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Comparison of Taylor's Series Approximation with Piecewise Linear Approximation in Obtaining an Optimum Multivariate Stratified Sampling Design: A Fuzzy Goal Programming Approach

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SUMMARY

In this paper a compromise allocation is obtained in multivariate stratified sampling survey. The problem has been formulated as a multiobjective nonlinear programming problem and approximated to a linear programming problem (LPP) using two separate methods (i) Piecewise linear approximation method and (ii) Taylor series approximation. The solutions of both the LPP have been obtained through fuzzy goal programming. A numerical example is also provided to show the applicability of the methods. A comparison of the approaches has also been made.

Keywords: Compromise allocation, Coefficient of variation, Multivariate stratified sampling, Fuzzy goal programming, Piecewise linear approximation.

1. INTRODUCTION

Optimum allocation problem was first discussed by Neyman (1934) for univariate stratified sampling. Usually, more than one variables are defined on each unit of the population. In such cases the problem of allocation becomes more complicated because usually an allocation which is optimum for one variable is not optimum for others, unless the variables are strongly correlated. A compromise criterion is then needed that gives an allocation, which is optimum for all characteristics in some sense. Many authors have discussed criteria for obtaining a usable compromise allocation. Among them are Neyman (1934), Dalenius (1957), Ghosh (1958), Yates (1960), Aoyama (1963), Folks and Antle (1965), Ahsan and Khan (1977, 1982), Bethel (1989), Jahan *et al*. (1994), Khan *et al*. (1997), Ansari *et al*. (2009).

Goal Programming is one of the most used and well known decision making techniques. Goal Programming was introduced by Charnes and Cooper (1961). The interesting philosophy and high applicability of GP in handling real world decision making problems with multiobjectives structures made it very useful and widespread. This leads to further development of GP for different decision making problems. Goal Programming approach in fuzzy environment has been first introduced by Narasimhan (1980). In this model the fuzzy goals are characterized by introducing associated tolerance bounds. The membership functions for each character are then rehabilitated into membership goals by taking maximum possible membership value means unity as the target level and introducing lower and upper deviational variables to each of them.

The method of separable programming was first introduced by Miller (1963). Nonlinear separable programming is that in which both the objective and constraint functions can be expressed as the sum of one variable nonlinear functions, and each nonlinear term

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involves only one variable. The scheme is to replace each nonlinear function with a piecewise linear approximation (see Bazaraa *et al*. 1993).

This paper deals with the problem of obtaining a usable compromise allocation by minimizing the sum of coefficient of variation (*CV*) subject to the budgetary and other restrictions. The benefit of using the *CV's* is that they are unit free therefore, they can be added for minimization. The problem is formulated as a multiobjective non linear programming problem and the solution is obtained through fuzzy goal programming. The nonlinear membership goals are transformed into linear membership goals by using piecewise linear approximation and Taylor series approximation. A comparison of the two methods of approximations shows that the Taylor's series approximation gives better result than the piecewise linear approximation. A numerical example is also provided to illustrate the computational details.

2. FORMULATION OF THE PROBLEM

Let a population of size N be divided into L strata of sizes N_k; $k = 1, 2, ..., L$. Let independent without replacement simple random samples of sizes n_k; $k = 1, 2, ..., L$ be drawn from the k^{th} strata. Further let p characteristics be defined on each unit of the population.

