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JOURNAL OF THE INDIAN SOCIETY OF AGRICULTURAL STATISTICS 70(3) 2016 243–254

Long Memory in Conditional Variance

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Received 12 November 2015; Revised 12 July 2016; Accepted 22 September 2016

SUMMARY

Presence of long memory in return and volatility of the spot price of gram in Delhi market has been investigated. GPH method resulted strong evidence of long range dependence in the volatility processes for the series. Accordingly, FIGARCH model has been applied for forecasting the volatility of gram price. GARCH model and several extensions of GARCH models such as TARCH, EGARCH, Component GARCH and Asymmetric component GARCH have been applied for modelling and forecasting of return series. Evaluation of forecasting has been carried out separately in six moving windows by the help of mean squares prediction error (MSPE), mean absolute prediction error (MAPE) and relative mean absolute prediction error (RMAPE). The residuals of the fitted models were used for diagnostic checking. Diebold Mariano test was conducted for different pairs of models to test for the difference in predictive accuracy. It is found that FIGARCH model has better predictive accuracy as compared to all other models. It is also observed that component GARCH and asymmetric component GARCH models have better predictive accuracy than GARCH, TARCH and EGARCH models whereas there is no significant difference in the predictive accuracy of GARCH, TARCH and EGARCH models. The R software package has been used for data analysis.

Keywords: Conditional heteroscedasticity, Gram price, Return series, Stationarity, Validation.

1. INTRODUCTION

The concept of long-term dependency was developed by Hurst and Mandelbrot (1963). They developed the rescaled range (R/S) method to test for persistent long-term dependency. Later on in 1980's Granger and Joyeaux (1980) and Geweke and Porter-Hudak (1983) developed fractional integration as an alternative, in which the differencing parameter is allowed to be a fraction. In the literature, several studies have illustrated the existence of long-range dependency and the applicability of fractional differencing (Jin and Frechette 2004). There has been a large amount of research on long memory in economic and financial time series. For modelling the time series in presence of long memory, the autoregressive fractionally integrated moving-average (ARFIMA) model is used. ARFIMA model searches for a non-integer parameter, d, to difference the data to capture long memory. The existence of non-zero d is an indication of long memory and its departure from zero measures the strength of long memory. Paul (2014) and Paul *et al.* (2015a, 2015b) have applied ARFIMA model for forecasting of agricultural commodity prices. However, ARFIMA model is based on some crucial assumptions like linearity, stationarity and homoscedastic errors. Further, time series data quite often exhibits features like long memory in volatility; which cannot be explained by ARFIMA model. Sometime asymmetric phenomenon arises with economic series, which tend to behave differently when economy is moving into recession rather than when coming out of it. Many financial time series shows periods of stability followed by unstable periods with high volatility. Volatility is generally measured in terms of the conditional standard deviation of the underlying asset return. Modelling the volatility of a time series can improve the efficiency and the accuracy of forecast. In time series literature,

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models which attempt to explain the changes in conditional variance are generally known as conditional heteroscedastic models. Some of the volatility models that have been extensively used in the literature are Autoregressive Conditional heteroscedastic (ARCH) model of Engle (1982), Generalized ARCH (GARCH) model (Bollerslev 1986 and Taylor 1986), Exponential GARCH (EGARCH) model of Nelson (1991) and Fractionally Integrated GARCH (FIGARCH) model of Baillie *et al.* (1996). Huge amount of empirical and theoretical research work has been already done for GARCH and related models. The GARCH model assumes that negative and positive shocks of equal magnitude have identical impacts on the conditional variance. In order to accommodate differential impacts on conditional variance between positive and negative shocks, Glosten *et al.* (1992) proposed the asymmetric GARCH, or GJR model. As the positive and negative shocks on conditional volatility, called leverage effect, are asymmetric, Nelson (1991) proposed the EGARCH model. Some applications of GARCH family of models may be found in Paul *et al.* (2009), Ghosh *et al*. (2010a, 2010b) and Paul *et al*. (2014).

In terms of volatility persistence, a GARCH model features an exponential decay in the autocorrelation of conditional variances. However, a shock in the volatility series seems to have "long memory" and impacts on future volatility over a long horizon. Fung *et al.* (1994) described that a long memory process could allow conditional heteroscedasticity, which could be the explanation of non-periodic cycles. It seems a long memory model is more flexible than an ARCH model in terms of capturing irregular behaviour. Therefore, Baillie *et al.* (1996) proposed the FIGARCH (*p*, *d*, *q*) model where a full description of the properties of the process and the appropriate quasi-maximum likelihood estimation (QMLE) method can be found. Baillie *et al.* (2007) explained that the long memory refers to the presence of very slow hyperbolic decay in the autocorrelations functions. Therefore, econometrically, the long memory is between the usual exponential rates of decay associated with the class of stationary and invertible ARMA models, and the alternative extreme of infinite persistence associated with integrated, unit root processes. FIGARCH model is capable of explaining and representing the observed temporal dependencies of the financial market volatility in a much better way than other types of GARCH models (2004). Jin and Frechette (2004) applied FIGARCH model for describing fourteen agricultural future price series. When estimating the parameters of a FIGARCH model, generally, the value of parameter *d* is estimated first and one uses these estimates to obtain the estimation of other parameters (Lopes and Mendes 2006, H¨ardle and Mungo 2008). The same procedure was followed in Paul *et al*. (2015c) for forecasting agricultural commodity prices in India using ARFIMA-FIGARCH model. In the present investigation, an attempt has been made to apply different extensions of GARCH model along with FIGARCH model for modelling and forecasting of spot return price of gram in Delhi Market. The paper is organized as follows: section 2 deals with the concept of long memory process; section 3 deals with GARCH model and its important extensions; section 4 describes the details of FIGARCH model, its estimation process and forecasting and section 5 deals with the results and discussion followed by conclusions in section 6.

