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Hedge Ratio based on Ordinary Least Squares (OLS) Vs State Space Methodologies

Amit Saha[1], S.P. Bhardwaj[1], K.N. Singh[1], Kanchan Sinha[1], Avnish Grover[2] and Bishal Gurung[1]
[1]ICAR-Indian Agricultural Statistics Research Institute, New Delhi
[2]University of Delhi, Delhi

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SUMMARY

Risk in agriculture is imminent due to the sudden fluctuations in prices of essential commodities. To cope up with these fluctuations, hedging is usually employed to reduce risk. Studies based on hedging carried out in developed markets like the United States and Europe have finally been employed in emerging markets such as China, India, Brazil, Russia and many other Asian countries. Our study deals with estimating the optimal hedge ratio, which is defined as the relationship between the price of the spot commodity and that of the hedging commodity. Hedging is done in order to minimize the price risk of agricultural commodities due to volatility in prices. Several techniques to estimate the hedge ratio have been proposed in literature. The conventional approach is based on fixed coefficient models which gives a constant optimal hedge ratio. Previous studies have shown that the expected relationship between economic or financial variables may be better captured by a time-varying parameter model rather than a fixed coefficient model. Therefore, the optimal hedge ratio can be a time varying rather than a constant. To this end, a powerful technique called "Kalman filter" has been applied for estimating time-varying hedge ratio. As illustration, three contracts of soybean market were also carried out. It was seen that time-varying hedge ratio performed better than constant hedge ratio for the data under consideration.

Keywords: Optimal Hedge Ratio, Constant coefficient, Time-Varying coefficient, Kalman filter, State Space Modelling.

1. INTRODUCTION

The price risk of agricultural commodities is mainly due to volatility of prices in agricultural commodities. Volatility is a measure of variation in prices of agricultural commodities over time. It can be defined as the rate at which the price of a commodity increases or decreases for a given set of returns of the underlying commodity. If the price of a commodity fluctuates rapidly in a short time span, it is said to have high volatility and if it fluctuates slowly in a longer life span, it has low volatility. One method of estimating volatility is by calculating the standard deviation of commodities annualised logarithmic returns over a given period of time. Agricultural prices mainly vary because production and consumption are variable. So, the hedging of risk has therefore become a very important issue to minimize the risk.

However, it is crucial that the optimal quantity of hedging to be used is determined. Thus, the calculation of the optimal hedge ratio plays a critical role in the hedging process. At present, there are several methods that for estimating the optimal hedge ratio, both on a static and dynamic basis (Chen *et al*. 2003).

Myers (1989) showed that empirical ARCH models are no better than simpler regression models. Falkenstein and Hanweck (1996) developed a multi-futures weighted regression method in an attempt to use the information from two or more points on the yield curve for the hedge. However, they did not compare this method to the typical regression method to see whether the weighted regression procedure is superior methods versus the less costly traditional regression procedure. Lien *et al*. (2000) compared the performances of the hedge ratios estimated from the OLS method and the constant – correlation VGARCH (Vector Generalized Auto Regressive Conditional Heteroscedasticity (VGARCH) model. They found that the OLS hedge ratio performs better than the

Corresponding author: Amit Saha
E-mail address: amitsahauni@gmail.com

VGARCH hedge ratio. Choudhry (2004) investigated the hedging effectiveness of Australian, Hong Kong and Japanese stock futures markets. Their study compared the hedging effectiveness of futures markets by OLS method and the Generalized Auto Regressive Conditional Heteroscedasticity (GARCH) model with consideration of the time varying distribution of the cash and futures price changes. So, there is still scope for further research in this interesting area. To this end, we propose the use of Kalman filter, to obtain estimates of time-varying regression coefficients.

The paper in divided as follows. Section 1 deals with introduction and is followed by Section 2, where some preliminaries on hedging, constant coefficient method, time-varying method and Kalman filter is elucidated in detail. Further, in Section 3, we provide illustration of both the methods. Lastly, in Section 4, concluding remarks and future works on this area is provided.