Notations

For k^{th} stratum and j^{th} characteristics

N_k - No. of units in the k^{th} stratum

n_k - No. of sampled units from k^{th} stratum

y_{jki} - Value of the i^{th} unit from k^{th} stratum for j^{th} characteristics

$$\bar{Y}_{jk} = \frac{1}{N_k}\sum_{i=1}^{N_k} y_{jki}$$

$$\bar{y}_{jk} = \frac{1}{n_k}\sum_{i=1}^{n_k} y_{jki}$$

and $W_k = \frac{N_k}{N}$ - Stratum weight

Further, define

$S_{jk}^2 = \frac{1}{N_k - 1}\sum_{i=1}^{N_k}(y_{jki} - \bar{Y}_{jk})^2$ - Stratum variance

$\bar{Y}_j = \frac{1}{N}\sum_{k=1}^{L}\sum_{k=1}^{N_k} y_{jki} = \sum_{k=1}^{L} W_k \bar{Y}_{jk}$ - Population mean of the j^{th} charateristics

$\bar{y}_{st,j} = \frac{1}{n}\sum_{k=1}^{L}\sum_{i=1}^{n_k} y_{jki} = \sum_{k=1}^{L} W_k \bar{y}_{jk}$ -Stratified sampling mean for the j^{th} characteristics

$V(\bar{y}_{st,j}) = \sum_{k=1}^{L} W_k^2\left(\frac{1}{n_k} - \frac{1}{N_k}\right)S_{jk}^2$ - Sampling variance of stratified sampling mean (1)

Assuming a linear cost function, the total cost C is given as

$$C = c_0 + \sum_{k=1}^{L} c_k n_k \tag{2}$$

where

c_0 - overhead cost

c_k - per unit cost of measurement in the k^{th} stratum

To obtain the optimum compromise solution with a linear cost given in (2), we have to solve the following multiobjective non-linear programming problem (MNLPP)

$$\left.\begin{array}{ll} \text{Minimize} & \begin{pmatrix} (CV)_1^2 \\ \vdots \\ (CV)_p^2 \end{pmatrix} \\ \text{Subject to} & \sum_{k=1}^{L} c_k n_k \le C_0 \\ \text{and} & 2 \le n_k \le N_k\,;\ k = 1, 2, \ldots L \end{array}\right\} \tag{3}$$

where $C_0 = C - c_0$ and $(CV)_j^2$ is the squared population coefficient of variation of $\bar{y}_{st,j}$, that is,

$$(CV)_j^2 = (CV(\bar{y}_{st,j}))^2$$

$$= \frac{V(\bar{y}_{st,j})}{\bar{Y}_j^2}$$

$$= \frac{\sum_{k=1}^{L} W_k^2\left(\frac{1}{n_k} - \frac{1}{N_k}\right)S_{jk}^2}{\bar{Y}_j^2};\ j = 1, 2, ..., p \tag{4}$$

The individual optimum allocations that minimize the squared CV for fixed cost are the solutions to the following p NLPP's

$$\left.\begin{aligned} &\text{Minimize} \quad \frac{\sum_{k=1}^{L} W_k^2 \left(\frac{1}{n_k} - \frac{1}{N_k}\right) S_{jk}^2}{\bar{Y}_j^2} \\ &\text{Subject to} \quad \sum_{k=1}^{L} c_k n_k \le C_0 \\ &\text{and} \quad 2 \le n_k \le N_k \,;\, k = 1, 2, \ldots L \end{aligned}\right\} j = 1, 2, \ldots, p \tag{5}$$

Ignoring the terms independent of n_k, we get

$$\left.\begin{aligned} &\text{Minimize} \quad Z_j = \frac{\sum_{k=1}^{L} W_k^2 S_{jk}^2}{\bar{Y}_j^2} \\ &\text{Subject to} \quad \sum_{k=1}^{L} c_k n_k \le C_0 \\ &\text{and} \quad 2 \le n_k \le N_k \,;\, k = 1, 2, \ldots L \end{aligned}\right\} j = 1, 2, \ldots, p \tag{6}$$

3. FUZZY GOAL PROGRAMMING FORMULATION

First, we formulate the fuzzy programming model of NLLP in (6) by transforming the objective function Z_j into fuzzy goals by means of assigning an imprecise aspiration level for each objective. Let Z_j^* be the optimal solution of the objective function. Then the fuzzy goals appear in the form

$$Z_1 \tilde{\ge} Z_1^*$$
$$\vdots$$
$$Z_j \tilde{\ge} Z_j^*$$

Using the individual best solution, we find the upper and lower tolerance limit U_j and L_j for each objective function.