2. LONG MEMORY PROCESS

Long memory in time-series can be defined as autocorrelation at long lags Robinson (2003). The acf of a time-series y_t is defined as

$$\rho_k = \mathrm{cov}\left(y_t, y_{t-1}\right) / \mathrm{var}\left(y_t\right) \qquad (1)$$

for integer lag *k*. A covariance stationary time-series process is expected to have autocorrelations such that $\lim_{k\to\infty} \rho_k = 0$. Most of the well-known class of stationary and invertible time-series processes have autocorrelations that decay at the relatively fast exponential rate, so that $\rho_k \approx |m|^k$, where |m|<1 and this property is true, for example, for the well-known stationary and invertible ARMA (*p*, *q*) process. For long memory processes, the autocorrelations decay at an hyperbolic rate which is consistent with $\rho_k \approx Ck^{2d-1}$, as *k* increases without limit, where *C* is a constant and *d* is the long memory parameter.

Suppose that $\{Y_t\}$ is a stationary process with the spectral density function (SDF) denoted by $S_Y(.)$, then $\{Y_t\}$ is a stationary long memory process if there exist constants a and C_S satisfying $-1<a<0$ and $C_S>0$ such that

$$\lim_{f\to 0} S_Y(f)/(C_S|f|^a)=1 \quad (2)$$

In other words, a stationary long memory process has an SDF $S_Y(.)$ such that $S_Y(.) \approx C_S|f|^a$, with the approximation improving as f approaches zero. An alternative definition can be stated in terms of the auto covariance sequence (ACVS) $\{S_{Y,\tau}\}$ for $\{Y_t\}$. $\{Y_t\}$ is a stationary long memory process if there exist constants b and C_S satisfying $-1<b<0$ and $C_S>0$ such that

$$\lim_{\tau\to 0} S_{y,\tau}/\left(C_s\tau^b\right)=1 \quad (3)$$

where b is related to a in (2) via $b = -a-1$.

2.1 Long Memory Tests

Long memory is an important empirical feature of any financial variables. The presence of long memory in the data implies the existence of nonlinear forms of dependency between the first and the second moments, and thus the potential of time-series predictability. Testing for long memory property is an essential task since any evidence of long memory would support the use of Long Memory (LM)-based volatility models such as FIGARCH.

We test for long memory components in the return series and volatility of gram using the Geweke and Porter-Hudak (1983) (GPH) statistic. For long memory in the volatility process, this test is applied to the logarithm of squared returns series of gram, which is commonly regarded as a proxy of conditional volatility (Lobato and Savin 1998, Choi and Hammoudeh 2009).

Let r_t be the return series. The *GPH estimator* of the long memory parameter d for r_t can be then determined using the following periodogram:

$$\log\left[I(w_j)\right]=\beta_0+\beta_1\log\left[4\sin^2\left(\frac{w_j}{2}\right)\right]+\varepsilon_j \quad (4)$$

where $w_j=2\pi j/T,\ j=1,2,...,n$; ε_j is the residual term and w_j represents the $n=\sqrt{T}$ Fourier frequencies. $I(w_j)$ denotes the sample periodogram defined as

$$I(w_j)=\frac{1}{2\pi T}\left|\sum_{t=1}^{T} r_t e^{-w_j t}\right|^2$$

where r_t is assumed to be a covariance stationary time series. The estimate of d, say $\hat{d}_{GPH}$, is $-\hat{\beta}_1$.

3. GARCH MODEL

The ARCH(q) model for the series $\{\varepsilon_t\}$ is defined by specifying the conditional distribution of ε_t given the information available up to time $t-1$. Let ψ_{t-1} denote this information. ARCH (q) model for the series $\{\varepsilon_t\}$ is given by

$$\varepsilon_t \mid \psi_{t-1} \sim N(0,h_t) \quad (5)$$

$$h_t = a_0+\sum_{i=1}^{q} a_i\,\varepsilon_{t-i}^2 \quad (6)$$

where $a_0>0$, $a_i\geq 0$ for all i and $\sum_{i=1}^{q} a_i<1$ are required to be satisfied to ensure non negativity and finite unconditional variance of stationary $\{\varepsilon_t\}$ series.

Bollerslev (1986) and Taylor (1986) proposed the Generalized ARCH (GARCH) model independently of each other, in which conditional variance is also a linear function of its own lags and has the following form

$$\varepsilon_t=\xi_t h_t^{1/2}$$

$$h_t=a_0+\sum_{i=1}^{q} a_i\,\varepsilon_{t-i}^2+\sum_{j=1}^{p} b_j\,h_{t-j}$$

$$=a_0+a(L)\varepsilon_t^2+b(L)h_t \quad (7)$$

where $\xi_t \sim$ IID(0,1). A sufficient condition for the conditional variance to be positive is $a>0$, $a_i\geq 0$, $i = 1, 2, ..., q$, $b_j\geq 0$, $j = 1, 2, ..., p$ and $a(L)$ and $b(L)$ are lag operator such that $a(L) = a_1L + a_2L^2 + ... + a_qL^a$ and $b(L) = b_1L + b_2L^2 + ... + b_qL^a$. For $p = 0$, the process reduces to an ARCH(q) and for $p = q = 0$, ε_t is simply a white noise process. The GARCH (p, q) process is weakly stationary if and only if

$$\sum_{i=1}^{q} a_i+\sum_{j=1}^{p} b_j<1.$$

The conditional variance defined by (3) has the property that the unconditional acf of ε_t^2, if it exists, can decay slowly. For the ARCH family, the decay rate is too rapid compared to what is typically observed in financial time-series, unless the maximum lag q is long. As (7) is a more parsimonious model of the conditional variance than a high-order ARCH model, most users prefer it to the simpler ARCH alternative. Huge amount of empirical and theoretical research work has been already done for GARCH and related models. There are several important extensions of GARCH models such as: Threshold ARCH (TARCH) model, Exponential GARCH (EGARCH) model, Component GARCH model, Asymmetric component GARCH model etc.