2. SOME PRELIMINARIES

In this section, Hedging and its different types are discussed. Then constant hedge ratio along with the time-varying hedge ratio with its estimation using Kalman filter is also discussed here.

2.1 Hedging

Hedging is a risk management strategy used in limiting or offsetting probability of loss from fluctuations in the prices of commodities. It was defined by Shepherd as "it is executing opposite sales or purchases in the futures market to offset the purchases or sales of physical products made in the cash market". Hoffman defined it as "the practice of buying or selling futures to offset an equal and opposite position in the cash market and thus avoid the risk of uncertain changes in price".

Hedging is based on two basic assumptions:

- The future and cash commodity prices move up and down together, i.e., the basis of price changes remains unchanged.
- The mechanics of hedging includes the making of simultaneous transactions, but of opposite nature, in the futures and cash market.

The basis reflects the relationship between cash price and futures price (In futures trading, the term "cash" refers to the underlying product). The "basis" is obtained by subtracting the futures price from the cash price. The "basis" can be a positive or negative number. A positive basis is said to be "over" as the cash price is higher than the futures price. A negative basis is said to be "under" as the cash price is lower than the futures price.

There are mainly two types of hedging, viz., long and short hedge. Long Hedge is done to protect short position in the underlying commodity against a possible price rise. The long hedge is a hedging strategy used by manufacturers and producers to lock in the price of a product or commodity to be purchased in some time in the future. Hence, the long hedge is also known as input hedge. The long hedge involves taking up a long future position. If the price of commodity rises; the gain in the value of the long futures position will be able to offset the increase in purchasing costs. Short hedge is done to protect long position in the underlying commodity against a possible price rise. The short hedge is a hedging strategy used by manufacturers and producers to lock in the price of a product or commodity to be delivered sometime in the future. Hence, the short hedge is also known as output hedge. The short hedge involves taking up a short future position while owning the underlying product or commodity to be delivered. If, the price of underlying commodity falls, the gain in the value of the short futures position will be able to offset the drop in revenue from the sale of the underlying commodity. In real life, it may so happen that the commodity which is being hedged may not have a corresponding contract in futures segment. For example, futures are not available for Rice, in which the seller holds 5000 contracts. After a lot of observation and research, he found that Wheat futures are available for trade which moves in tandem with Rice. So, in order to hedge his position in Rice, he can go short on Wheat. Such a hedge is called Cross hedge.

2.2 Constant Hedge Ratio

The Minimum-Variance Hedge Ratio (Benninga *et al*. 1983, 1984) has been suggested as slope coefficient of the OLS regression in which changes in spot prices is regressed on changes in futures price. Many studies have been proposed on the empirical estimation of the optimal hedge ratio employing various techniques. If the spot and futures prices are not co-integrated and the conditional variance-covariance matrix is time invariant, it has been shown that a constant optimal hedge ratio can be obtained from the slope coefficient h in the regression:

$$\Delta S_t = \alpha + h \times \Delta F_t + \varepsilon_t \qquad (1)$$

where ΔS_t and ΔF_t are the spot and futures return respectively, h is the optimal hedge ratio and ε_t is the error term in the OLS equation.

Hence, the simple method of the Ordinary Least Squares regression in which the coefficient estimate for the future price gives the hedge ratio by regressing the spot on the future price. Many researchers have defined hedging effectiveness as the extent of reduction in variances as a risk minimization problem. Using OLS regression for estimating the hedge ratio and assessing hedging effectiveness based on its R-square, has been criticized mainly on two grounds (Park and Switzer 1995). First, the hedge ratio estimated using OLS regression is based on assumption of unconditional distribution of spot and futures prices; whereas, the use of conditional distributions is more appropriate because hedging decision made by any hedger is based on all the information available at that time. Second, OLS model is based on assumption that the relationship between spot and future prices is time invariant but empirically it has been found that the joint distribution of spot and futures prices are time variant (Mandelbrot 1963). Thus, in order to improve the estimation of the hedge ratio, it is necessary to consider the possible time-varying nature of the second moments.