The fuzzy goals are characterized by their membership functions. The membership function of objective function for each characteristic is given below

$$\mu_j(\bar{n}) = \begin{cases} 1 & \text{if } Z_j(\bar{n}) \le L_j \\ \dfrac{U_j - Z_j(\bar{n})}{U_j - L_j} & \text{if } L_j \le Z_j(\bar{n}) \le U_j \\ 0 & \text{if } Z_j(\bar{n}) \ge U_j \end{cases}$$

$$j = 1, 2, \ldots, p \tag{7}$$

In the fuzzy goal programming (FGP) formulation, the defined membership function in equation (7) for each characteristics are transformed into membership goals by introducing under and over deviational variables and assigning the highest membership value means unity as the aspiration level to each of them.

Under the above circumstances, the membership goals are

$$\frac{U_j = Z_j(\bar{n})}{U_j - L_j} + d_j^- - d_j^+ = 1;\; j = 1, 2, \ldots, p \tag{8}$$

where $d_j^+, d_j^- \ge 0$ represent the under and over deviational variables.

The weighted fuzzy goal programming objective will then be

$$\text{minimize } Z = \sum_{j=1}^{p} w_j d_j^-$$

Subject to the goal expression in equation (8) and the system constraints (6), where $w_k > 0$ is the weight of unwanted deviational variable associated with the j^{th} goal as defined by Pal *et al*. (2003) as

$$w_j = \frac{1}{U_j - L_j};\; j = 1, 2, \ldots, p$$

(see Pal *et al*. 2003).

4. PIECEWISE LINEARIZATION OF NONLINEAR GOALS

The goal in (8) can be written as

$$\frac{1}{U_j - L_j}\left[U_j - \sum_{k=1}^{L} f_{jk}(n_k)\right] + d_j^- - d_j^+ = 1$$

To linearize the non linear function $f_{jk}(n_k)$, the grid points for the variable n_k (k = 1, 2, ... L) are chosen as a_{kr} (r = 0, 1, ..., r_k). Further, we introduced a new variable γ_{kr} (r = 0, 1, ..., r_k), so that n_k can be expressed as

$$n_k = \sum_{r=0}^{r_k} a_{kr}\, \gamma_{kr}$$

where $\sum_{r=0}^{rk} \gamma_{kr} = 1\, (\gamma_{kr} \ge 0)$ with grid points a_{k0}, a_{k1}, ..., a_{kr_k}.

So, the piecewise linear form of nonlinear function f_{jk} (n_k) nominated as F_{jk} can be written as

$$F_{jk} = \sum_{r=0}^{n_k} \gamma_{kr} f_{jk}(a_{kr}) \tag{9}$$

Then, the linear FGP model by using expression in (9) of the MNLPP (6) can be written as

$$\left.\begin{aligned} &\textit{Minimize } Z = \sum_{j=1}^{p} w_j d_j^- \\ &\textit{so as to satisfy} \\ &\qquad \frac{1}{U_j - L_j}\left[U_j - \sum_{k=1}^{L} F_{jk}\right] + d_j^- - d_j^+ = 1 \\ &\qquad \textit{where } F_{jk} = \sum_{r=0}^{n_k} \gamma_{kr} f_{jk}(a_{kr}) \\ &\textit{Subject to } \sum_{k=1}^{L} c_k \sum_{r=0}^{n_k} a_{kr}\gamma_{jk} \le C_0 \\ &\qquad \sum_{r=0}^{n_k} \gamma_{kr} = 1 (\gamma_{kr} \ge 0) \\ &\qquad k = 1, 2, \ldots L \\ &\qquad r = 0, 1, \ldots, n_k \\ &\qquad j = 1, 2, \ldots, p \end{aligned}\right\} \tag{10}$$

