{\left(1+w^2-2\rho r w\right)^{n-1}}dw \tag{3.21}$$

I_1 will be evaluated by applying Mellin Transform as stated earlier.

Putting $-\rho r=\mathrm{Cos}\,\theta$ and considering $s=n$ and $\lambda=n-1$ in the expression (3.21), I_1 is obtained by applying Mellin Transform as

$$I_1 = 2^{n-\frac{1}{2}}\left(Sin\ \theta\right)^{n-\frac{3}{2}}\Gamma(n-\frac{1}{2})\mathrm{B(n,n\text{-}2)}\ \mathrm{P}_{\frac{1}{2}}^{\frac{3}{2}-n}\left(Cos\ \theta\right)$$

$$= 2^{n-\frac{1}{2}}\left[\left(1+\ \mathrm{Cos}\ \theta\right)\left(1-\ \mathrm{Cos}\ \theta\right)\right]^{\frac{2n-3}{4}}\ \Gamma(n-\frac{1}{2})\ \mathrm{B(n,n\text{-}2)}\ \mathrm{P}_{\frac{1}{2}}^{\frac{3}{2}-n}\left(Cos\ \theta\right)$$

$$= 2^{n-\frac{1}{2}}\left[\left(1+\ \mathrm{Cos}\ \theta\right)\left(1-\ \mathrm{Cos}\ \theta\right)\right]^{\frac{2n-3}{4}}\ \Gamma(n-\frac{1}{2})\ \mathrm{B(n,n\text{-}2)}\frac{1}{\Gamma(n-\frac{1}{2})}\left[\frac{\left(1+Cos\ \theta\right)}{\left(1-Cos\ \theta\right)}\right]^{\frac{3-2n}{4}}\ {}_2F_1\left(-\frac{1}{2},\ \frac{3}{2};\ \mathrm{n}\text{-}\frac{1}{2};\ \frac{1}{2}\text{-}\frac{1}{2}\mathrm{Cos}\ \theta\right)$$

$$= 2^{n-\frac{1}{2}}\ \mathrm{B(n,n\text{-}2)}\ \left(1\text{-}\ \mathrm{Cos}\ \theta\right)^{\frac{2n-3}{2}}\ {}_2F_1\left(-\frac{1}{2},\ \frac{3}{2};\ \mathrm{n}\text{-}\frac{1}{2};\ \frac{1}{2}\text{-}\frac{1}{2}\mathrm{Cos}\theta\right)$$

$$= 2^{n-\frac{1}{2}}\ \mathrm{B(n,n\text{-}2)}\ \left(1+\rho \mathrm{r}\right)^{\frac{2n-3}{2}}\ \sum_{\mathrm{k}=0}^{\infty}\frac{\left(\text{-}\frac{1}{2}\right)_k\left(\frac{3}{2}\right)_k}{\left(n-\frac{1}{2}\right)_k}\ \frac{\left(1+\rho \mathrm{r}\right)^k}{2^k\ \mathrm{k!}}$$

$$= 2^{n-\frac{1}{2}}\ \mathrm{B(n,n\text{-}2)}\ \sum_{\mathrm{k}=0}^{\infty}\frac{\left(\text{-}\frac{1}{2}\right)_k\left(\frac{3}{2}\right)_k}{\left(n-\frac{1}{2}\right)_k}\ \frac{\left(1+\rho \mathrm{r}\right)^{\frac{2n+2k-3}{2}}}{2^k\ \mathrm{k!}} \tag{3.22}$$

