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JOURNAL OF THE INDIAN SOCIETY OF AGRICULTURAL STATISTICS 66(3) 2012 427-440

Estimation of Finite Population Variance using Partial Jackknifing

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Received 03 March 2010; Revised 16 August 2012; Accepted 21 August 2012

SUMMARY

In this paper, a new idea of partial jackknifing to estimate the variance of the ratio type estimator of the finite population variance due to Isaki (1983) in the presence of random non-response has been introduced. The proposed estimator has been compared with three different estimators of the variance through an empirical study.

Keywords: Estimation of variance, Jackknifing, Auxiliary information.

1. INTRODUCTION

An excellent literature on jackknifing while estimating population mean and variance has been well documented by Singh *et al.* (2008), Arnab and Singh (2006), Upadhyaya *et al*. (2004), and Quenouille (1956); among others. Thus, any review of those papers in not given here. Let Y and X be the study and auxiliary variables, respectively, in a population Ω consisting of N units. Let y_i and x_i for $i = 1, 2, ..., N$ be the ith values of the study variable Y and the auxiliary variable X, respectively. Motivated by Isaki (1983), by using information on the auxiliary variable X; consider the problem of estimation of the finite population variance

$$\sigma_y^2 = \{2N(N-1)\}^{-1}\sum_{i\neq j}^{N}\sum_{=1}^{N}(y_i - y_j)^2 \tag{1}$$

Next consider selecting a simple random and without replacement sample (SRSWOR) s of n units from the population Ω. Let (y_i, x_i), $i = 1, 2, ..., n$ be the values of the study variable and auxiliary variable in the SRSWOR sample s of n units. Assume only response on the study variable y_i for $i = 1, 2, ..., r$ respondents is available in the sub-sample s_1 of s, while the information on the auxiliary variable x_i, $i = 1, 2, ..., n$ is available in the entire sample s. In other words, the sub-sample s_1 of the responding units consists of data values (y_i, x_i), $i = 1, 2, ... r$ and the sub-sample $s_2 = s - s_1$ consists of data values $(\dagger, x_i)$, $i = 1, 2, ... (n - r)$, where † denotes a missing value. Detail about such a non-response mechanism in a real life can be seen in Rueda *et al*. (2007).

Let $s_y^2 = \{2r(r-1)\}^{-1}\sum_{i\neq}^{r}\sum_{j=1}^{r}(y_i - y_j)^2$ and $s_x^2 = \{2r(r-1)\}^{-1}\sum_{i\neq}^{r}\sum_{j=1}^{r}(x_i - x_j)^2$, be the sample variances of the study variable and auxiliary variable, respectively, based on the responding units in the sub-sample s_1 and $s_x^{*2} = \{2n(n-1)\}^{-1}\sum_{i\neq}^{n}\sum_{j=1}^{n}(x_i - x_j)^2$, be the sample variance of the auxiliary variable based on the entire sample s. Following Isaki (1983), an analogous of the ratio type estimator, in two-phase

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sampling, of the finite population variance σ_y^2 in (1) is given by

$$\hat{\sigma}_R^2 = s_y^2 \left(\frac{s_x^{*2}}{s_x^2} \right) \tag{2}$$

Following Cochran (1978) and applying the concept of two-phase sampling, an approximate variance of the ratio estimator $\hat{\sigma}_R^2$ is given by

$$V(\hat{\sigma}_R^2) \approx \left(\frac{1}{n} - \frac{1}{N}\right)\left(\mu_{40}^{\Diamond} - \frac{(4N-1)}{N}\sigma_y^4\right) + \left(\frac{1}{r} - \frac{1}{n}\right)\left[\mu_{40}^{\Diamond} + \left(\frac{\sigma_y^2}{\sigma_x^2}\right)^2 \mu_{04}^{\Diamond} - 2\left(\frac{\sigma_y^2}{\sigma_x^2}\right)\mu_{22}^{\Diamond}\right] - \frac{2(N-1)}{N}\left(\frac{1}{r} - \frac{1}{n}\right)(2\mu_{20}^2 - B\mu_{11}^2) \tag{3}$$

where

$$\mu_{40}^{\Diamond} = \frac{1}{2N(N-1)} \sum_{i \neq j}^{N} \sum_{=1}^{N} (Y_i - Y_j)^4 = \mu_{40} + \frac{3(N-1)}{N}\mu_{20}^2;$$

$$\mu_{04}^{\Diamond} = \frac{1}{2N(N-1)} \sum_{i \neq j}^{N} \sum_{=1}^{N} (x_i - x_j)^4 = \mu_{04} + \frac{3(N-1)}{N}\mu_{02}^2;$$

$$\mu_{22}^{\Diamond} = \frac{1}{2N(N-1)} \sum_{i \neq j}^{N} \sum_{=1}^{N} (y_i - y_i)^2 (x_i - x_j)^2 = \mu_{22} + \frac{(N-1)}{N}(\mu_{20}\mu_{02} + 2\mu_{11}^2); \text{ with}$$

$$\mu_{ab} = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{Y})^a (x_i - \bar{X})^b,$$

$$\bar{Y} = N^{-1}\sum_{i=1}^{N} y_i, \bar{X} = N^{-1}\sum_{i=1}^{N} x_i, \sigma_y^2 = \mu_{20}, \text{ and}$$

$\sigma_x^2 = \mu_{02}$ etc., have their usual meanings.