5. FIRST ORDER TAYLOR SERIES APPROXIMATION

We transform the non linear membership function $\mu_j(\bar{n})$ into equivalent linear membership function at individual best solution by first order Taylor series approximation.

$$\begin{aligned} \mu_j(\bar{n}) \cong\ & \mu_j(n_{k,j}^*) + (n_1 - n_{1,j}^*)\frac{\delta}{\delta n_1}\mu_j(n_{k,j}^*)_{\text{at } \bar{n}=n_{k,j}^*} + \ldots \\ & + (n_L - n_{L,j}^*)\frac{\delta}{\delta n_L}\mu_j(n_{k,j}^*)_{\text{at } \bar{n}=n_{k,j}^*} \\ & = \hat{\mu}_j(\bar{n});\ j = 1, 2, \ldots, p \end{aligned} \tag{11}$$

Then, the linear FGP model by using expression in (11) of the NLPP (6) can be expressed as

$$\left.\begin{aligned} &\textit{Minimize } Z = \sum\nolimits_{j=1}^{p} w_j d_j^- \\ &\textit{Subject to} \\ &\qquad \hat{\mu}_j(\bar{n}) + d_j^- - d_j^+ = 1 \\ &\qquad \sum\nolimits_{k=1}^{L} c_k n_k \le C_0 \\ &\textit{and} \quad 2 \le n_k \le N_k;\ k = 1, 2, \ldots, L \end{aligned}\right\} j = 1, 2, \ldots, p \tag{12}$$

6. NUMERICAL ILLUSTRATION

In order to demonstrate the feasibility of the proposed computational method, the following data has been used. The data are from 1997 Agricultural Censuses in Iowa State conducted by National Agricultural Statistics Service, USDA, Washington D.C. (source: http://www.agcensus.usda.gov/) reported by Khan *et al*. (2010). The 99 counties in Iowa State are divided into four strata. Two characteristic are defined here. First, the quantity of corn harvested X_1 and second is the quantity of oats harvested X_2, are given below.

Table 1. Data for four strata and two characteristics

k	N_k	W_k	S_{1k}^2	S_{2k}^2
1	8	0.0808	21601503189.8	1154134.2
2	34	0.3434	19734615816.7	7056074.8
3	45	0.4545	27129658750.0	2082871.3
4	12	0.1212	17258237358.5	732004.9

The values of $\bar{X}_1$ and $\bar{X}_2$ are 405654.19 and 2116.70 respectively. The costs of measurement c_h in the four strata are assumed to be $c_1 = 10$, $c_2 = 5$, $c_3 = 3$, and $c_4 = 7$ units. The total amount available for measurement is $C_0 = (C - c_0) = 280$ units, where $c_0 = 70$ units is the expected overhead cost and $C = 350$ units is the total budget of the survey.

After substituting the values of W_k, S_{1k}^2, S_{2k}^2, $\bar{X}_1, \bar{X}_2, c_0$ and c_k in expression (5), the NLPP for first characteristic the optimal solution provided by LINGO is

$n_{k,1}^*$ = (3.315297, 19.04115, 38.15016, 5.312976) with Z_1^* =.00218389

Similarly, for the second characteristic the optimal solution provided by LINGO is given as

$n_{k,2}^*$ = (2.063321, 30.66842, 28.46800, 2.945812) with Z_2^* = .0110586

where $n_{k,1}^*$ and $n_{k,2}^*$ denote the optimum allocations for the first and second characteristic in four strata.

Further we compute the upper and lower tolerance for each objective

For first characteristic

U_1 = .002595784 and L_1 = .00218389

Similarly, for second characteristic

$U_2 = .01322948$ and $L_2 = .0110586$

The membership functions are

$$\mu_1(\bar{n}) = \frac{1}{.0004118940}[.002595784 - Z_1(\bar{n})]$$

and $$\mu_2(\bar{n}) = \frac{1}{.00217088}[.01322948 - Z_2(\bar{n})]$$

Each of the objective function can be expressed as the sum of the separable functions which are shown in the Table 2.