2.3 Time-Varying Hedge Ratio

As the constant hedge ratio is found to be inefficient, we propose to make the hedge ratio vary over the time as this will result in more accurate forecasting properties. Further, the static estimation of the hedge ratio may be downward bias due to the misspecification of the regression equation (1) as the dynamic nature of hedge ratio results in the non-whiteness of the error terms.

The time-varying hedge ratio can be written as:

$$\Delta S_t = \alpha + h_t \times \Delta F_t + u_t \qquad 2(a)$$

$$h_t = h_{t-1} + v_t, \qquad t = 1(1)T \qquad 2(b)$$

where, T is the final time period.

Equation (2) is a TVP model where the hedge ratio is assumed to vary over time. The equation 2(a) is known as the observation (or measurement) equation and the equation 2(b) as the state (or transition) equation. The state equation describes the dynamics of the coefficient h, which is assumed to follow an autoregressive process of the first degree. The error terms u and v are assumed to be independent white noise processes.

2.4 Kalman Filter Approach

State space modelling includes the State transition equation, eq. (3), which allows the state variable, $\boldsymbol{\alpha}_t$ to change through time, and the measurement equation, eq. (4), which relates the state variable to an observation Y_t.

$$\boldsymbol{\alpha}_{t+1} = \mathbf{F}_t \boldsymbol{\alpha}_t + \mathbf{G}_t \boldsymbol{\varepsilon}_t \qquad (3)$$

$$Y_t = \mathbf{H}_t' \boldsymbol{\alpha}_t + v_t \qquad (4)$$

It is assumed that $\{\varepsilon_t\}$ of eq. (3) and $\{v_t\}$ of eq. (4) are independent, zero mean, Gaussian white noise process with

$$E[v_t v_t'] = R_t \text{ and } E[\varepsilon_t \varepsilon_t'] = \mathbf{Q}_t \qquad (5)$$

The Kalman Filter (KF) is a recursive algorithm for sequentially updating the state vector given past information $\boldsymbol{\Psi}_t$.

Denote

$$\hat{\boldsymbol{\alpha}}_{t|t-1} = E\{\boldsymbol{\alpha}_t \mid \Psi_{t-1}\} \text{ and } \hat{\boldsymbol{\alpha}}_{t|t} = E\{\boldsymbol{\alpha}_t \mid \Psi_t\} \text{ for } t = 0,1,2, \ldots \qquad (6)$$

and assume $\hat{\boldsymbol{\alpha}}_{0|-1} = E\{\boldsymbol{\alpha}_0\}$ and $\boldsymbol{\Sigma}_{0|-1} = \mathbf{P}_0$. The state vector $\boldsymbol{\alpha}_t$ and its mean squared error $\boldsymbol{\Sigma}_t = E[(\boldsymbol{\alpha}_t - \hat{\boldsymbol{\alpha}}_t)(\boldsymbol{\alpha}_t - \hat{\boldsymbol{\alpha}}_t)']$ are recursively estimated by:

$$\hat{\boldsymbol{\alpha}}_{t|t} = \hat{\boldsymbol{\alpha}}_{t|t-1} + \boldsymbol{\Sigma}_{t|t-1} \mathbf{H}_t (\mathbf{H}_t' \boldsymbol{\Sigma}_{t|t-1} \mathbf{H}_t + R_t)^{-1} (Y_t - \mathbf{H}_t' \hat{\boldsymbol{\alpha}}_{t|t-1}) \qquad (7)$$

$$\boldsymbol{\Sigma}_{t|t} = \boldsymbol{\Sigma}_{t|t-1} - \boldsymbol{\Sigma}_{t|t-1} \mathbf{H}_t (\mathbf{H}_t' \boldsymbol{\Sigma}_{t|t-1} \mathbf{H}_t + \mathbf{R}_t)^{-1} \mathbf{H}_t' \boldsymbol{\Sigma}_{t|t-1} \qquad (8)$$

