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JOURNAL OF THE INDIAN SOCIETY OF
AGRICULTURAL STATISTICS 73(1) 2019 27–32

Construction of Balanced Sampling Plans Excluding Adjacent Units

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Received 19 January 2018; Revised 09 November 2018; Accepted 15 November 2018

SUMMARY

Balanced sampling plans excluding adjacent units are useful for sampling from populations in which the nearer units provide similar observations due to natural ordering of the units in time or space. The ordering of units in the population may be circular or linear. For these plans, all the first order inclusion probabilities are equal whereas second order inclusion probabilities for pairs of adjacent units at a distance less than or equal to α are zero and constant for all other pairs of non-adjacent units which are at a distance greater than α. In this article, we present 13 new balanced sampling plans excluding adjacent units for one dimensional population with circular and 111 with linear ordering of units in the parametric range $N \leq 50$, $n \leq 7$, $\lambda \leq 7$, $\alpha \leq 5$.

Keywords: Balanced sampling plans excluding adjacent units, Linear programming approach, Polygonal designs.

1. INTRODUCTION

Simple random sampling is a basic selection procedure which provides equal chance of selection to all possible samples in the sample space. There do occur many situations where providing equal probability of selection to all possible samples is not a very desirable feature and controls may be desirable for selection procedures which may provide the basis of preferability of the samples. There may arise a situation when the units in the population are ordered in time or space. Because of this natural ordering of the units, there may be some positive correlation between the nearer units. As a result, observations from nearer units are expected to be similar. Considering aspects like time, cost and effectiveness, it is desirable to avoid nearer units in a sample.

Balanced sampling plans excluding contiguous units are useful for such situations when the nearer units provide similar observations. These plans were introduced by Hedayat *et al.* (1988) for a given circular population of size N, for which a sample of size n is obtained without replacement such that the second-order inclusion probabilities are zero for contiguous units and constant for non-contiguous units. Balanced sampling plans excluding contiguous units are those sampling plans in which pair of contiguous units never appear in a sample whereas all other pairs appear equally often in the samples.

Stufken (1993) has generalized the concept of balanced sampling plans excluding contiguous units by excluding all those pairs whose distance is less than or equal to α. These plans were termed as Balanced Sampling Plans Excluding Adjacent units or BSA (α) plans by Stufken (1993). Here, two units are called adjacent when their distance is less than a specified number α whose choice depends on the experimenter. Both BSEC plans and BSA plans may be uniformly called as BSA (α) plans. It is obvious that a BSEC plan is a BSA (1).

Stufken *et al.* (1999) introduced polygonal designs and showed that polygonal designs are equivalent to BSA plans. A polygonal design is an arrangement of N symbols in b blocks of size n with r replications and distance α such that (i) any two symbols i, j with distance less than or equal to α do not appear together in a block (ii) any other pair of symbols i, j with distance greater than α appear together in precisely λ blocks.

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In this case, the parameters of the design are N, b, r, n, λ and α. The parameter of the design satisfies the following necessary conditions:

i. $Nr = bn;$

ii. $\lambda(N - 2\alpha - 1) = r(n - 1)$ (1)

If $\alpha = 0$, a polygonal design reduces to a balanced incomplete block design. Henceforth, we shall use polygonal designs or BSA(α) interchangeably.

Most of the works on BSA plans assume one dimensional population though there are some works on two dimensional BSA plans, e.g., Bryant *et al.* (2002), Wright (2008) and Gopinath *et al.* (2018). In this article, we restrict ourselves to one dimensional population.

For an one dimensional population, the structure of BSA plans depends on the assumption of ordering of units which may be circular or linear. Under the circular ordering, the distance between two units i and j is denoted by $\delta(i,j) = \text{Min}\{|i-j|, N-|i-j|\}$ and for the linear ordering, the distance between two units i and j is $\delta(i,j) = \text{Max}(i-j, j-i)$, $i \neq j = 1, 2, \ldots, N$. A BSA ($\alpha$) under circular and linear ordering of population units is denoted as cBSA (α) and lBSA(α), respectively.