Table 2. Separable functions associated with the each objective

$f_{11}(n_1)$	$0.0008570275/n_1$
$f_{12}(n_2)$	$0.01414221/n_2$
$f_{13}(n_3)$	$0.03405651/n_3$
$f_{14}(n_4)$	$0.001540599/n_4$
$f_{21}(n_1)$	$0.001681746/n_1$
$f_{22}(n_2)$	$0.1857142/n_2$
$f_{23}(n_3)$	$0.09603103/n_3$
$f_{24}(n_4)$	$0.002399941/n_4$

After introducing the under and over deviational variable the nonlinear membership goals are

$$\frac{1}{.0004118940}[.002595784 - f_{11}(n_1) - f_{12}(n_2) - f_{13}(n_3) - f_{14}(n_4)] + d_1^- - d_1^+ = 1$$

and $$\frac{1}{.00217088}[.01322948 - f_{21}(n_1) - f_{22}(n_2) - f_{23}(n_3) - f_{24}(n_4)] + d_2^- - d_2^+ = 1 \quad (13)$$

These non linear membership goals are approximated with linear form. Let the grid points for each variables n_1, n_2, n_3, n_4 are

$a_{10} = 2,\ a_{11} = 3.5,\ a_{12} = 5,\ a_{13} = 6.5,\ a_{14} = 8$

$a_{20} = 2,\ a_{21} = 10,\ a_{22} = 18,\ a_{23} = 26,\ a_{24} = 34$

$a_{30} = 2,\ a_{31} = 12.75,\ a_{32} = 23.5,\ a_{33} = 34.25,\ a_{34} = 45$

$a_{40} = 2,\ a_{41} = 4.5,\ a_{42} = 7,\ a_{43} = 95,\ a_{44} = 12$

Using these values the proposed linear FGP in (10) becomes

$$\text{Minimize } Z = \frac{1}{.0004118940} d_1^- + \frac{1}{.00217188} d_2^-$$

so as to satisfy

$$\frac{1}{.0004118940}[.002595784 - F_{11} - F_{12} - F_{13} - F_{14}] + d_1^- - d_1^+ = 1$$

$$\frac{1}{.00217088}[.01322948 - F_{21} - F_{22} - F_{23} - F_{24}] + d_2^- - d_2^+ = 1$$

$$F_{11} = (.0004285138)\gamma_{10} + (.0002448650)\gamma_{11} + (.0001714055)\gamma_{12} + (.0001318504)\gamma_{13} + (.0001071284)\gamma_{14}$$

$$F_{12} = (.007071105)\gamma_{20} + (.001414221)\gamma_{21} + (.0007856783)\gamma_{22} + (.0005439312)\gamma_{23} + (.0004159474)\gamma_{24}$$

$$F_{13} = (.01702826)\gamma_{30} + (.002671099)\gamma_{31} + (.001449213)\gamma_{32} + (.0009943507)\gamma_{33} + (.0007568113)\gamma_{34}$$

$$F_{14} = (.0007702995)\gamma_{40} + (.0003423553)\gamma_{41} + (.0002200856)\gamma_{42} + (.0001621683)\gamma_{43} + (.0001283832)\gamma_{44}$$

$$F_{21} = (.0008408730)\gamma_{10} + (.0004804989)\gamma_{11} + (.0003363492)\gamma_{12} + (.0002587302)\gamma_{13} + (.0002102182)\gamma_{14}$$

$$F_{22} = (.09285710)\gamma_{20} + (.01857142)\gamma_{21} + (.01031746)\gamma_{22} + (.007142854)\gamma_{23} + (.005462182)\gamma_{24}$$

$$F_{23} = (.04801552)\gamma_{30} + (.007531845)\gamma_{31} + (.004086427)\gamma_{32} + (.002803826)\gamma_{33} + (.002134023)\gamma_{34}$$

$$F_{24} = (.001199970)\gamma_{40} + (.0005333202)\gamma_{41} + (.0003428487)\gamma_{42} + (.0002526254)\gamma_{43} + (.0001999951)\gamma_{44}$$