Using the recursive filter equations (3) and (4), we can obtain $\hat{\boldsymbol{\alpha}}_{t+1|t}$ as

$$\hat{\boldsymbol{\alpha}}_{t+1|t} = \mathbf{F}_t \hat{\boldsymbol{\alpha}}_{t|t} \qquad (9)$$

and

$$\boldsymbol{\Sigma}_{t+1|t} = \mathbf{F}_t \boldsymbol{\Sigma}_{t|t} \mathbf{F}_t' + \mathbf{G}_t \mathbf{Q}_t \mathbf{G}_t' \qquad (10)$$

Eq. (9) can also be written as

$$\hat{\alpha}_{t+1|t} = F_t\hat{\alpha}_{t|t-1} + F_t\Sigma_{t|t-1}H_t \ (H_t'\Sigma_{t|t-1}H_t + R_t)^{-1} (Y_t - H_t'\hat{\alpha}_{t|t-1}) \qquad (11)$$

Which implies that the time update rules for each forecast of state are weighted average of the previous forecast $\hat{\alpha}_{t|t-1}$ and the forecast error $(Y_t - H_t'\hat{\alpha}_{t|t-1})$. After obtaining $\hat{\alpha}_{t|t-1}$, one may predict Y_t by the optimal predictor $\hat{Y}_{t|t-1}$, where

$$\hat{Y}_{t|t-1} = H_t'\hat{\alpha}_{t|t-1} \qquad (12)$$

And the conditional error variance due to predictor $\hat{Y}_{t|t-1}$ is

$$H_t'\Sigma_{t|t-1}H_t + R_t \qquad (13)$$

3. ILLUSTRATION

Our study is based on time-series data of contracts of Soybean from 3rd November 2014 to 14th May 2015 in three markets (Indore, Nagpur and Kota) which shows considerable fluctuations leading to volatility. Hedger can go through the long hedge to protect him from the losses due to volatility in price. We apply both constant as well as time-varying method to estimate the hedge ratio.

3.1 By using Ordinary Least Square Method (OLS)

OLS regression (equation [1]) has been used to calculate the hedge ratio and hedging effectiveness. The slope of the regression equation gives the hedge ratio and R^2 gives the hedging effectiveness.

For all futures contracts, the hedge ratio is higher than 0.92 for all three futures. Hedge ratio estimated from OLS method provides variance reduction for future of all three futures which indicates that the hedge provided by these contracts is effective.

Table 1. Parameter estimation through OLS

Market	Parameter Estimable	Standard Error	Pr>|t|
Future(Indore)	0.97	0.002	<.01
Future(Nagpur)	0.92	0.003	<.01
Future (Kota)	0.97	0.002	<.01

3.2 Model Comparison

Further, hedge ratio is also estimated by using a powerful technique of Kalman filter. Here, it is assumed that the coefficients vary over time rather than a constant value.

Using the Kalman filter equation in (3) and (4), we obtain the optimal estimates of time-varying coefficients by updating the state equation For comparing purpose we have calculate MSE for both the model that is, constant and time-varying approach, and the same has been produced in Table 2.

Table 2. MSE of three types of contracts by Kalman Filter and OLS method

Markets	Time Varying (Kalman Filter)	Constant (OLS)
Indore	1575.96	13143.04
Nagpur	1432.45	11043.83
Kota	8872.76	19871.76

To study the changing hedge ratio, graph is also provided of the estimated hedge ratio of both constant coefficient as well as time-varying coefficient.

The hedge ratio is very volatile (in Fig.1) from 3rd November 2014 to 14th May 2015. Time varying hedge ratio is below the fixed hedge ratio from 5th November 2014 to 16th January 2015 and some days of May (5th,6th,13th, 14th) and above the fixed hedge ratio at rest of the time. It can be seen from Fig. 1, the hedge ratio is less than 1 for most of the period. This indicates that the gain in the futures market is equal to the loss in the cash market over this time period compare to other time period. Hedging is effective during this time period. The hedge ratio is more than 1 at rest of the period. Hedging is not effective during this time period because it shows that the gains in the futures market have been lower than loss in the stock market over this time period. If, hedger uses the constant hedge ratio over the whole time period then hedger may face losses.