Substituting the value of I_1 from expression (3.22) in to the expression (3.20), we get

$$E\left(\hat{\beta}\middle|A^c\right)\Pr\left(A^c\right) = Q\int_{r_{\alpha_1}}^{r_{\alpha_2}} r\left(1-r^2\right)^{\frac{n-4}{2}}\ 2^{\left(n-\frac{1}{2}\right)}B\left(n,\ \mathrm{n\text{-}2}\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)_k\left(\frac{3}{2}\right)_k}{\left(n-\frac{1}{2}\right)_k}\frac{\left(1+\rho r\right)^{\frac{2n+2k-3}{2}}}{2^k k!}dr$$

$$= Q2^{n-\frac{1}{2}}B\left(n,\ \mathrm{n\text{-}2}\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)_k\left(\frac{3}{2}\right)_k}{\left(n-\frac{1}{2}\right)_k 2^k k!}\int_{r_{\alpha_1}}^{r_{\alpha_2}} r\left(1-r^2\right)^{\frac{n-4}{2}}\left(1+\rho r\right)^{\frac{2n+2k-3}{2}}dr$$

$$= Q2^{n-\frac{1}{2}}B\left(n,\ \mathrm{n\text{-}2}\right)\sum_{k=0}^{\infty}\ \sum_{i=0}^{\left(\frac{2n+2k-3}{2}\right)}\rho^i\begin{pmatrix}2n+2k-3\\ \mathrm{i}\end{pmatrix}\frac{\left(-\frac{1}{2}\right)_k\left(\frac{3}{2}\right)_k}{\left(n-\frac{1}{2}\right)_k 2^k k!}\int_{r_{\alpha_1}}^{r_{\alpha_2}} r^{i+1}\left(1-r^2\right)^{\frac{n-4}{2}}dr$$

Note that

$$\int_{r_{\alpha_1}}^{r_{\alpha_2}} r^{i+1}\left(1-r^2\right)^{\frac{n-4}{2}}dr = \frac{1}{2}\int_{r_{\alpha_1}^2}^{r_{\alpha_2}^2}\left(r^2\right)^{\frac{i}{2}}\left(1-r^2\right)^{\frac{n-4}{2}}dr^2$$

when r_{α_1} is positive

$$= \frac{1}{2}\int_0^{r_{\alpha_2}^2}\left(r^2\right)^{\frac{i}{2}}\left(1-r^2\right)^{\frac{n-4}{2}}dr^2$$ when r_{α_2} is negative.

In the present situation, it is assumed that r_{α_2} is positive and further it is assumed that r_{α_1} is positive. So, we may write the above expression as

$$E\left(\hat{\beta}\middle|A^c\right)\Pr\left(A^c\right) = D_1\sum_{k=0}^{\infty}\ \sum_{\mathrm{i}=0}^{\frac{\mathrm{2n+2k\text{-}3}}{2}}\frac{\left(-\frac{1}{2}\right)_k\left(\frac{3}{2}\right)_k}{\left(n-\frac{1}{2}\right)_k 2^k k!}\begin{pmatrix}\frac{\mathrm{2n+2k\text{-}3}}{2}\\ \mathrm{i}\end{pmatrix}\rho^i\int_{r_{\alpha_1}^2}^{r_{\alpha_2}^2}\left(r^2\right)^{\frac{i}{2}}\left(1-r^2\right)^{\frac{n-4}{2}}dr^2 \tag{3.23}$$

where $D_1 = Q2^{n-\frac{3}{2}}B\left(n,n-2\right)$

$$= \frac{2^{2n-\frac{9}{2}}\left(n-2\right)\left(1-\rho^2\right)^{\frac{n-1}{2}}B\left(n,n-2\right)}{\pi}$$

Let $I_1' = \int_{r^2_{\alpha_1}}^{r^2_{\alpha_2}}\left(r^2\right)^{i/2}\left(1-r^2\right)^{\frac{n-4}{2}}dr^2$