Subject to

$$10(2\gamma_{10} + 3.5\gamma_{11} + 5\gamma_{12} + 6.5\gamma_{13} + 8\gamma_{14}) + 5(2\gamma_{20} + 10\gamma_{21} + 18\gamma_{22} + 26\gamma_{23} + 34\gamma_{24}) + 3(2\gamma_{30} + 12.75\gamma_{31} + 23.5\gamma_{32} + 34.25\gamma_{33} + 45\gamma_{34}) + 7(2\gamma_{40} + 4.5\gamma_{41} + 7\gamma_{42} + 9.5\gamma_{43} + 12\gamma_{44}) \le 280$$

$$\gamma_{10} + \gamma_{11} + \gamma_{12} + \gamma_{13} + \gamma_{14} = 1$$

$$\gamma_{20} + \gamma_{21} + \gamma_{22} + \gamma_{23} + \gamma_{24} = 1$$

$$\gamma_{30} + \gamma_{31} + \gamma_{32} + \gamma_{33} + \gamma_{34} = 1$$

$$\gamma_{40} + \gamma_{41} + \gamma_{42} + \gamma_{43} + \gamma_{44} = 1$$

$$\gamma_{kr} \ge 0;\ k = 1, 2, 3, 4 \text{ and } r = 0, 1, 2, 3, 4$$

(14)

The above model in (14) provides optimum compromise solution using the optimization software LINGO (2001) as:

$n_1 = 2$, $n_2 = 24$, $n_3 = 34$, $n_4 = 5$ with $Z_1 = .002327555$ and $Z_2 = .01188340$.

Now we formulate the fuzzy goal programming model in (12) using Taylor's Series approximation as

$$\left.\begin{array}{l} \textit{Minimize } Z = \dfrac{1}{.0004118940} d_1^- + \dfrac{1}{.00217088} d_2^- \\ \text{so as to satisfy} \\ \quad 6.5123 n_1 + 6.41463 n_2 + 6.36585 n_3 + 6.43902 n_4 \\ \quad - 420.79849 + d_1 = 1 \\ \quad 6.27189 n_1 + 6.18433 n_2 + 6.14747 n_3 + 6.21659 n_4 \\ \quad - 395.912558 + d_2 = 1 \\ \qquad 10 n_1 + 5 n_2 + 3 n_3 + 7 n_4 \le 280 \\ \qquad\qquad 2 \le n_1 \le 8 \\ \qquad\qquad 2 \le n_2 \le 34 \\ \qquad\qquad 2 \le n_3 \le 45 \\ \qquad\qquad 2 \le n_4 \le 12 \\ \textit{and} \qquad n_4 \text{ are integers; } k = 1, 2, ..., L \end{array}\right\} \tag{15}$$

The optimum compromise solution of the above model is obtained using LINGO as:

$n_1 = 2$, $n_2 = 33$, $n_3 = 28$, $n_4 = 2$ with $Z_1 = .002843669$ and $Z_2 = .01109823$.

7. CONCLUSION

Fuzzy goal programming plays a very important role in many real life problems where coefficients are not known exactly. Goal programming is one of the most useful, multiobjective decision making problem. But in real life situations, the acceptability of goal programming may be constrained by two main difficulties. One is imprecise (non-specific) goals defined by decision makers and another is the need to minimize all goals simultaneously. According to fuzzy set theory the fuzziness is not randomness. To overcome such situation fuzzy programming has been used in goal programming where opinions are based on decision makers own decisions. The compromise allocations for the deterministic problem by using piece-wise linear approximation and the Taylor's Series approximation are given in Table 3.