3.1 Testing for ARCH Effects

Let ε_t be the residual series. The squared series $\{\varepsilon_t^2\}$ is then used to check for conditional heteroscedasticity, which is also known as the ARCH effects. The test for conditional heteroscedasticity used here is the LM test, which is equivalent to usual F-statistic for testing H_0: $a_i = 0$, $i = 1, 2, \ldots, q$ in the linear regression

$$\varepsilon_t^2 = a_0 + a_1 \varepsilon_{t-1}^2 + \ldots + a_q \varepsilon_{t-q}^2 + e_t, \quad t = q+1, \ldots, T \qquad (8)$$

where e_t denotes error term, q is prespecified positive integer, and T is sample size. Let $SSR_0 = \sum_{t=q+1}^{T} \left(\varepsilon_t^2 - \bar{\omega}\right)^2$, where $\bar{\omega} = \sum_{t=q+1}^{T} \varepsilon_t^2 / T$ is sample mean of $\{\varepsilon_t^2\}$, and $SSR_1 = \sum_{t=q+1}^{T} \hat{e}_t^2$, where $\hat{e}_t$ is least squares residual of (8). Then, under H_0:

$$F = \frac{(SSR_0 - SSR_1)/q}{SSR_1(T-q-1)}$$

is asymptotically distributed as chi-squared distribution with q degrees of freedom.

4. FIGARCH PROCESS

The GARCH (p, q) process may also be expressed as an ARMA (m, p) process in ε_t^2

$$[1 - a(L) - b(L)]\varepsilon_t^2 = a_0 + [1 - b(L)]v_t$$

where $m = \max\{p, q\}$ and $v_t = \varepsilon_t^2 - h_t$. The $\{v_t\}$ process can be interpreted as the "innovations" for the conditional variance, as it is a zero-mean martingale. Therefore, an integrated GARCH (p, q) process can be written as

$$[1 - a(L) - b(L)](1 - L)\varepsilon_t^2 = a_0 + [1 - b(L)]v_t \qquad (9)$$

The fractionally integrated GARCH or FIGARCH class of models is obtained by replacing the first difference operator $(1 - L)$ in (9) with the fractional differencing operator $(1 - L)^d$, where d is a fraction $0 < d < 1$. Thus, the FIGARCH class of models can be obtained by considering

$$[1 - a(L) - b(L)](1 - L)^d \varepsilon_t^2 = a_0 + [1 - b(L)]v_t \qquad (10)$$

Such an approach can develop a more flexible class of processes for the conditional variance that are capable of explaining and representing the observed temporal dependencies of the financial market volatility in a much better way than other types of GARCH models (Davidson 2004).

The ARFIMA (p, d, q) class of models for the discrete time real-valued process $\{y_t\}$ introduced by Granger and Joyeux (1980), Granger (1980, 1981) and Hosking (1981) is defined by

$$a(L)(1 - L)^d y_t = b(L)\xi_t \qquad (11)$$

where a(L) and b(L) are polynomials in the lag operator of orders p and q respectively, and ξ_t is a mean-zero serially uncorrelated process. For the ARFIMA models, the fractional parameter d lies between -1/2 and 1/2, (Hosking 1981). The ARFIMA model is nothing but the fractionally integrated ARMA for the mean process. Analogous to the ARFIMA(p, d, q) process defined in (11) for the mean, the FIGARCH (p, d, q) process for ε_t^2 can be defined as

$$a(L)(1 - L)^d \varepsilon_t^2 = a_0 + [1 - b(L)]v_t \qquad (12)$$

where $0 < d < 1$, and all the roots of $a(L)$ and $[1 - b(L)]$ lie outside the unit circle. In the case of ARFIMA model, the long memory operator is applied to unconditional mean μ of y_t which is constant. But this is not true in the case of FIGARCH model, where it is not applied to α_0, but on squared errors.

Rearranging the terms in (10), an alternative representation for the FIGARCH(p, d, q) model may be obtained as

$$\left[1-b(L)\right]h_t = a_0 + \left[1-b(L)-a(L)(1-L)^d\right]\varepsilon_t^2 \quad (13)$$

where, $v_t = \varepsilon_t^2 - h_t$.

4.1 Estimation of FIGARCH Model

The estimation of parameters of FIGARCH model is generally carried out using the maximum likelihood method (which is most efficient) with normality assumption for z_t. But the normality assumption can be questioned with some empirical evidence and therefore the use of quasi-maximum likelihood estimator is preferred.

The FIGARCH model is estimated by using the quasi-maximum likelihood (QML) estimation method allowing for asymptotic normality distribution, based on the following log-likelihood function

$$LL_T(\varepsilon_t,\theta) = -\frac{1}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\left[\log(h_t) + \frac{\varepsilon_t^2}{h_t}\right] \quad (14)$$

where $\theta' \equiv \left(a_0, d, b_1, b_2, \ldots, b_p, a_1, a_2, \ldots, a_q\right)$.

The likelihood function is maximized conditional on the start-up values. For the FIGARCH(p, d, q) model with $d > 0$, the population variance does not exist. In most practical applications with high frequency financial data, the standardized innovations $\xi_t = h_t^{-1/2}\varepsilon_t$ are leptokurtic and not normally distributed through time. In these situations the robust quasi-MLE (QMLE) procedures discussed by Weiss (1986) and Bollerslev and Wooldridge (1986) may give better results while doing inference. When estimating the parameters of a FIGARCH model, generally, the value of parameter d is estimated first and one uses these estimates to obtain the estimation of other parameters (Lopes and Mendes 2006, H¨ardle and Mungo 2008).

4.2 Forecasting by FIGARCH Model

Now consider the problem of forecasting using a FIGARCH model (Tayafi and Ramanathan 2012). The one-step ahead forecast of h_t is given by

$$h_t(1) = a_0\left[1-b_1\right]^{-1} + \lambda_1\varepsilon_t^2 + \lambda_2\varepsilon_{t-1}^2 + \ldots$$

where, $\lambda_k \approx \left[(1-b_1)\Gamma(d)^{-1}\right]k^{d-1}$

Similarly, the two-step ahead forecast is given by

$$h_t(2) = a_0\left[1-b_1\right]^{-1} + \lambda_1\varepsilon_{t+1}^2 + \lambda_2\varepsilon_t^2 + \ldots$$

Here ε_{t+1}^2 is unobservable and to be estimated by its conditional expectation $h_t(1)$, which is a function of past ε_t^2.