Since r^2 follows Beta distribution of first kind, then

$$I_1' = \int_0^{r^2_{\alpha_2}}\left(r^2\right)^{i/2}\left(1-r^2\right)^{\frac{n-4}{2}}dr^2 - \int_0^{r^2_{\alpha_1}}\left(r^2\right)^{i/2}\left(1-r^2\right)^{\frac{n-4}{2}}dr^2$$

$$= B\left(\frac{i+2}{2},\frac{n-2}{2}\right)\left[I_{r^2_{\alpha_2}}\left(\frac{i+2}{2},\frac{n-2}{2}\right) - I_{r^2_{\alpha_1}}\left(\frac{i+2}{2},\frac{n-2}{2}\right)\right] \tag{3.24}$$

where

$$I_{r^2_{\alpha_j}}\left(\frac{i+2}{2},\frac{n-2}{2}\right) = \frac{1}{B\left(\frac{i+2}{2},\frac{n-2}{2}\right)}\int_0^{r^2_{\alpha_j}}\left(r^2\right)^{i/2}\left(1-r^2\right)^{\frac{n-4}{2}}dr^2$$

for j=1,2 and $B\left(\frac{i+2}{2},\frac{n-2}{2}\right)=\int_0^1 \left(r^2\right)^{i/2}\left(1-r^2\right)^{\frac{n-4}{2}} dr^2$

Therefore, form the expression (3.24), it follows that

$$E\left(\hat{\beta}\middle|A^c\right)\Pr\left(A^c\right)=D_1\sum_{k=0}^{\infty}\sum_{i=0}^{\frac{2n+2k-3}{2}}\binom{\frac{2n+2k-3}{2}}{i}\frac{\left(\frac{1}{2}\right)_k\left(\frac{3}{2}\right)_k}{\left(n-\frac{1}{2}\right)_k 2^k k!}$$

$$B\left(\frac{i+2}{2},\frac{n-2}{2}\right)\rho^i\left[I_{r^2_{\alpha_2}}\left(\frac{i+2}{2},\frac{n-2}{2}\right)-I_{r^2_{\alpha_1}}\left(\frac{i+2}{2},\frac{n-2}{2}\right)\right] \tag{3.25}$$

It is clear that when r_{α_1} is negative, only change required in expression (3.25) is that the $\mathrm{I}_{r^2_{\alpha_1}}\left(\frac{i+2}{2},\frac{n-2}{2}\right)$ will not appear

Similarly, we have

$$E\left(\hat{\beta}^2\middle|A^c\right)\Pr\left(A^c\right)=E\left(r^2\nu\middle|r\geq r_{\alpha_1},r\leq r_{\alpha_2}\right)\Pr\left(r\geq r_{\alpha_1},r\leq r_{\alpha_2}\right)$$

$$=\int_{r_{\alpha_1}}^{r_{\alpha_2}} r^2E\left(\nu/r\right)f(r)dr$$

$$=\int_{r_{\alpha_1}}^{r\alpha_2}\int_0^{\infty} r^2\nu f(r\nu)d\nu dr$$

accordingly as in (3.19)

$$=Q\int_{r_{\alpha_1}}^{r_{\alpha_2}}\int_0^{\infty}\frac{r^2\left(1-r^2\right)^{\frac{n-4}{2}}\nu^{\frac{n-1}{2}}}{\left(1+\nu-2\rho r\sqrt{\nu}\right)^{n-1}}d\nu dr$$

$$=Q\int_{r_{\alpha_1}}^{r_{\alpha_2}} r^2\left(1-r\right)^{\frac{n-4}{2}}\left[\int_0^{\infty}\frac{\nu^{\frac{n-1}{2}}}{\left(1+\nu-2\rho r\sqrt{\nu}\right)^{n-1}}d\nu\right]dr \tag{3.26}$$

Let $\mathrm{I}_2=\int_0^{\infty}\frac{\nu^{\frac{n-1}{2}}}{\left(1+\nu-2\rho r\sqrt{\nu}\right)^{n-1}}d\nu$

Putting $\nu=\mathrm{w}^2$ we have as under

$$\mathrm{I}_2=2\int_0^{\infty}\frac{w^n}{\left(1+w^2-2\rho rw\right)^{n-1}}dw \tag{3.27}$$

I_2 will be evaluated in similar way by applying Mellin Transform. Putting –ρr =Cosθ and considering s=n+1 and λ=n-1 in the expression (3.27), I_2 is obtained as follows.