Last column of Table 3 shows that the Taylor's series approximation gives more efficient results as compared to the piecewise linear approximation in terms of the trace value of the coefficient of variation.

Table 3. Value of the Coefficient of Variation under different approaches

Allocations	n_1 n_2 n_3 n_4	Coefficient of Variation	Total
Piecewise linear approximation	2 24 34 5	k = 1 0.002327555 k = 2 0.01188340	0.0142
Taylor series approximations	2 33 28 2	k = 1 0.00284366 k = 2 0.01109823	0.01393

This work can also be extended for other multivariate sampling designs.

REFERENCES

Ahsan, M.J. and Khan, S.U. (1977). Optimum allocation in multivariate stratified random sampling using prior information. *J. Ind. Statist. Assoc.*, **15**, 57-67.

Ahsan, M.J. and Khan, S.U. (1982). Optimum allocation in multivariate stratified random sampling with overhead cost. *Metrika*, **29**, 71-78.

Ansari, A.H., Najmussehar and Ahsan, M.J. (2009). On multiple response stratified random sampling design. *J. Statist. Sci.*, **1(1)**, 45-54.

Aoyama, H. (1963). Stratified random sampling with optimum allocation for multivariate populations. *Ann. Inst. Statist. Math.*, **14**, 251-258.

Bazaraa, M.S., Sherali, H.D., and Shetty, C.M. (1993). *Nonlinear Programming Theory and Algorithms*. John Wiley and Sons, New York.

Bethel, J. (1989). Sample allocation in multivariate surveys. *Survey Methodology,* **15**, 47-57.

Charnes, A. and Cooper, W.W. (1961). *Management Models and Industrial Applications of Linear Programming*. **1** and **2,** John Wiley, New York.

Dalenius, T. (1957). *Sampling in Sweden: Contributions to the Methods and Theories of Sample Survey Practice*. Stockholm, Almqvist OchWiksell.

Folks, J.L. and Antle, C.E. (1965). Optimum allocation of sampling units to the strata when there are R responses of sampling units of interest. *J. Amer. Statist. Assoc.*, **60**, 225-233.

Ghosh, S.P. (1958). A note on stratified random sampling with multiple characters. *Bull. Cal. Statist. Assoc.*, **8**, 81-89.

Jahan, N., Khan, M.G.M. and Ahsan, M.J. (1994). A generalized compromise allocation. *J. Ind. Statist. Assoc.*, **32**, 95-101.

Khan, M.G.M., Maiti, T. and Ahsan, M.J. (2010). An optimal multivariate stratified sampling design using auxiliary information: an integer solution using goal programming approach. *J. Off. Statist.*, **26(4)**, 695-708.

Khan, M.G.M., Ahsan, M.J. and Jahan, N. (1997). Compromise allocation in multivariate stratified sampling: an integer solution. *Naval Res. Logistics*, **44(1)**, 69-79.

LINGO (2001): *LINGO User's Guide*. LINDO Systems Inc., Chicago, Illinois, USA.

Miller, C.E. (1963). The simplex method for local separable programming. In: *Recent Advances in Mathematical programming (R.L. Groves and P. Wolfe Eds.)*, 89-100.

Narasimhan, R. (1980). On fuzzy goal programming – Some comments. *Decision Sci.*, **11**, 532-538.

Neyman, J. (1934). On the two different aspects of the representative methods: the method of stratified sampling and the method of purposive selection. *J. Roy. Statist. Soc.*, **97**, 558-606.

Pal, B.B., Moitra, B.N. and Maulik, U. (2003). A Goal Programming procedure for fuzzy multiobjective linear fractional programming problem. *Fuzzy Sets Sys.*, **139**, 395-405.

Yates, F. (1960). *Sampling Methods for Census and Surveys*. Charles Griffin and Co. Ltd., London.