In section 2, the ratio estimator $\hat{\sigma}_R^2$ in (2) is shown as a special case of the proposed imputing method. In section 3, the problem of estimation of variance $V(\hat{\sigma}_R^2)$ given in (3) of the ratio estimator $\hat{\sigma}_R^2$ in (2) by using partial jackknifing has been considered.

2. IMPUTATION AND RESULTANT ESTIMATOR

A new method is suggested to impute the squares of the differences between the consecutive values of the sample s as follows

$$(\hat{y}_{io} - \hat{y}_{jo})^2 = \begin{cases} (y_i - y_j)^2 \text{ for } i, j \in s_1 \\ \hat{B}(x_i - x_j)^2 \text{ for } i, j \in s_2 \end{cases} \tag{4}$$

where $\hat{B}$ is given by

$$\hat{B} = \frac{\sum_{i \neq}^{r} \sum_{j=1}^{r} (y_i - y_i)^2}{\sum_{i \neq}^{r} \sum_{j=1}^{r} (x_i - x_i)^2} \tag{5}$$

Define a point estimator of the finite population variance σ_y^2 as

$$\hat{\sigma}_R^2 = \frac{1}{2n(n-1)} \sum_{i \neq}^{n} \sum_{j=1}^{n} (\hat{y}_{io} - \hat{y}_{jo})^2 \tag{6}$$

Then using (4) and (5); the point estimator $\hat{\sigma}_R^2$ in (6) of the finite population variance σ_y^2 becomes

$$\hat{\sigma}_R^2 = \frac{1}{2n(n-1)} \sum_{i \neq}^{n} \sum_{j=1}^{n} (\hat{y}_{io} - \hat{y}_{jo})^2$$

$$= \frac{1}{2n(n-1)} \left[\sum_{i \neq j \in s_1} \sum (y_i - y_j)^2 + \hat{B} \sum_{i \neq j \in s_2} \sum (x_i - x_j)^2 \right]$$

$$= \frac{1}{2n(n-1)} \left[\sum_{i \neq j \in s_1} \sum (y_i - y_j)^2 + \left(\frac{\sum_{i \neq j \in s_1} \sum (y_i - y_j)^2}{\sum_{i \neq j \in s_1} \sum (x_i - x_j)^2} \right) \sum_{i \neq j \in s_2} \sum (x_i - x_j)^2 \right]$$

$$= \frac{1}{2n(n-1)}\left[\sum_{i\neq j}\sum_{\in s_1}(y_i-y_j)^2 + \left(\frac{\sum\sum_{i\neq j\in s_1}(y_i-y_j)^2}{\sum\sum_{i\neq j\in s_1}(x_i-x_j)^2}\right)\left(\sum_{i\neq j}\sum_{\in s}(x_i-x_j)^2 - \sum_{i\neq j}\sum_{\in s_1}(x_i-x_j)^2\right)\right]$$

$$= \frac{1}{2n(n-1)}\sum_{i\neq}^{r}\sum_{j=1}^{r}(y_i-y_j)^2\left[\frac{\sum_{i\neq}^{n}\sum_{j=1}^{n}(x_i-x_j)^2}{\sum_{i\neq}^{r}\sum_{j=1}^{r}(x_i-x_j)^2}\right] \quad (7)$$

Thus, the following theorem is proposed.

Theorem 1. The point estimator $\hat{\sigma}_R^2$ of the finite population variance σ_y^2 based on imputed squares of the differences becomes

$$\hat{\sigma}_R^2 = s_y^2\left(\frac{s_x^{*2}}{s_x^2}\right) \quad (8)$$

Proof. It follows from (7).

Note that the estimator $\hat{\sigma}_R^2$ in (8) is the same as the analogous of the ratio estimator of σ_y^2 in (2) due to Isaki (1983). In the next section, now consider the problem of estimation of variance $V\left(\hat{\sigma}_R^2\right)$ in (3) of the ratio estimator $\hat{\sigma}_R^2$ in (8) by using a new method of partial jackknifing.