$$\mathrm{I}_2=2^{n-\frac{1}{2}}\left(\sin\theta\right)^{n-\frac{3}{2}}\Gamma(n-\frac{1}{2})B(n+1,n-3)P^{\frac{3}{2}-n}_{\frac{3}{2}}(Cos\theta)$$

On simplification as done earlier, I_2 is given by

$$\mathrm{I}_2=2^{n-\frac{1}{2}}B\left(n+1,n-3\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{2}\right)_k\left(\frac{5}{2}\right)_k}{\left(n-\frac{1}{2}\right)_k 2^k k!}\left(1+\rho r\right)^{\frac{2n+2k-3}{2}} \tag{3.28}$$

From the expressions (3.26) and (3.28) we get

$$E\left(\hat{\beta}^2\middle|A^c\right)\Pr\left(A^c\right)=Q2^{n-\frac{3}{2}}B\left(n+1,n-3\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{2}\right)_k\left(\frac{5}{2}\right)_k}{\left(n-\frac{1}{2}\right)_k 2^k k!}$$

$$\int_{r^2_{\alpha_1}}^{r^2_{\alpha_2}}\left(r^2\right)\left(1-r^2\right)^{\frac{n-4}{2}}\left(1+\rho r\right)^{\frac{(2n+2k-3)}{2}}dr$$

$$=Q2^{n-\frac{1}{2}}B\left(n+1,n-3\right)\sum_{k=0}^{\infty}\sum_{i=0}^{\frac{(2n+2k-3)}{2}}\binom{\frac{(2n+2k-3)}{2}}{i}\rho^i\frac{\left(-\frac{3}{2}\right)_k\left(\frac{5}{2}\right)_k}{\left(n-\frac{1}{2}\right)_k 2^k k!}$$

$$\int_{r^2_{\alpha_1}}^{r^2_{\alpha_2}} r^{2+i}\left(1-r^2\right)^{\frac{n-4}{2}}dr$$

Under the assumption that r_{α_1} and $r_{\alpha 2}$ are positive, it follows that

$$E\left(\hat{\beta}^2\middle|A^c\right)\Pr\left(A^c\right)=Q2^{n-\frac{1}{2}}B\left(n+1,n-3\right)\sum_{k=0}^{\infty}\sum_{i=0}^{\frac{(2n+2k-3)}{2}}\binom{\frac{(2n+2k-3)}{2}}{i}$$

$$\rho^i\frac{\left(-\frac{3}{2}\right)_k\left(\frac{5}{2}\right)_k}{\left(n-\frac{1}{2}\right)_k 2^k k!}\int_{r^2_{\alpha_1}}^{r^2_{\alpha_2}}\left(r^2\right)^{\frac{i+1}{2}}\left(1-r^2\right)^{\frac{n-4}{2}}dr^2 \tag{3.29}$$

Let $\mathrm{I}_2'=\int_{r^2_{\alpha_1}}^{r^2_{\alpha_2}}\left(r^2\right)^{\frac{i+1}{2}}\left(1-r^2\right)^{\frac{n-4}{2}}dr^2$

$$=\int_0^{r^2_{\alpha_2}}\left(r^2\right)^{\frac{i+1}{2}}\left(1-r^2\right)^{\frac{n-4}{2}}dr^2-\int_0^{r^2_{\alpha_1}}\left(r^2\right)^{\frac{i+1}{2}}\left(1-r^2\right)^{\frac{n-4}{2}}dr^2$$

$$=B\left(\frac{i+3}{2},\frac{n-2}{2}\right)\left[\mathrm{I}_{r^2_{\alpha_2}}\left(\frac{i+3}{2},\frac{n-2}{2}\right)-\mathrm{I}_{r^2_{\alpha_1}}\left(\frac{i+3}{2},\frac{n-2}{2}\right)\right] \tag{3.30}$$