Therefore,

$$h_t(2) = a_0\left[1-b_1\right]^{-1} + \lambda_1 h_t(1) + \lambda_2\varepsilon_t^2 + \ldots$$

In general, the l-step ahead forecast is

$$h_t(l) = a_0\left[1-b_1\right]^{-1} + \lambda_1 h_t(l-1) + \ldots$$
$$+ \lambda_l h_t(1) + \lambda_l\varepsilon_t^2 + \lambda_{l+1}\varepsilon_{t-1}^2 + \ldots$$

For all practical purpose, we stop at a large M and this leads to the forecasting equation

$$h_t(l) \approx a_0\left[1-b_1\right]^{-1} + \sum_{i=1}^{l-1}\lambda_i h_t(l-i) + \sum_{j=0}^{M}\lambda_{l+j}\varepsilon_{t-j}^2$$

The parameters will have to be replaced by their corresponding estimates.

5. RESULTS AND DISCUSSION

Daily time series data for spot prices of gram in Delhi Market during 1 January, 2009 to 31 July, 2013 has been considered. The return series are computed as differences in log prices. The data is collected form Ministry of Consumer's Affairs, Government of India. The data for the period January 1, 2009 to June 30, 2013 have been used for model building and the remaining data have been used for model validation. The summary statistics for percentage return and squared percentage return series have been computed and reported in Table 1. A perusal of table 1 indicates that both series are positively skewed and platykurtic. The daily unconditional volatility of returns and the squared return, as measured by standard deviations, are 1.46 and 7.86 respectively.

The time series plot of percentage log return series and squared percentage log return series have been exhibited in Fig. 1 and 2 respectively. A perusal of the plot indicates that the dataset is stationary. In order to test for stationarity, two tests namely Augmented Dickey-Fuller unit root test and Philips-Peron unit root test are used. The results of the tests are reported in Table 2. Table 2 indicates that both the return as well as squared return spot price series data is stationary.

Table 1. Descriptive statistics for returns and squared returns

Mean	0.006	2.131
Minimum	-7.380	0.000
Maximum	17.070	290.180
Standard deviation	1.460	7.855
Skewness	0.688	26.300
Kurtosis	11.601	931.144

Table 2. Test for stationarity

Series	ADF Test	PP Test
Return series	-21.345	-40.968
Squared return series	-15.089	-40.101

5% Critical Value for ADF and PP test –2.864

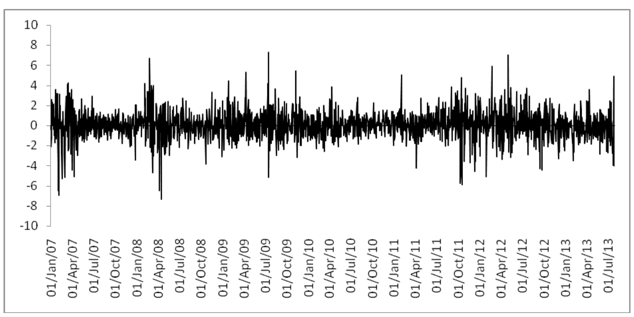


Fig. 1. Log returns series

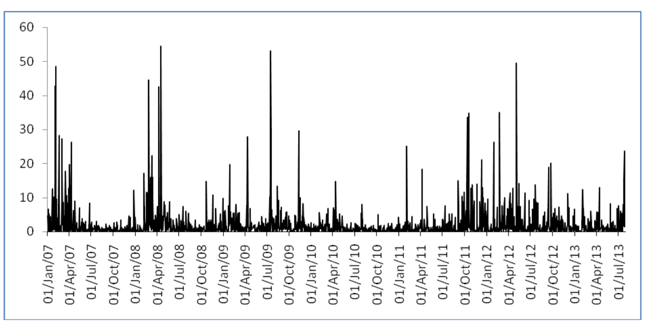


Fig. 2. Squared log returns series

Presence of ARCH effect has been tested for both the series. It is found that in squared return series; there is significant presence of ARCH effect; whereas in the return series, there is no ARCH effect.

5.1 Autocorrelation

The distributional characteristics of the return series can be investigated further by analyzing the behavior of their autocorrelation functions. The results, displayed in Fig. 3 shows that the autocorrelation functions of the returns are small and have no particular form. Most of them stay inside the 95% confidence intervals. This is suggestive of their short memory property. The autocorrelation functions of the squared returns are however larger, and they remain significant for many lags. More importantly, they exhibit a slow decay, indicating that the time series are strongly auto correlated up to a long lag. Periodogram of return and squared return series are displayed in Fig. 4.

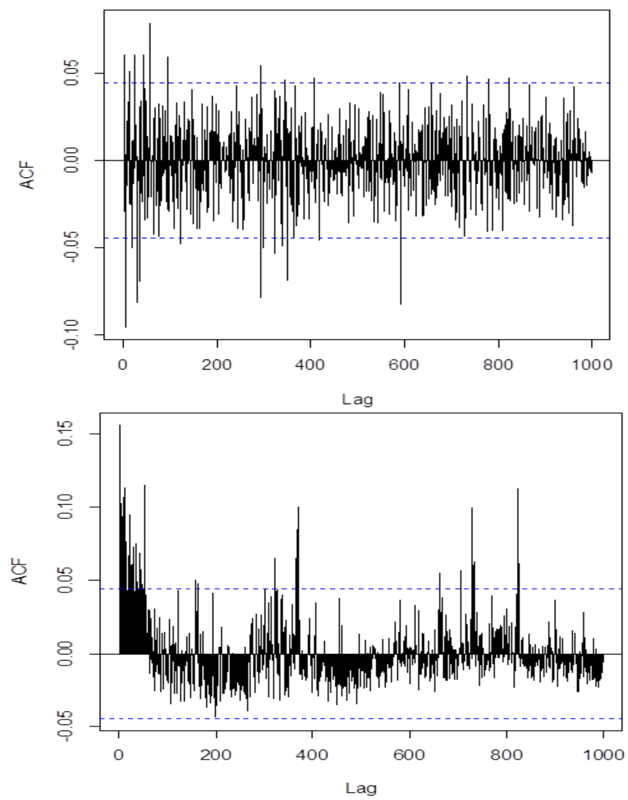
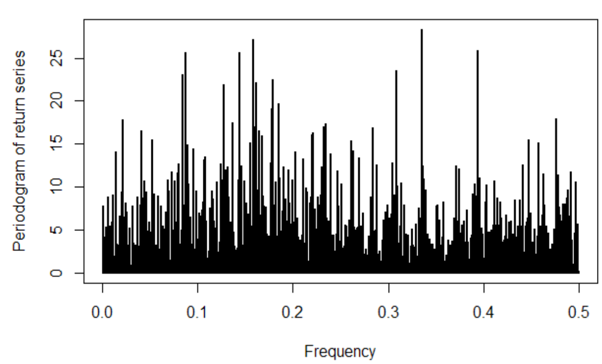