3. PARTIAL JACKKNIFED ESTIMATOR OF THE VARIANCE

Consider the partial jackknifing of the ratio estimator $\hat{\sigma}_R^2$ in (8) as follows

$$\hat{\sigma}_R^2(i,j) = \begin{cases} s_y^2(i,j)\left(\dfrac{s_x^{*2}(i,j)}{s_x^2(i,j)}\right) & \text{if } i, j\in s_1 \\ s_y^2\left(\dfrac{s_x^{*2}(i,j)}{s_x^2}\right) & \text{if } i, j\in s_2 \end{cases} \quad (9)$$

where

$$s_y^2(i,j) = \frac{2r(r-1)s_y^2-(y_i-y_j)^2}{2r(r-1)-2} \quad \text{for } i, j\in s_1 \quad (10)$$

Note that $s_y^2\ (i,j)$ in (10) is not a sample variance after dropping two units y_i and y_j from the given sample, but it eliminates a partial effect of two units from the sample variance s_y^2; thus it is named as a partial jackknifed estimator of variance. It is explained with the help of following example.

Example 1. Consider a sample consisting of $r = 5$ units say, $y_1 = 15$, $y_2 = 17$, $y_3 = 12$, $y_4 = 25$ and $y_5 = 56$. Now consider a symmetric matrix of order 5 × 5 as given in Table 1.

Table 1. Squared differences $(y_i - y_j)^2$

	y_1	y_2	y_3	y_4	y_5
y_1	–	4	9	100	1681
y_2	4	–	25	64	1521
y_3	9	25	–	169	1936
y_4	100	64	169	–	961
y_5	1681	1521	1936	961	–

Obviously

$$s_y^2 = \frac{1}{2r(r-1)}\sum_{i\neq}^{r}\sum_{j=1}^{r}(y_i-y_j)^2 = \frac{12940}{2\times5\times4} = 323.50 \quad (11)$$

and $$s_y^2(2,\ 4) = \frac{12876}{2\times5\times4-2} = 338.84 \quad (12)$$

Clearly $s_y^2(2,\ 4)$ is not a sample variance of $y_1 = 15$, $y_3 = 12$ and $y_5 = 56$. Thus, it has been named $s_y^2(2,\ 4)$ as a partial jackknifed estimator of variance. It is easy to verify that

$$\frac{1}{r(r-1)}\sum_{i\neq}^{r}\sum_{j=1}^{r}s_y^2(i,\ j) = s_y^2 \quad (13)$$

Thus the average of the partial jackknifed estimators of variance remains the same as the original sample variance. Note that the average of the jackknifed sample means also remains equal to the original sample mean, thus partial jackknifing of sample variance and jackknifing of sample mean give similar findings.

Now in the same fashion, obtain the partial jackknifed estimators of variances for the auxiliary variable as follows:

$$s_x^2(i,j) = \frac{2r(r-1)s_x^2 - (x_i - x_j)^2}{2r(r-1)-2} \quad \text{for } i, j \in s_1 \quad (14)$$

and

$$s_x^{*2}(i,j) = \frac{2n(n-1)s_x^{*2} - (x_i - x_j)^2}{2n(n-1)-2} \quad \text{for } i, j \in s \quad (15)$$

One can check that

$$\hat{\sigma}_R^2(i,j) - \hat{\sigma}_R^2$$

$$= \begin{cases} -\left(\dfrac{s_x^{*2}(i,j)}{s_x^2(i,j)}\right)\dfrac{\left\{(y_i - y_j)^2 - \dfrac{s_y^2}{s_x^2}(x_i - x_j)^2\right\}}{2r(r-1)-2} + \left(\dfrac{s_y^2}{s_x^2}\right)\dfrac{\{2s_x^{*2} - (x_i - x_j)^2\}}{2n(n-1)-2} & \text{if } i, j \in s_1 \\ \left(\dfrac{s_y^2}{s_x^2}\right)\dfrac{\{2s_x^{*2} - (x_i - x_j)^2\}}{2n(n-1)-2} & \text{if } i, j \in s_2 \end{cases} \quad (16)$$

Note that the expression (16) is exact. The complete jackknifing of the sample variances in (10), (14) and (15) is feasible, but that will not give the exact expression (16). Hence, the introduction of a new idea of partial jackknifed estimator of variance remains useful in the present investigation.

A suggestion is being made to use an adjusted partial jackknifed estimator of variance $V\left(\hat{\sigma}_R^2\right)$ in (3) of the estimator $\hat{\sigma}_R^2$ as

$$\hat{v}_J(\hat{\sigma}_R^2) = \frac{n(n(n-1)-1)}{r(r-1)} \sum_{i \neq j}^{n} \sum_{=1}^{n} \left[\hat{\sigma}_R^2(i,j) - \hat{\sigma}_R^2\right]^2 + 2\left(\frac{1}{r} - \frac{1}{n}\right)\left(2\hat{B}^2\hat{\mu}_{02}^{*2} - \hat{B}\hat{\mu}_{11}^2\right) \quad (17)$$

where

$$\hat{\mu}_{02}^* = \frac{1}{2n(n-1)} \sum_{i \neq j}^{r} \sum_{=1}^{r} (x_i - x_j)^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \overline{x}_n)^2;$$

$$\hat{\mu}_{11} = \frac{1}{2r(r-1)} \sum_{i \neq j}^{r} \sum_{=1}^{r} (y_i - y_j)(x_i - x_j)$$

$$= \frac{1}{r-1} \sum_{i=1}^{r} (y_i - \overline{y}_r)(x_i - \overline{x}_r).$$

Further, note that two more situations such that $i \in s_1$, $j \in s_2$ and $j \in s_1$, $i \in s_2$ could also be considered. The main purpose of estimating the variance of the ratio type estimator due to Isaki (1983) gets resolved and hence the other two cases are not considered, but any researcher could play with these two cases.