In Fig. 2, by using time-varying approach, the hedge ratio is very volatile in the whole time period. Time varying hedge ratio is below the fixed hedge ratio from 5th November 2014 to 5th December 2014 and 12th December to 22nd January and above the fixed hedge ratio at rest of the time. It can be seen that the hedge ratio is less than 1 for most of the period. This indicates that the gain in the futures market is equal to the loss in the cash market over this time period. Hedging is effective during this time period. The hedge ratio is more than 1 during 15th April and 24th

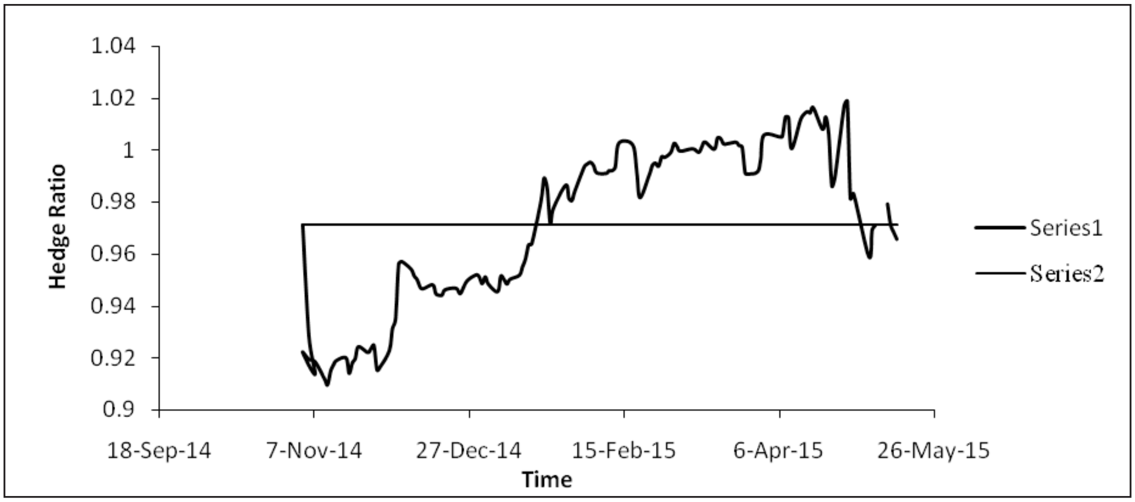

Fig. 1: Graphical Plot of Constant (series2) and Time Varying Hedge Ratio (series1) of Indore market