Fig. 3. Autocorrelation function for log returns and squared log returns series



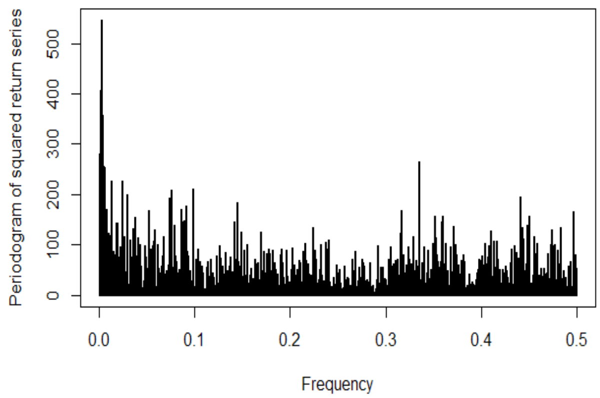


Fig. 4. Periodogram of return and squared return series

5.2 Results of long memory tests

We apply the GPH tests for testing long memory to the raw and squared returns of the spot prices of gram. The obtained results are reported in Table 3. For the (raw) return series, the test shows no evidence of LM patterns for return series; as the null hypothesis of no persistence is not rejected.

Table 3. Results of LM tests for returns and squared returns

Long Memory Parameter	Return	Squared Return
D	0.000045	0.3163
SE	0.000039	0.00007
Z	1.159	4060
P-value	0.246	<0.001

The result for squared return is different from that of the returns. Indeed, long memory property is found to be highly significant for the squared returns. Since squared returns are a good proxy for volatility, these findings thus suggest that the conditional volatility of return would tend to be range-dependent, persist and decay slowly. Intuitively, this volatility persistence can be appropriately modeled by a FIGARCH process because it allows for long memory behavior and slow decay of the impact of a volatility shock.

It is, however, important to note that the estimate of the LM parameter d is less than 0.5 for squared return indicating the stationarity of the process.

5.3 Fitting of different extension of GARCH model and FIGARCH model

Important extensions of GARCH models like TARCH model, EGARCH model, Component GARCH model, Asymmetric component GARCH model and FIGARCH model have been explored for modelling the return series. Minimum Schwarzbayesian information criteria (SBIC) value has been used for choosing the best model. The models fitted for the present data sets are AR(1)-GARCH(1,1) Model, AR(1)-TARCH(1,1) Model, AR(1)-EGARCH(1,1) Model, AR(1)-Component GARCH(1,1) Model, AR(1)-Asymmetric component GARCH(1,1) Model andAR(1)-FIGARCH(1,d,1).The parameters estimates of above fitted models are reported in Table 4 and 5. A perusal of Table 4 and 5 indicate that, all the parameters are statistically significant. The long memory parameter, d is less than 0.5 ensures the stationarity of the model.

Table 4. Parameter estimate of GARCH family of models

	Coefficient	Std. Error	z-Statistic	Probability
AR(1)-GARCH(1,1) Model				
Mean Equation				
C	0.012	0.034	0.374	0.700
AR(1)	0.108	0.024	4.438	<0.001
Variance Equation				
C	0.035	0.007	4.958	<0.001
ARCH(1)	0.070	0.007	9.668	<0.001
GARCH(1)	0.912	0.007	117.681	<0.001
AR(1)-TARCH(1,1) Model				
Mean Equation				
C	0.031	0.033	0.954	0.340
AR(1)	0.110	0.023	4.762	<0.001
Variance Equation				
C	-0.092	0.010	-9.007	<0.001
|RES|/ SQR[GARCH](1)	0.133	0.014	9.460	<0.001
RES/SQR [GARCH](1)	0.033	0.007	4.494	<0.001
EGARCH(1)	0.983	0.003	283.885	<0.001
AR(1)-EGARCH(1,1) Model				
Mean Equation				
C	0.025	0.034	0.743	0.457
AR(1)	0.109	0.024	4.482	<0.001
Variance Equation				
C	0.032	0.006	4.981	<0.001
ARCH(1)	0.080	0.008	9.346	<0.001
(RESID<0)* ARCH(1)	-0.030	0.010	-2.962	0.003
GARCH(1)	0.918	0.007	117.478	<0.001
AR(1)-Component GARCH(1,1) Model				
Mean Equation				
C	0.019	0.033	0.577	0.563
AR(1)	0.113	0.026	4.232	<0.001

Variance Equation				
Perm: C	2.013	0.324	6.200	<0.001
Perm: [Q-C]	0.989	0.003	286.120	<0.001
Perm: [ARCH-GARCH]	0.048	0.008	5.634	<0.001
Tran: [ARCH-Q]	0.078	0.020	3.773	<0.001
Tran: [GARCH-Q]	0.591	0.127	4.649	<0.001
AR(1)-Asymmetric component GARCH(1,1) Model				
Mean Equation				
C	0.021	0.033	0.632	0.526
AR(1)	0.114	0.026	4.333	<0.001
Variance Equation				
Perm: C	1.994	0.327	6.094	<0.001
Perm: [Q-C]	0.989	0.003	284.277	<0.001
Perm: [ARCH-GARCH]	0.048	0.008	5.524	<0.001
Tran: [ARCH-Q]	0.090	0.024	3.642	0.0003
Tran: (RES<0)* [ARCH-Q]	-0.032	0.029	-1.118	0.2632

Table 5. Parameter estimate of AR(1)-FIGARCH (1, *d*, 1) model

Mean Equation				
	Coefficient	**Std. Error**	***t*-Statistic**	**Probability**
Constant	0.016	0.032	0.513	0.608
AR(1)	0.111	0.026	4.333	<0.001
Variance equation				
Constant	0.160	0.063	2.529	0.011
d-Figarch	0.418	0.081	5.156	<0.001
ARCH (Phi1)	0.303	0.072	4.216	<0.001
GARCH (Beta1)	0.631	0.097	6.506	<0.001

5.4 Diagnostic Checking

The model verification is concerned with checking the residuals of the model to see if they contained any systematic pattern which still could be removed to improve the chosen FIGARCH Model. This has been done through examining the autocorrelations and partial autocorrelations of the residuals of various lags. For this purpose, autocorrelations of the residuals were computed and it was found that none of these autocorrelations was significantly different from zero at any reasonable level. This proved that the selected FIGARCH model was an appropriate model for capturing the volatility present in the data under study.