In the next section, three new estimators of $V(\hat{\sigma}_R^2)$ as natural competitors of the proposed partial jackknife estimator of variance $\hat{v}_J(\hat{\sigma}_R^2)$ in (17) have been considered.

4. NEW ESTIMATORS OF THE VARIANCE

A design-consistent linearization variance estimator of the estimator $(\hat{\sigma}_R^2)$ given by a standard formula

$$\hat{v}_1(\hat{\sigma}_R^2) = \left(\frac{1}{r} - \frac{1}{n}\right)\hat{\sigma}_d^2 + \left(\frac{1}{n} - \frac{1}{N}\right)\left(\hat{\mu}_{40}^{\Diamond} - \frac{(4N-1)}{N}\hat{\mu}_{20}^2\right) - \frac{2(N-1)}{N}\left(\frac{1}{r} - \frac{1}{n}\right)\left(2\hat{\mu}_{20}^2 - \hat{B}\hat{\mu}_{11}^2\right) \quad (18)$$

where

$$\hat{\sigma}_d^2 = \frac{1}{2r(r-1)} \sum_{i \neq j}^{r} \sum_{=1}^{r} d_{ij}^2 \text{ with}$$

$$d_{ij} = (y_i - y_j)^2 - \hat{B}(x_i - x_j)^2, \text{ and}$$

$$\hat{\mu}_{20} = \frac{1}{2r(r-1)} \sum_{i \neq j}^{r} \sum_{=1}^{r} (y_i - y_j)^2 = \frac{1}{r-1} \sum_{i=1}^{r} (y_i - \overline{y}_r)^2.$$

To motivate the new linearization variance estimator of $\hat{\sigma}_R^2$, at first express $\mu_{40}^{\Diamond}$ as

$$\mu_{40}^{\Diamond} = \frac{1}{2N(N-1)} \sum_{i \neq j}^{N} \sum_{=1}^{N} (y_i - y_j)^4$$

$$= \frac{1}{2N(N-1)} \sum_{i \neq j}^{N} \sum_{=1}^{N} [D_{ij}^2 + B^2 (x_i - x_j)^4$$

$$+ 2BD_{ij}(x_i - x_j)^2]$$

$$= \sigma_D^2 + B^2 \mu_{04}^{\Diamond} + 2B\sigma_{DX^2} \tag{19}$$

where

$$\sigma_D^2 = \frac{1}{2N(N-1)} \sum_{i \neq j}^{N} \sum_{=1}^{N} D_{ij}^2,$$

$$\mu_{04}^{\Diamond} = \frac{1}{2N(N-1)} \sum_{i \neq j}^{N} \sum_{=1}^{N} (x_i - x_j)^4,$$

$$\sigma_{DX^2} = \frac{1}{2N(N-1)} \sum_{i \neq j}^{N} \sum_{=1}^{N} D_{ij} (x_i - x_j)^2 \text{ and}$$

$D_{ij} = (y_i - y_j)^2 - B(x_i - x_j)^2.$

Also express μ_{20} as

$$\mu_{20} = \frac{1}{N} \sum_{i=1}^{N} (y_i - \bar{Y})^2 = \frac{1}{2N(N-1)} \sum_{i \neq j}^{N} \sum_{=1}^{N} (y_i - y_j)^2$$

$$= \frac{1}{2N(N-1)} \sum_{i \neq j}^{N} \sum_{=1}^{N} [D_{ij} + B(x_i - x_j)^2]$$

$$= B\mu_{02} \tag{20}$$

Similarly,

$$\hat{\mu}_{40}^{\Diamond} = \hat{\sigma}_d^2 + \hat{B}^2 \hat{\mu}_{04}^{\Diamond} + 2\hat{B}\ \hat{\sigma}_{dx^2} \tag{21}$$

and

$$\hat{\mu}_{20} = \hat{B}\ \hat{\mu}_{02} \tag{22}$$

It follows from (19), (20), (21) and (22) that alternative estimators of $\mu_{02}^{\Diamond}$ and $\mu_{04}^{\Diamond}$, those make more complete use of the sample data, can be obtained by using

$$\hat{\mu}_{ob}^{*} = \frac{1}{2n(n-1)} \sum_{i \neq j}^{n} \sum_{=1}^{n} (x_i - x_j)^b \quad \text{for } b = 2, 4.$$