There is a lot of interest in the existence and construction of polygonal designs for given parameters N, n and α. A number of authors (Hedayat *et al.*, 1988, Colbourn and Ling, 1999, Stufken *et al.*, 1999, Stufken and Wright 2008, Mandal *et al.,* 2008, Mandal *et al.,* 2011, Tahir *et al.*, 2012, Gupta *et al.*, 2012, Mandal *et al.*, 2014, Kumar *et al.*,2016) have obtained a large number of polygonal designs, still there are gaps in the design parameters. Additional efforts are, therefore, required to obtain polygonal designs for given combinations of N, b, n and α.

Wright and Stufken (2008) and Mandal *et al.* (2008) presented linear programming approaches to obtain smaller cBSAs which then can be utilized to obtain more cBSAs and lBSAs. Moreover, most of the methods produce cBSAs which are cyclic in nature, i.e., the support of the plan can be obtained by cyclically developing initial generator samples modulo N. However, there may exist non-cyclic cBSAs with smaller support sizes for a given N, n and α and such BSAs need to be identified.

In this article, we present several new cBSAs and lBSAs with smaller support sizes. We obtain these BSAs by using an algorithm developed by Kumar *et al.* (2016). An important feature of the proposed algorithm is that it can construct cyclic or non-cyclic cBSAs and lBSAs.

2. METHODS OF CONSTRUCTION

Kumar *et al.* (2016) developed an algorithm to obtain cBSAs and lBSAs. We describe the algorithm of Kumar *et al.* (2016) in brief in the sequel for completeness. Details may be seen from Kumar *et al.* (2016).

The algorithm tries to obtain the incidence matrix **N** of the required polygonal design. First, the user need to input N, b, n, λ, α and r for cBSA and r_1, r_2,...,r_N for lBSA. In the first step, the first row of the incidence matrix **N** is obtained by randomly allotting 1 to r columns (blocks) in case of cBSA and r_1 columns (blocks) in case of lBSA out of b available columns of the **N** matrix. Next row of the **N** matrix is obtained in such a way that the desired concurrence of the second row with the first row is achieved and this is done with the help of an integer linear programming formulation. Similarly, the third row is obtained such that desired concurrences of the third row with the first row and the second row are achieved. This process is continued till all N rows are obtained. There may be a chance that at some row, no solution is obtained. Suppose that at i^{th} $(2 \leq i \leq N)$ row, there is no solution for the integer linear programming formulation. In that case, m^{th} row of matrix **N** is deleted where m is a randomly selected number between 1 to $(i - 1)$ and an alternative solution to m^{th} row is obtained by using another integer linear programming formulation and then the solution to i^{th} row is obtained. For further details of the algorithm, see Kumar *et al.*(2016).

3. RESULTS

In this section, we describe the results of polygonal designs for circular ordering of population units and linear ordering of population units. Polygonal designs for circular ordering and linear ordering of population units have been obtained in the range $\mathfrak{R} = \{N \leq 50, n \leq 7, \lambda \leq 7, \alpha \leq 5\}$, using the algorithm of Kumar *et al.* (2016). We partition the parametric range $\mathfrak{R}$ as $\mathfrak{R} = \mathfrak{R}_1 + \mathfrak{R}_2$, where $\mathfrak{R}_1 = \{N \leq 30, n \leq 5, \lambda \leq 5, \alpha \leq 5\}$ is already covered by Kumar *et al.* (2016), and

$\mathfrak{R}_2$ represent the remaining parametric range in $\mathfrak{R}$ not covered by them.

3.1 Polygonal designs for circular ordering of population units

Polygonal designs within parametric range $\mathfrak{R} = \{N \leq 50,\ n \leq 7,\ \lambda \leq 7,\ \alpha \leq 5\}$ satisfying the necessary parametric relations are obtained. In the above parametric range, a total of 3363 parameters satisfy the necessary conditions (1) of existence of polygonal designs. Distribution of these 3363 designs for α = 1, 2, 3, 4 and 5 along with number of designs obtained through the algorithm, number of non-existent designs, number of designs for which either the solution is unknown or non-existence is not proved and new designs obtained is given in Table 3.1.1 for both $\mathfrak{R}_1$ and $\mathfrak{R}_2$.