5.5 Validation

One-step ahead moving window forecasts of percentage log return series for the period July 01, 2013 to July 31, 2013 (total 27 data points excluding market holidays) in respect of above

Table 6. Validation of models

Model	**MAE**	**MSPE**	**RMAPE (%)**	**MAE**	**MSPE**	**RMAPE (%)**	**MAE**	**MSPE**	**RMAPE (%)**
	5-step ahead			**10-step ahead**			**15-step ahead**		
GARCH	0.182	0.053	11.14	0.165	0.044	11.20	0.135	0.028	11.40
TARCH	0.183	0.054	11.76	0.165	0.045	11.80	0.138	0.028	12.40
EGARCH	0.182	0.053	11.49	0.165	0.044	11.50	0.136	0.028	12.00
Component GARCH	0.189	0.057	11.72	0.171	0.047	11.70	0.141	0.030	12.10
Asymmetric Component GARCH	0.192	0.059	11.94	0.173	0.049	12.00	0.143	0.031	12.30
FIGARCH	0.162	0.042	8.92	0.143	0.032	8.70	0.126	0.027	8.80
	20-step ahead			**25-step ahead**			**27-step ahead**		
GARCH	0.129	0.025	10.90	0.139	0.026	12.10	0.136	0.027	12.50
TARCH	0.132	0.027	11.20	0.148	0.028	14.00	0.149	0.029	15.10
EGARCH	0.131	0.026	11.00	0.145	0.027	13.30	0.145	0.028	14.20
Component GARCH	0.134	0.027	11.40	0.147	0.029	13.10	0.145	0.029	13.80
Asymmetric Component GARCH	0.137	0.028	11.40	0.150	0.030	13.50	0.148	0.030	14.20
FIGARCH	0.120	0.024	9.10	0.125	0.025	9.60	0.112	0.022	8.80

fitted model are computed. Total 6 moving windows have been considered: 5-step, 10-step, 15-step, 20-step, 25-step and 27-step ahead. For measuring the accuracy in fitted time series model, Mean square prediction error (RMSPE), Mean absolute prediction error (MAPE) and Relative mean absolute prediction error (RMAPE) are computed by using the formulae given below and are reported in Table 6.

$$\text{MAPE} = 1/h\sum_{i=1}^{h}\left|y_{t+i} - \hat{y}_{t+i}\right|$$

$$\text{MSPE} = 1/h\sum_{i=1}^{h}\left\{\left(y_{t+i} - \hat{y}_{t+i}\right)^2\right\}$$

$$\text{RMAPE} = 1/h\sum_{i=1}^{h}\left\{\left|y_{t+i} - \hat{y}_{t+i}\right| / y_{t+i}\right\}\times 100$$

where, h denotes the window length.

A perusal of Table 6 depicts that, irrespective of criteria used for model evaluation, FIGARCH model outperforms other models considered in this paper.

5.6 Diebold-Mariano Test

Diebold-Mariano test (Diebold and Mariano 1995) has also been applied for comparison of forecasting performance between different extension of GARCH model and FIGARCH model. A brief description of the test is given below. Let $\{y_t\}$ denote the series to be forecast and let $y^1_{t+h|t}$ and $y^2_{t+h|t}$ denote two competing forecasts of y_{t+h} based on information up to time t. The forecast errors from the two models are $\varepsilon^1_{t+h|t} = y_{t+h} - y^1_{t+h|t}$ and $\varepsilon^2_{t+h|t} = y_{t+h} - y^2_{t+h|t}$ The accuracy of each forecast is measured by a particular loss function $L\left(y_{t+h}, y^i_{t+h|t}\right) = L\left(\varepsilon^i_{t+h|t}\right)$, $i = 1, 2$.

Some popular loss functions are

Squared error loss: $L\left(\varepsilon^i_{t+h|t}\right) = \left(\varepsilon^i_{t+h|t}\right)^2$

Absolute error loss: $L\left(\varepsilon^i_{t+h|t}\right) = \left|\varepsilon^i_{t+h|t}\right|$

To determine if one model predicts better than another we may test null hypotheses

$$H_0 : E\left[L\left(\varepsilon^1_{t+h|t}\right)\right] = E\left[L\left(\varepsilon^2_{t+h|t}\right)\right]$$

against the alternative

$$H_1 : E\left[L\left(\varepsilon^1_{t+h|t}\right)\right] \neq E\left[L\left(\varepsilon^2_{t+h|t}\right)\right]$$

The Diebold-Mariano test is based on the loss differential $L\left(\varepsilon^1_{t+h|t}\right) - L\left(\varepsilon^2_{t+h|t}\right)$

The null of equal predictive accuracy is then H_0: $E[d_t]=0$

The Diebold-Mariano test statistic is

$$S = \frac{\bar{d}}{\left(LRV_{\bar{d}} / T\right)^{1/2}}$$

where

$$LRV_{\bar{d}} = \gamma_0 + 2\sum_{j=1}^{\infty}\gamma_j, \qquad \gamma_j = \text{cov}\left(d_t, d_{t-j}\right)$$

$LRV_{\bar{d}}$ is a consistent estimate of the asymptotic (long-run) variance of $\sqrt{T}\bar{d}$. Diebold and Mariano (1995) showed that under the null of equal predictive accuracy S ~ $N(0, 1)$.

The results of the test for different pairs of models are reported in Table 7. It is clear that FIGARCH model has better predictive accuracy as compared to all other models explored in this paper. It is also observed that component GARCH and asymmetric component GARCH models have better predictive accuracy than GARCH, TARCH and EGARCH models whereas there is no significant difference in the predictive accuracy of GARCH, TARCH and EGARCH models.