Thus, the second linearization estimator of the variance $V(\hat{\sigma}_R^2)$ is considered as

$$\hat{v}_2(\hat{\sigma}_R^2) = \left(\frac{1}{r} - \frac{1}{N}\right)\hat{\sigma}_d^2 + \left(\frac{1}{n} - \frac{1}{N}\right)\hat{B}^2 \left(\hat{\mu}_{04}^{*} - \frac{(4N-1)}{N}\hat{\mu}_{02}^{*2}\right)$$

$$+2\left(\frac{1}{n} - \frac{1}{N}\right)\hat{B}\hat{\sigma}_{dx^2}$$

$$-2\left(\frac{N-1}{N}\right)\left(\frac{1}{r} - \frac{1}{n}\right)(2\hat{B}^2\hat{\mu}_{02}^{*2} - \hat{B}\hat{\mu}_{11}^2) \tag{23}$$

where

$$\hat{\sigma}_{dx^2} = \frac{1}{2r(r-1)} \sum_{i \neq j}^{r} \sum_{=1}^{r} d_{ij} (x_i - x_j)^2.$$

Assuming $\dfrac{s_x^{*2}(i, j)}{s_x^2(i, j)} \approx \dfrac{s_x^{*2}}{s_x^2}$ in (16); one can obtain from (16) and (17)

$$\hat{v}_J(\hat{\sigma}_R^2) \cong \left(\frac{s_x^{*2}}{s_x^2}\right)\frac{\hat{\sigma}_d^2}{r} + 2\left(\frac{s_x^{*2}}{s_x^2}\right)\frac{\hat{B}\hat{\sigma}_{dx^2}}{n} + \frac{\hat{B}^2(\hat{\mu}_{04}^{*} - \hat{\mu}_{02}^{*2})}{n}$$

$$+\ 2\left(\frac{1}{r} - \frac{1}{n}\right)\left(2\hat{B}^2\hat{\mu}_{02}^{*2} - \hat{B}\hat{\mu}_{11}^2\right). \tag{24}$$

Ignoring the finite population correction and comparing (23) and (24), it now follows that $\hat{v}_J(\hat{\sigma}_R^2)$ is also design-consistent since $\dfrac{s_x^{*2}}{s_x^2} \cong 1$ for large n. It follows from (23) and (24) that another design-consistent linearization variance estimator of $V(\hat{\sigma}_R^2)$ when the finite population corrections are not ignorable is given by

$$\hat{v}_3(\hat{\sigma}_R^2) = \left(\frac{s_x^{*2}}{s_x^2}\right)^2 \left(\frac{1}{r} - \frac{1}{N}\right)\hat{\sigma}_d^2 + 2\left(\frac{s_x^{*2}}{s_x^2}\right)\left(\frac{1}{n} - \frac{1}{N}\right)\hat{B}\hat{\sigma}_{dx^2}$$

$$+\left(\frac{1}{n} - \frac{1}{N}\right)\hat{B}^2\left(\hat{\mu}_{04}^{*} - \frac{4N-1}{N}\hat{\mu}_{02}^{*2}\right)$$

$$-2\left(\frac{N-1}{N}\right)\left(\frac{1}{r} - \frac{1}{n}\right)\left(2\hat{B}^2\hat{\mu}_{02}^{*2} - \hat{B}\hat{\mu}_{11}^2\right). \tag{25}$$

The new variance estimator (25) resembles the robust variance estimator in single phase sampling.

In the next section, these four estimators of the variance $V(\hat{\sigma}_R^2)$ are compared by using a simulation study.

5. SIMULATION STUDY

In the simulation study, consider the use of the model

$$M : y_i = Rx_i + e_i x_i^g \tag{26}$$

where $x_i \sim$ Gamma (θ_1, θ_2), R is the regression coefficient of y on x; $e_i \sim N\left(0, \sigma^2\right)$ being independent of x_i and g is any real number. The IMSL subroutine *RNGAM*(*N*, *THETA1*, *X*) has been used to generate a Gamma variable X with single parameter θ_1, then used the subroutine *SSCAL*(*N*, *THETA*2, *X*, 1) to convert it into a gamma variable X with two parameters (θ_1, θ_2). The subroutine *RNNOR*(*N*, *E*) is used to generate the error term from a standard normal distribution. A similar model has been used for generating two-phase samples by Ramasubramanian *et al.* (2007) and Singh and Arnab (2010) among others. Royal (1970) used a similar model to generate these types of populations under which the ratio estimator is the best among a wide class of estimators. The mean, variance and coefficient of variation of x_i are, respectively, given by $\bar{X} = \theta_1\theta_2, \sigma_x^2 = \theta_1\theta_2^2$, and $C_x = \frac{\sigma_x}{\bar{X}} = \theta_2^{-1}$. Further, the mean and variance of y_i are given by $\bar{Y} = R\,\bar{X}$, and $\sigma_y^2 = R^2\sigma_x^2 + \bar{X}\sigma^2$ respectively.