From the expressions (3.29) and (3.30), we get

$$E\left(\hat{\beta}^2 \middle| A^c\right)\Pr\left(A^c\right) = D_2 \sum_{k=0}^{\infty} \sum_{i=0}^{\frac{(2n+2k-3)}{2}} \frac{\left(-\frac{3}{2}\right)_k \left(\frac{5}{2}\right)_k}{\left(n-\frac{1}{2}\right)_k 2^k k!} \binom{\frac{(2n+2k-3)}{2}}{i} \rho^i$$

$$B\left(\frac{i+3}{2}, \frac{n-2}{2}\right)\left[\mathrm{I}_{r^2_{\alpha_2}}\left(\frac{i+3}{2}, \frac{n-2}{2}\right) - \mathrm{I}_{r^2_{\alpha_1}}\left(\frac{i+3}{2}, \frac{n-2}{2}\right)\right] \quad (3.31)$$

Where

$$D_2 = Q 2^{n-\frac{3}{2}} B(n+1, n-3)$$

$$= \frac{2^{n-\frac{9}{2}}(n-2)\left(1-\rho^2\right)^{\frac{n-1}{2}} B\left(n+1, n-3\right)}{\pi}$$

Here also if r_{α_1} is negative the term $\mathrm{I}_{r^2_{\alpha_1}}\left(\frac{i+3}{2}, \frac{n-2}{2}\right)$ in the expression (3.31) will not appear. Thus, substituting the value of conditional expectations from the expressions (3.25) and (3.31) into the expression (3.9), and after little simplification we get the required variance of $\hat{\mu}_y$ as stated in the theorem 3.1

Hence it proves the theorem.

4. COMPARISON OF THE ESTIMATORS AND CONCLUSIONS

The comparison of the preliminary test estimator $\hat{\mu}_y$ is made with difference estimator T_1 and regression estimator T_2. It will be assumed that $n \geq 4$ and we shall begin by comparing $\hat{\mu}_y$ with T_1. Let e_1 (T_1, $\hat{\mu}_y$) be the relative efficiency of $\hat{\mu}_y$ over T_1, and it is given by

$$e_1\left(\hat{\mu}_y, T_1\right) = \frac{V\left(T_1\right)}{V\left(\hat{\mu}_y\right)} \quad (4.1)$$

Similarly, e_2 (T_2, $\hat{\mu}_y$) be the relative efficiency of $\hat{\mu}_y$ over T_2, and it is given by

$$e_2\left(\hat{\mu}_y, T_2\right) = \frac{V\left(T_2\right)}{V\left(\hat{\mu}_y\right)} \quad (4.2)$$

The theoretical comparison of the estimators is complex and not straight forward. In order to have the idea of the relative efficiencies, the values of e_1 and e_2 are computed for values of n=10 &15, ρ_0=0.7, α=0.05 and for different values of ρ and are presented in the Tables 1.

Table 1. The values of relative efficiency e_1 and e_2 for α=0.05, n=10, 15, ρ_0=0.7 and for different values ρ

n	ρ	e_1 (%)	e_2 (%)
10	0.66	187	213
	0.68	226	258
	0.70	297	339
	0.72	461	526
	0.74	1234	1406
15	0.66	258	279
	0.68	336	363
	0.70	507	550
	0.72	1199	1298
	0.74	*	*

* The computation of e_i's is not possible because variance of $\hat{\mu}_y$ comes out to be negative at $\rho = 0.74$.

It is obvious from the results of the Table-1 the preliminary test estimator $\hat{\mu}_y$ is more efficient than T_1 and T_2. Moreover, there is more gain in precision of $\hat{\mu}_y$ with respect to T_2. Finally it can be concluded from the above results that a preliminary test regression type estimator (PTRTE) can be used to obtain more precise estimate of population mean when a guess value of ρ is available and its significance is tested to use either of the estimators T_1 and T_2 in the PTRTE $\left(\hat{\mu}_y\right)$.

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