From Table, 3.1.1, it can be seen that out of these 3363 designs, 963 designs are non-existent as per the Theorem numbers 4.3(1) of Stufken *et al.* (2008) and Result 2.1 of Parsad *et al.* (2007). Out of 1560 designs obtained, 13 designs (3 in $\mathfrak{R}_1$ and 10 in $\mathfrak{R}_2$) are new in the sense that their solution was not available in the literature earlier. The parameters of these 13 new designs are given in Table 3.1.2. Out of these 13 new designs, there are 3 designs which falls in $\mathfrak{R}_1$ and those are at Sl No. 3, 4 and 5 of Table 3.1.2

Table 3.1.2. Parameters of new designs under circular ordering of population units

Sl. No.	N	b	r	n	λ	α	Remarks
1	24	84	21	6	5	1	
2	21	343	49	3	7	3	
3	28	98	14	4	2	3	
4	28	147	21	4	3	3	
5	28	245	35	4	5	3	
6	32	200	25	4	3	3	
7	33	121	11	3	1	5	
8	33	242	22	3	2	5	2 copies of design at Sl. No.7
9	33	363	33	3	3	5	3 copies of design at Sl. No. 7
10	33	484	44	3	4	5	4 copies of design at Sl. No.7
11	33	605	55	3	5	5	5 copies of design at Sl. No. 7
12	33	726	66	3	6	5	6 copies of design at Sl. No. 7
13	33	847	77	3	7	5	7 copies of design at Sl. No. 7

Table 3.1.1. Distribution of polygonal designs for circular ordering in parametric range $\mathfrak{R}$

	$\alpha = 1$		$\alpha = 2$		$\alpha = 3$		$\alpha = 4$		$\alpha = 5$		Total	
	$\mathfrak{R}_1$	$\mathfrak{R}_2$	$\mathfrak{R}_1$	$\mathfrak{R}_2$	$\mathfrak{R}_1$	$\mathfrak{R}_2$	$\mathfrak{R}_1$	$\mathfrak{R}_2$	$\mathfrak{R}_1$	$\mathfrak{R}_2$	$\mathfrak{R}_1$	$\mathfrak{R}_2$
Number of parametric combinations	231	467	233	496	216	483	180	428	172	457	**1032**	**2331**
Number of designs obtained	188	260	140	233	90	207	70	169	45	158	**533**	**1027**
Number of designs exists but not obtained	20	131	32	137	22	116	3	68	0	0	**77**	**452**
Number of non-existing designs	19	26	61	77	101	107	88	149	127	208	**396**	**567**
Number of designs for which solution is unknown	4	49	0	49	0	42	0	42	0	84	**4**	**266**
Number of new designs	0	1	0	0	3	2	0	0	0	7	**3**	**10**

Table 3.2.1. Distribution of polygonal designs for linear ordering in parametric range $\mathfrak{R}$

	$\alpha = 1$		$\alpha = 2$		$\alpha = 3$		$\alpha = 4$		$\alpha = 5$		Total	
	$\mathfrak{R}_1$	$\mathfrak{R}_2$	$\mathfrak{R}_1$	$\mathfrak{R}_2$	$\mathfrak{R}_1$	$\mathfrak{R}_2$	$\mathfrak{R}_1$	$\mathfrak{R}_2$	$\mathfrak{R}_1$	$\mathfrak{R}_2$	$\mathfrak{R}_1$	$\mathfrak{R}_2$
Number of parametric combinations	188	372	177	361	163	352	151	345	138	334	**817**	**1764**
Number of designs obtained	176	234	144	206	118	188	105	182	95	178	**638**	**988**
Number of designs exist but not obtained	1	5	6	5	7	10	0	0	0	0	**14**	**20**
Number of non-existing designs	10	22	25	47	38	71	44	99	43	115	**160**	**354**
Number of designs for which solution is unknown	1	77	2	65	0	53	0	60	0	38	**3**	**293**
Number of new designs	0	34	0	38	0	30	2	4	0	3	**2**	**109**

From Table 3.1.2, it can easily be observed that out of 13 new designs, 6 designs could be obtained by taking copies of other designs.

In the present investigation, the polygonal designs have been obtained within parameter range $N \leq 50$, $n \leq 7$, $\alpha \leq 5$ but the algorithm is general in nature and can be used for obtaining polygonal designs outside this range.