Now consider the four estimators of the variance $V(\hat{\sigma}_R^2)$ of the ratio estimator $\hat{\sigma}_R^2$ of the finite population variance σ_y^2 as $\hat{v}_1 = \hat{v}_1(\hat{\sigma}_R^2)$, $\hat{v}_2 = \hat{v}_2\left(\hat{\sigma}_R^2\right)$, $\hat{v}_3 = \hat{v}_3\left(\hat{\sigma}_R^2\right)$ and $\hat{v}_4 = \hat{v}_J(\hat{\sigma}_R^2)$. Then $\Theta = 2000$ samples each of size $n = 200$ units have been selected from the population of size $N = 4000$ units by using IMSL subroutine RNSRI. From the given sample of $n = 200$ units, then a sub-sample of $r = 180$ (or 190) units has been selected by using the same ISML subroutine RNSRI. Note that the IMSL subroutine RNSRI selects SRSWOR sample from a given population.

The empirical percent relative bias (RB) in the k^{th} estimator $\hat{v}_k$ for $k = 1, 2, 3, 4$ was computed as

$$\text{RB}(\hat{v}_k)_1 = \frac{\frac{1}{\Theta}\sum_{i=1}^{\Theta}\hat{v}_{k/i} - V(\hat{\sigma}_R^2)}{V(\hat{\sigma}_R^2)} \times 100\% = \text{RB}(k)_1 \tag{27}$$

The percent relative efficiency (RE) of the k^{th} estimator $\hat{v}_k$, for $k = 2, 3, 4$ with respect to the first estimator $\hat{v}_1$ was computed as

$$\text{RE}\,(\hat{v}_1, \hat{v}_k)_1 = \frac{\sum_{i=1}^{\Theta}[\hat{v}_{1/i} - V\left(\hat{\sigma}_R^2\right)]^2}{\sum_{i=1}^{\Theta}[\hat{v}_{k/i} - V\left(\hat{\sigma}_R^2\right)]^2} \times 100\% = \text{RE}\,(k)_1 \tag{28}$$

As required by one of the reviewer, the RB has also been computed as

$$\text{RB}\,(\hat{v}_k)_2 = \frac{\frac{1}{\Theta}\sum_{i=1}^{\Theta}\hat{v}_{k/i} - \text{MSE}}{\text{MSE}} \times 100\% = \text{RB}\,(k)_2 \tag{29}$$

and the percent relative efficiency (RE) of the k^{th} estimator $\hat{v}_k$, for $k = 2, 3, 4$ with respect to the first estimator $\hat{v}_1$ has also been computed as

$$\text{RE}\,(\hat{v}_1, \hat{v}_k)_2 = \frac{\sum_{i=1}^{\Theta}[\hat{v}_{1/i} - \text{MSE}]^2}{\sum_{i=1}^{\Theta}[\hat{v}_{k/i} - \text{MSE}]^2} \times 100\% = \text{RE}\,(k)_2 \tag{30}$$

where $$\text{MSE} = \frac{1}{\Theta}\sum_{i=1}^{\Theta}(\hat{\sigma}_{R/i}^2 - \sigma_y^2)^2 \tag{31}$$

In addition, the ratio, $\mathbb{R}$, of the approximate variance $V\left(\hat{\sigma}_R^2\right)$ in (3) and the simulated MSE in (31) has also been computed as

$$\mathbb{R} = \frac{\text{MSE}}{V\left(\hat{\sigma}_R^2\right)} \tag{32}$$

The FORTRAN codes used in the simulation are given in the Appendix. Thus, a very limited results are presented in Table 2 and Table 3, and other results as per desire can be obtained by using the codes if required.

In Table 2, for g = 0.0, R = 0.5 and r = 180, the value of the ratio $\mathbb{R}$ remains 1.02035 indicating the simulated MSE in (32) and the approximate variance $V\left(\hat{\sigma}_R^2\right)$ in (3) are approximately same. The $RE(2)_1$, $RE(2)_2$ value $RE(3)_1$ and $RE(3)_2$, $RE(4)_1$ and $RE(4)_2$ remain approximately 107.5%, 106.7%, 162.1%, 107.6%, 106.8% and 164.7%, respectively. In this situation, the criterion suggested in (28) and (30) provide almost the same relative efficiency values. For g = 0.0, R = 1.0 and r = 180, the value of the ratio $\mathbb{R}$ becomes 1.0062 which indicates the approximate variance $V\left(\hat{\sigma}_R^2\right)$ remains almost same the simulated MSE value. Although the rest of Table 2 can be read in the same way, but one point is remarkable that for g = 0.5, R = 0.5 and r = 180, the ratio $\mathbb{R}$ remains 4.8727. This value of the ratio $\mathbb{R}$ has been verified by executing the FORTRAN codes several times, so it could happen. In Table 3, for g = 0.0, R = 0.5 and r = 180, the percent relative bias remains less than 10% in case of all the four estimators. It is remarkable that similar findings, as reported in Table 2 and Table 3, have been observed for several other choices of parameters by executing the program again and again.