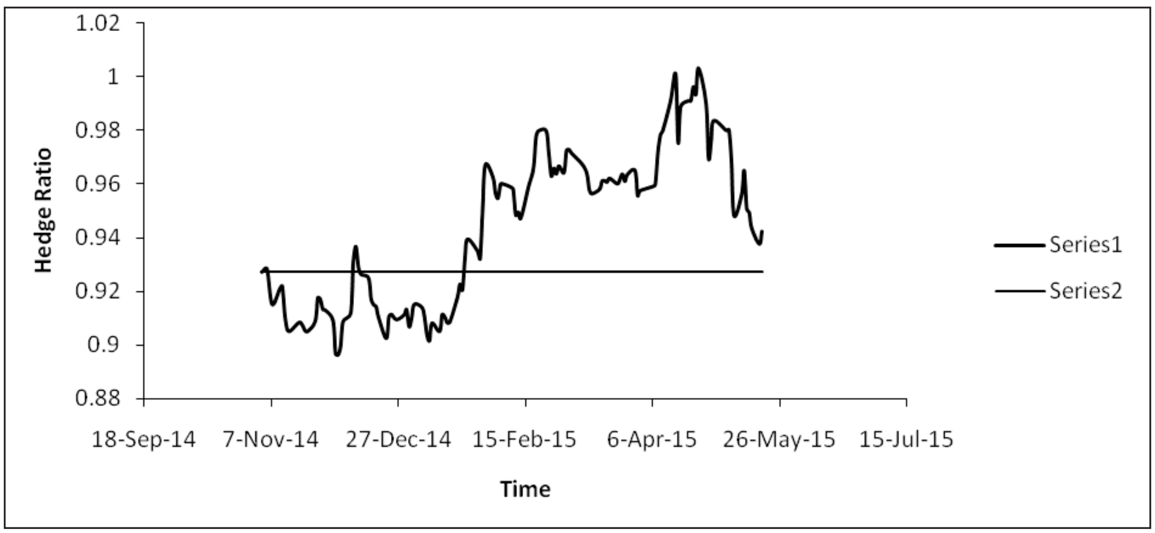

Fig. 2: Graphical Plot of Constant (series 2) and Time Varying Hedge Ratio (series 1) of Nagpur market

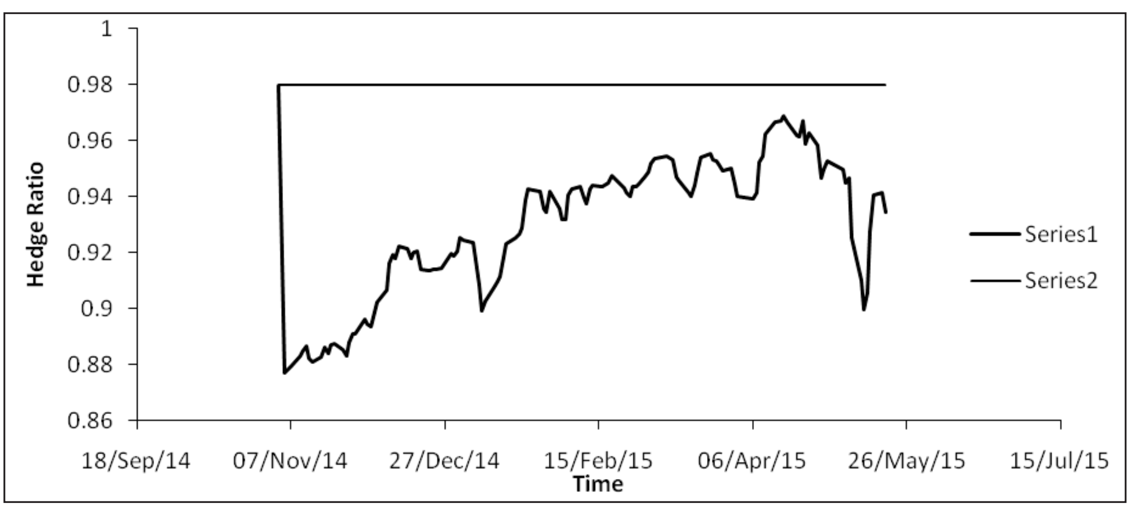

Fig. 3: Graphical Plot of Constant (series 2) and Time Varying Hedge Ratio (series 1) of Kota market

April. Hedging is effective in most of the time in Case of Nagpur Future. Hedge ratio is much lower on 1st week of December (0.89). Hedging is most effective during this period.

In Fig. 3, the hedge ratio is very volatile from 5th November to 4th December and lower volatile in rest of the time compare to previous period. Here, the time varying hedge ratio is below the fixed hedge ratio for all the period.

It can also be seen from Fig. 3, that up to 4th December, the value of the hedge ratio has a decreasing trend. This indicates that the returns in the futures market have been greater than returns in the stock market over this time period. Hedger can omit his total risk during the whole time period in Kota future market.

4. CONCLUDING REMARKS

In this article, two methods of estimation of hedge ratios are studied, constant coefficient and time-varying coefficient methods. The constant coefficient hedge ratio is easily estimated using Ordinary least square (OLS); while time-varying coefficient hedge ratio is estimated by making use of state-space methodology. The parameters of state-space model are estimated using Kalman filter, a powerful technique. For illustration purpose, three contracts of soybean are taken. It is seen that hedge ratio estimated using Kalman filter performs better than constant coefficient method. Attempts will be made in future to obtain the hedge ratio using more powerful techniques such as unscented filter, MCMC and particle filtering.

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