3.2 Polygonal designs for linear ordering of population units

Polygonal designs within parameter range $\mathfrak{R} = \{N \leq 50, n \leq 7, \lambda \leq 7, \alpha \leq 5\}$ satisfying the necessary parametric relations under the assumption of linear ordering of units are obtained. In the above parametric range, a total of 2581 parametric combinations satisfy necessary conditions of existence. Distribution of these 2581 designs for α = 1, 2, 3, 4 and 5 along with number of designs obtained through the algorithm, number of non-existent designs, number of designs for which either the solution is unknown or non-existence is not proved and new designs obtained is given in Table 3.2.1 for both $\mathfrak{R}_1$ and $\mathfrak{R}_2$.

From Table 3.2.1, it can be seen that out of these 2581 designs, 514 designs are non-existent as per Theorems 6.1, 6.2 and Table 7 of Stufken *et al.* (2008).

Out of 1626 designs obtained, 111 designs are new in the sense that their solution was not available in the literature earlier. The parameters of 111 new designs are given in Table 3.2.2. Out of these 111 new designs, there are 2 designs which falls in $\mathfrak{R}_1$ and those are at Sl No. 1 and 2 for α = 4 of Table 3.2.2

From Table 3.2.2, one can easily see the parameters of 34, 38,30,6 and 3 new polygonal designs for linear ordering of units for α = 1, 2, 3, 4 and 5 respectively. The modified algorithm is general in nature and can be used for obtaining polygonal designs outside this parametric range $N \leq 50$, $n \leq 7$, $\lambda \leq 7$, $\alpha \leq 5$ also. Layout of all these designs and also those presented in Table 3.2.1 are available with the authors and can be obtained by sending an E-mail to Rajender.parsad@icar.gov.in or bn.mandal@icar.gov.in.

4. CONCLUDING REMARKS

In the present investigation, the algorithm developed by Kumar *et al.* (2016) has been used

Table 3.2.2. Parameters of newly obtained designs under linear ordering of population units

For α = 1

Sl. No.	N	b	r_1	r_2	n	λ	α
1	19	51	17	16	6	5	1
2	20	57	18	17	6	5	1
3	22	70	20	19	6	5	1
4	22	60	20	19	7	6	1
5	23	77	21	20	6	5	1
6	23	231	42	40	4	6	1
7	23	66	21	20	7	6	1
8	24	253	44	42	4	6	1
9	25	276	46	44	4	6	1
10	25	92	23	22	6	5	1
11	27	325	50	48	4	6	1
12	28	351	52	50	4	6	1
13	29	126	27	26	6	5	1
14	29	378	54	52	4	6	1
15	30	406	56	54	4	6	1
16	31	174	29	28	5	4	1
17	31	435	58	56	4	6	1
18	32	186	30	29	5	4	1
19	32	465	60	58	4	6	1
20	34	528	64	62	4	6	1
21	35	561	66	64	4	6	1
22	36	595	68	66	4	6	1
23	36	238	34	33	5	4	1
24	37	420	35	34	3	2	1
25	37	315	35	34	4	3	1
26	38	444	36	35	3	2	1
27	38	666	72	70	4	6	1
28	39	703	74	72	4	6	1
29	40	494	38	37	3	2	1
30	41	520	39	38	3	2	1
31	41	780	78	76	4	6	1
32	46	660	44	43	3	2	1
33	47	690	45	44	3	2	1
34	49	752	47	46	3	2	1

For α = 2

Sl. No.	N	b	r_1	r_2	r_3	n	λ	α
1	22	190	38	36	34	4	6	2
2	23	210	40	38	36	4	6	2
3	24	231	42	40	38	4	6	2
4	25	253	44	42	40	4	6	2
5	28	325	50	48	46	4	6	2
6	29	351	52	50	48	4	6	2