6. CONCLUSION

The new imputation technique estimates the finite population variance and the partial jackknifing estimates the variance of the resultant ratio type estimator. Among the four estimators of the variance considered, the estimator based on partial jackknifing performs better from the smaller mean squared error

Table 2. Percent relative efficiencies and the ratio values

g	R	r	$\mathbb{R}$	$RE(2)_1$	$RE(3)_1$	$RE(4)_1$	$RE(2)_2$	$RE(3)_2$	$RE(4)_2$
0.0	0.5	180	1.0204	107.5	106.7	162.1	107.6	106.8	164.7
		190	1.0064	104.1	103.7	155.4	104.1	103.7	156.1
	1.0	180	1.0652	110.7	110.6	163.8	110.5	110.5	144.2
		190	1.0176	104.9	104.9	159.6	104.9	104.9	153.2
	1.5	180	1.1088	110.4	110.2	208.2	110.3	110.1	183.7
		190	1.0232	105.2	105.1	208.9	105.2	105.1	202.5
0.5	0.5	180	4.8727	101.8	100.4	79.1	100.4	100.4	134.2
		190	1.0062	100.9	100.2	101.2	100.9	100.2	102.4
	1.0	180	1.0366	105.9	104.5	159.8	106.0	104.6	164.5
		190	0.9884	102.7	102.2	182.6	102.8	102.2	181.9
	1.5	180	1.0454	109.5	109.8	250.1	109.5	109.7	247.2
		190	0.9962	104.6	104.8	259.4	104.6	104.8	260.1
1.0	0.5	180	0.9207	102.9	101.4	97.5	102.7	101.1	88.2
		190	1.0742	101.5	100.7	133.8	101.6	100.9	147.0
	1.0	180	1.0264	100.6	99.6	122.7	100.7	99.7	126.8
		190	1.0112	100.2	99.6	173.2	100.3	99.6	175.1
	1.5	180	0.9372	104.6	106.7	206.8	104.6	106.6	296.5
		190	0.9625	102.3	103.4	278.4	102.3	103.4	277.4

Table 3. Percent Relative Bias Values

g	R	r	$RB(1)_1$	$RB(2)_1$	$RB(3)_1$	$RB(4)_1$	$RB(1)_2$	$RB(2)_2$	$RB(3)_2$	$RB(4)_2$
0.0	0.5	180	0.1326	0.5698	0.7189	6.4511	–1.8640	–1.4356	–1.2895	4.3283
		190	–1.1118	–0.9491	–0.8838	4.4810	–1.7244	–1.5628	–1.4979	3.8337
	1.0	180	–1.2054	–0.9857	–0.9518	–9.7537	–7.2538	–7.0476	–7.0158	–8.1563
		190	–1.3883	–1.2591	–1.2430	–8.7723	–3.0983	–2.9714	–2.9555	–9.2678
	1.5	180	–1.4755	–1.2131	–1.1637	–6.0900	–4.1443	–3.9077	–5.8643	–8.8339
0.5	0.5	180	2.8407	3.4229	3.6760	5.4548	–7.8897	–7.8775	–7.8723	–6.8282
		190	0.7556	1.0564	1.1785	4.3524	0.1314	0.43033	0.5517	4.2634
	1.0	180	–0.0855	0.3220	0.4627	7.1605	–3.6127	–3.2196	–3.0775	6.2716
		190	–1.0800	–0.9352	–0.8817	2.2634	0.0791	0.2256	0.2798	3.4618
	1.5	180	–0.6740	–0.3986	–0.3672	–2.8920	–4.9848	–4.7214	–4.6913	–7.1066
		190	–1.4956	–1.3560	–1.3449	–9.4950	–1.1228	–0.9825	–0.9712	–9.1523
1.0	0.5	180	2.9020	3.4550	3.7103	3.8162	5.7595	8.3601	9.6374	7.0582
		190	0.6561	0.8963	1.0227	3.2975	–6.3010	–6.0803	–5.9626	8.4090
	1.0	180	2.0373	2.4724	2.5536	2.7946	–0.5925	–0.1685	–0.0894	2.4648
		190	–0.0136	0.1708	0.2137	4.9601	–1.1125	–0.9304	–0.8879	3.6963
	1.5	180	0.9120	1.1449	0.9862	9.7564	7.6707	7.9191	7.7497	7.1073

point of views, and shows acceptable relative bias (less than 10%) in all cases. In conclusion, in many situations the proposed partial jackknifed estimator can be used to estimate the variance of the ratio estimator of the finite population variance due to Isaki (1983).

7. FURTHER STUDY

As pointed out by the reviewers, the proposed estimator can be compared with the resampling variance estimator and also conditional properties of the proposed estimator on the lines of Royall and Cumberland (1981a, 1981b) can be investigated in future studies.