Table 7. Testing predictive accuracy by D-M test

Null-Hypothesis	Alternate Hypothesis	D-M Statistic	P Value
Predictive accuracy of GARCH and FIGARCH is equal	FIGARCH has better predictive accuracy than GARCH	3.430	<0.001
Predictive accuracy of TARCH and FIGARCH is equal	FIGARCH has better predictive accuracy than TARCH	2.119	0.017
Predictive accuracy of EGARCH and FIGARCH is equal	FIGARCH has better predictive accuracy than EGARCH	3.366	>0.001

Cont Table

Cont Table

Predictive accuracy of Component GARCH and FIGARCH is equal	FIGARCH has better predictive accuracy than Component GARCH	2.952	0.004
Predictive accuracy of Asymmetric Component GARCH and FIGARCH is equal	FIGARCH has better predictive accuracy than Asymmetric Component GARCH	3.291	<0.001
Predictive accuracy of GARCH and TARCH is equal	TARCH has better predictive accuracy than GARCH	-0.227	0.5897
Predictive accuracy of GARCH and EGARCH is equal	EGARCH has better predictive accuracy than GARCH	-0.570	0.7157
Predictive accuracy of GARCH and Component GARCH is equal	Component GARCH has better predictive accuracy than GARCH	3.245	0.0005
Predictive accuracy of GARCH and Asymmetric Component GARCH is equal	Asymmetric Component GARCH has better predictive accuracy than GARCH	3.352	0.0004
Predictive accuracy of TARCH and EGARCH is equal	EGARCH has better predictive accuracy than TARCH	-0.435	0.6682
Predictive accuracy of TARCH and Component GARCH is equal	Component GARCH has better predictive accuracy than TARCH	3.442	0.00029
Predictive accuracy of TARCH and Asymmetric Component GARCH is equal	Asymmetric Component GARCH has better predictive accuracy than TARCH	4.210	<0.001
Predictive accuracy of EGARCH and Component GARCH is equal	Component GARCH has better predictive accuracy than EGARCH	4.417	<0.001
Predictive accuracy of EGARCH and Asymmetric Component GARCH is equal	Asymmetric Component GARCH has better predictive accuracy than EGARCH	4.549	<0.001
Predictive accuracy of Component GARCH and Asymmetric Component GARCH is equal	Asymmetric Component GARCH has better predictive accuracy than Component GARCH	3.616	<0.001

Table cont.

To have a visual ideal of model fitting performance of different models, the graphs of squared residuals and estimated conditional variance of individual model has been displayed in Figs. 5 to 10. It is to be noted here that, as depicted by figure 10, FIGARCH model captures the volatility more accurately as compared to other models.

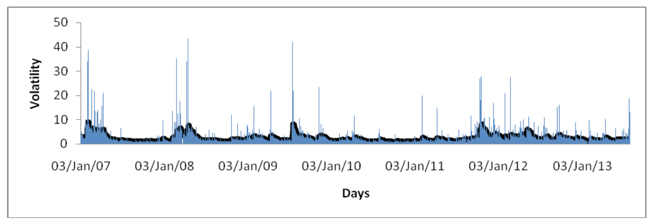


Fig. 5. Squared residuals vs. conditional variance of fitted AR(1)-GARCH(1,1) model

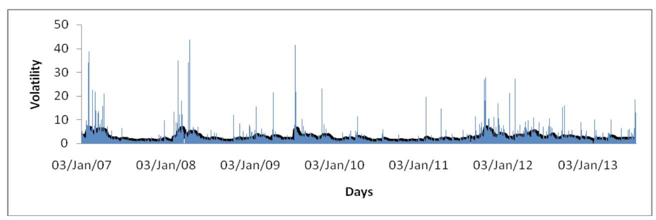


Fig. 6. Squared residuals vs. conditional variance of fitted AR(1)-TARCH(1,1) model

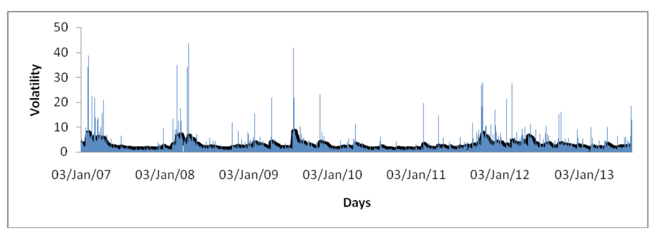


Fig. 7. Squared residuals vs. conditional variance of fitted AR(1)-EGARCH(1,1) model

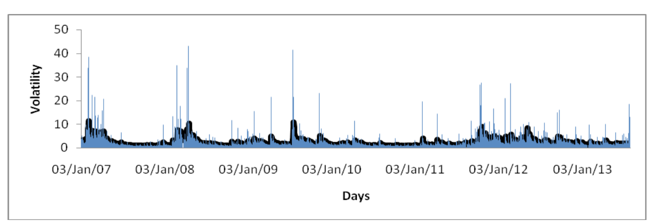


Fig. 8. Squared residuals vs. conditional variance of fitted AR(1)-component GARCH (1,1) model

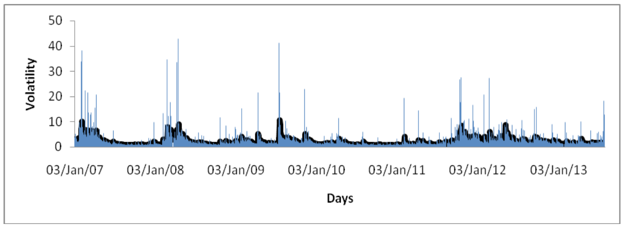


Fig. 9. Squared residuals vs. conditional variance of fitted AR(1)-asymmetric component GARCH (1,1) model

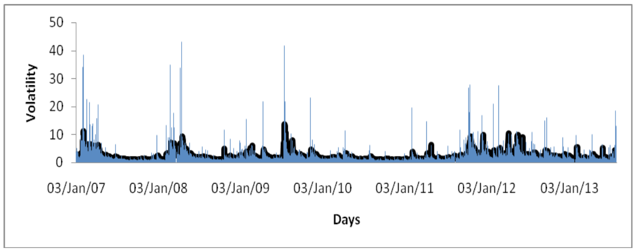


Fig. 10. Squared residuals vs. conditional variance of fitted AR(1)-FIGARCH (1,0.418,1) model