Sl. No.	N	b	r_1	r_2	r_3	n	λ	α
7	30	378	54	52	50	4	6	2
8	31	406	56	54	52	4	6	2
9	31	203	28	27	26	4	3	2
10	32	435	58	56	54	4	6	2
11	32	145	29	28	27	6	5	2
12	33	465	60	58	56	4	6	2
13	33	186	30	29	28	5	4	2
14	34	496	62	60	58	4	6	2
15	35	352	32	31	30	3	2	2
16	35	704	64	62	60	3	4	2
17	35	528	64	62	60	4	6	2
18	36	374	33	32	31	3	2	2
19	36	748	66	64	62	3	4	2
20	38	420	35	34	33	3	2	2
21	38	315	35	34	33	4	3	2
22	39	444	36	35	34	3	2	2
23	39	666	72	70	68	4	6	2
24	41	741	76	74	72	4	6	2
25	42	520	39	38	37	3	2	2
26	42	780	78	76	74	4	6	2
27	42	390	39	38	37	4	3	2
28	43	328	40	39	38	5	4	2
29	44	574	41	40	39	3	2	2
30	44	861	82	80	78	4	6	2
31	45	602	42	41	40	3	2	2
32	45	903	84	82	80	4	6	2
33	46	946	86	84	82	4	6	2
34	47	660	44	43	42	3	2	2
35	47	990	88	86	84	4	6	2
36	48	690	45	44	43	3	2	2
37	50	752	47	46	45	3	2	2
38	50	564	47	46	45	4	3	2

For $\alpha = 3$

Sl. No.	N	b	r_1	r_2	r_3	r_4	k	λ	α
1	36	1056	96	93	90	87	3	6	3
2	36	352	32	31	30	29	3	2	3
3	36	704	64	62	60	58	3	4	3
4	37	374	33	32	31	30	3	2	3
5	37	748	66	64	62	60	3	4	3
6	39	420	35	34	33	32	3	2	3
7	39	840	70	68	66	64	3	4	3
8	39	1260	105	102	99	96	3	6	3
9	39	315	35	34	33	32	4	3	3
10	40	444	36	35	34	33	3	2	3
11	40	888	72	70	68	66	3	4	3
12	40	1332	108	105	102	99	3	6	3
13	40	333	36	35	34	33	4	3	3
14	41	1406	111	108	105	102	3	6	3
15	42	494	38	37	36	35	3	2	3
16	42	988	76	74	72	70	3	4	3
17	42	1482	114	111	108	105	3	6	3
18	43	520	39	38	37	36	3	2	3
19	44	410	40	39	38	37	4	3	3
20	44	820	80	78	76	74	4	6	3
21	45	574	41	40	39	38	3	2	3
22	45	1148	82	80	78	76	3	4	3
23	45	861	82	80	78	76	4	6	3
24	46	602	42	41	40	39	3	2	3
25	46	903	84	82	80	78	4	6	3
26	48	660	44	43	42	41	3	2	3
27	48	495	44	43	42	41	4	3	3
28	48	990	88	86	84	82	4	6	3
29	49	690	45	44	43	42	3	2	3
30	49	1035	90	88	86	84	4	6	3

For $\alpha = 4$

Sl. No.	N	b	r_1	r_2	r_3	r_4	r_5	n	λ	α
1	29	200	24	23	22	21	20	3	2	4
2	29	400	48	46	44	42	40	3	4	4
3	29	600	72	69	66	63	60	3	6	4
4	30	650	75	72	69	66	63	3	6	4
5	31	234	26	25	24	23	22	3	2	4
6	31	468	52	50	48	46	44	3	4	4

For $\alpha = 5$

Sl. No.	N	b	r_1	r_2	r_3	r_4	r_5	r_6	n	λ	α
1	36	310	30	29	28	27	26	25	3	2	5
2	36	620	60	58	56	54	52	50	3	4	5
3	36	930	90	87	84	81	78	75	3	6	5

to obtain polygonal designs for circular and linear ordering of population units. The modified algorithm has been utilized to generate polygonal designs in the parametric range $N \leq 50$, $n \leq 7$, $\lambda \leq 7$, $\alpha \leq 5$. A total of 2400 designs satisfy the parametric conditions for existence of a circular polygonal design. Out of these 2400 designs, 1560 designs have been obtained. It is found that 13 designs are new and are not available in literature. In case of linear ordering of population units, 2067 designs satisfy the parametric conditions for existence of polygonal designs. Out of these 2067 parametric combinations, designs are obtained for

1626 parametric combinations and 111 are new under linear ordering of units. The number of designs for which solution is unknown is 270 in case of circular ordering and is 296 in case of linear ordering of population units, respectively. Further research efforts are required to either obtain these polygonal designs or to prove their non-existence.

ACKNOWLEDGEMENTS

Authors are thankful to the anonymous reviewer for giving valuable suggestions which have improved the presentation of the article.

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