6. CONCLUSION

Several papers in the literature have addressed the issue of volatility modeling for agricultural commodity prices, but very few of them have explicitly investigated the nature and causes of the observed volatility persistence. The present investigation is aimed to fill this gap by testing the relevance of long memory in modeling the return and volatility for the spot prices of gram. GPH test indicated the existence of long-term memory in the volatility processes. Several extensions of GARCH model along with FIGARCH were fitted to the present data. The sample ACFs of the volatility processes decay hyperbolically as the lag increases, indicating long-term memory exists in the squared log return seriesWe find that long memory is particularly strong and plays a dominant role in explaining the spot price return of Gram. Finally, our out-of-sample analysis using six moving windows indicates that the FIGARCH-based model outperforms other extensions of GARCH models in terms of MASPE, MAPE and RMAPE. To this end, Diebold-Mariano test was conducted to see the significant difference in the predictive accuracy of different models. Based on the analysis it can be concluded that, FIGARCH model has better predictive accuracy as compared to all other models applied here in the present data set. It is also observed that predictive accuracy of component GARCH and asymmetric component GARCH model are better than GARCH, TARCH and EGARCH model whereas there is no significant difference in the predictive accuracy of GARCH, TARCH and EGARCH models as far as the data under consideration.

ACKNOWLEDGEMENT

The authors are thankful to the reviewer for giving some constructive comments which in turn have helped in improving the quality of the manuscript.

REFERENCES

Baillie, R.T., Bollerslev, T. and Mikkelsen, H.O. (1996). Fractionally integrated generalized autoregressive conditional heteroskedasticity. *J. Econ.,* **74**, 3-30.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *J. Econ.,* **31**, 307-327.

Bollerslev, T. and Wooldridge, J.M. (1992). Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances. *Econ. Rev.*, **11**, 143-172.

Choi, K. and Hammoudeh, S. (2009). Long memory in oil and refined products markets. *Energy J.,* **30**, 97-116.

Davidson, J. (2004). Moment and memory properties of linear conditional heteroscedasticity models, and a new model. *J. Busi. Eco. Statist.*, **22**, 16-190.

Diebold, F.X. and Mariano, R.S. (1995). Comparing predictive accuracy. *J. Busi. Eco. Statist.,* **13**, 253-263.

Engle, R.F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of U.K. inflation. *Econometrica,* **50**, 987-1008.

Fung, H.G., Lo, W.C. and Peterson, J.E. (1994). Examining the dependence in intra-day stock index futures. *J. Futures Markets*. **14,** 405-419.

Geweke, J.P. and Porter-Hudack, S. (1983). The estimation and application of long memory time series models. *J. Time Series Anal.*, **4**, 221-238.

Ghosh, H., Paul, R.K. and Prajneshu (2010a). Nonlinear time series modeling and forecasting for periodic and ARCH effects. *J. Stat. Theory Pract.*, **4(1),** 27-44.

Ghosh, H., Paul, R.K. and Prajneshu (2010b). The GARCH and EGARCH nonlinear time-series models for volatile data: An application. *J. Statist. Appl.*, **5**(**2**), 177-193.

Granger, C.W.J. (1980). Long memory relationships and the aggregation of dynamic models. *J. Econ.*, **14**, 227-238.

Granger, C.W.J. (1981). Some properties of time-series data and their use in econometric model specification. *J. Econ.*, **16**, 121-130.

Granger, C.W.J. and Joyeux, R. (1980). An introduction to long-memory time-series models and fractional differencing. *J. Time-Series Anal.,* **4**, 221-238.

H¨ardle, W.K., and Mungo, J. (2008). Long memory persistence in the factor of implied volatility dynamics. *Inter. Res. J. Finance Eco.*, **18,** 213-230.

Hosking, J.R.M. (1981). Fractional differencing. *Biometrika,* **68**, 165-176.

Jin, H.J. and Frechette, D. (2004). Fractional integration in agricultural futures price volatilities. *Amer. J. Agric. Eco.*, **86,** 432-443.

Lobato, I.N. and Savin, N.E. (1998). Real and spurious long memory properties of stock market data. *J. Busi. Eco. Statist.*, **16**, 261-268.

Lopes, S.R.C., and Mendes, B.V.M. (2006). Bandwidth selection in classical and robust estimation of long memory. *Inter. J. Statist. Sys.*, **1**, 107-190.

Nelson, D. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica*, **59**, 347-370.

Paul, R.K., Prajneshu, and Ghosh, H. (2009). GARCH nonlinear time series analysis for modeling and forecasting of India's volatile spices export data. *J. Ind. Soc. Agril. Statist.*, **63(2),** 123-131.

Paul, R.K. (2014). Forecasting wholesale price of pigeon pea using long memory time-series models. *Agril. Eco. Res. Rev.*, **27(2),** 167-176.

Paul, R.K., Ghosh, H. and Prajneshu (2014). Development of out-of-sample forecast formulae for ARIMAX-GARCH model and their application. *J. Ind. Soc. Agril. Statist.*, **68(1),** 85-92.

Paul, R.K., Gurung, B. and Paul, A.K. (2015a). Modelling and forecasting of retail price of arhar dal in Karnal, Haryana. *Ind. J. Agril. Sci.*, **85(1),** 69-72.

Paul, R.K., Samanta, S. and Gurung, B. (2015b). Monte Carlo simulation for comparison of different estimators of long memory parameter: An application of ARFIMA model for forecasting commodity price. *Model Assisted Statist. Appl.*, **10(2),** 116-127.

Paul, R.K., Gurung, B, and Samanta, S. (2015c). Analyzing the effect of dual long memory process in forecasting agricultural prices in different markets of india. *Inter. J. Empirical Finance*, **4(4),** 235-249.

Robinson, P.M. (2003). *Time Series with Long Memory*. Oxford University Press, Oxford.

Tayafi, M. and Ramanathan, T.V. (2012). An overview of FIGARCH and related time-series models. *Austr. J. Statist.*, **141,** 175-196.

Taylor, S. (1986). *Modeling Financial Time-series*. John Wiley and Sons, Inc. New York.

Weiss, A. (1986). Asymptotic theory for ARCH models: Estimation and testing. *Econometric Theory*, **2**, 101-103.