ACKNOWLEDGEMENTS

The author is thankful to the Associate Editors and two educated referees for constructive comments on the original version of the manuscript. The authors are also thankful to Raymond Garcia III, Department of Language and Literature, Texas A&M University-Kingsville, Kingsville, TX, for editing the manuscript.

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Appendix

```
! PARTIAL JACKKNIFING
      USE NUMERICAL_LIBRARIES
      IMPLICIT NONE
      INTEGERNP,NS,NR,I,IR(5000),IT(5000),IIR(5000),K,J,KKKK,ISEED
      INTEGERID(5000),IRS(5000),NK,JJ,JJJ,IIII,NITR,NITRR,III
      INTEGER IN(5000),IM(5000), NITRP1, NITRP2,
      NITRP,IP(5000)
      REAL TH2, TH1, E(5000),XP(5000), G, R
      DOUBLE PRECISION
      YP(5000),ANP,ANS,ANR,SIGYP2,SIGXP2,YPMU40,
      1
      XPMU04,YPXPMU22,VARP,YF(5000),XF(5000),YS(5000),
      XS(5000)
      1, XSNR(5000),RAN(5000),XSS(5000),YSS(5000),
      AM20SS, AM02SS, AM40SS
      1, SUMYIYJ2,SUMXIXJ2,BHAT,D(300,300),SUMDIDJ2,
      SIGMDIJ2
      1, V1HAT,SIGDX2,AM02FF,AM04FF,V2HAT,V3HAT
      1, AM02F(300,300), AM02S(300,300), AM20S(300,300)
      1, VJACK(300,300),VJP1,VJP2,VJP,V4HAT
      1, VARV11,VARV21,VARV31,VARV41,RBV11,RBV21,
      RBV31,RBV41
      1,
      RE12,RE13,RE14,ANITR,SIGXYP,RHOXYP,VARE,SUMYP,
      SUMXP,YMP,XMP
      DOUBLE PRECISION VARVARE, SUMXF, XFM,
      VARXF, SUMYS,
      SUMXS,
      1 YSM, XSM, VARXS, VARYS,
      RAT,VARV12,VARV22,VARV32,VARV42,
      1 RBV12,RBV22,RBV32,RBV42,RE22,RE23,RE24,
      AM11SS
      CHARACTER*20 OUT_FILE
      WRITE(*,.(A).) .NAME OF THE OUTPUT FILE.
      READ(*,.(A20).) OUT_FILE
      OPEN(42, FILE=OUT_FILE, STATUS=.unknown.)
      NP = 4000
      ANP = NP
      WRITE(42, 198)NP
198 FORMAT(2X,I7)
      DO 8888 G = 0.0, 1.1, 0.5
      DO 8888 R = 0.5, 1.6, 0.5
      WRITE(*,345)G,R
345 FORMAT(2X,.g=.,F6.3,2X,.R=.,F6.3)
      TH1 = 2.3
      TH2 = 3.5
      ISEED = 13031963
      CALL RNSET (ISEED)
      CALL RNGAM(NP,TH1,XP)
      CALL SSCAL (NP,TH2,XP,1)
      ISEED = 13031963
      CALL RNSET (ISEED)
      CALL RNNOR(NP,E)
      DO 111 I =1, NP
      YP(I) = R*XP(I)+E(I)*XP(I)**G
111 CONTINUE
      SIGXYP = 0.0
      SIGYP2 = 0.0
      SIGXP2 = 0.0
      DO 11 I = 1, NP
      DO 11 J = 1,NP
      IF(I.NE.J) THEN
      SIGXYP = SIGXYP+ (YP(I)-YP(J))*(XP(I)-XP(J))
      SIGYP2 = SIGYP2 + (YP(I)-YP(J))**2
      SIGXP2 = SIGXP2 + (XP(I)-XP(J))**2
      YPMU40 = YPMU40 + (YP(I)-YP(J))**4
      XPMU04 = XPMU04 + (XP(I)-XP(J))**4
      YPXPMU22 = YPXPMU22+(YP(I)-YP(J))**2*(XP(I)-
      XP(J))**2
      ELSE
      CONTINUE
      ENDIF
11 CONTINUE
      SIGXYP = SIGXYP/(2*ANP*(ANP-1))
      SIGYP2 = SIGYP2/(2*ANP*(ANP-1))
      SIGXP2 = SIGXP2/(2*ANP*(ANP-1))
      SUMYP = 0.0
      SUMXP = 0.0
      DO 45 I=1, NP
      SUMYP = SUMYP + YP(I)
45 SUMXP = SUMXP + XP(I)
      YMP = SUMYP/ANP
      XMP = SUMXP/ANP
      YPMU40 = 0.0
      XPMU04 = 0.0
      YPXPMU22 = 0.0
      DO 43 I =1, NP
      YPMU40 = YPMU40 + (YP(I)-YMP)**4
      XPMU04 = XPMU04 + (XP(I)-XMP)**4
      YPXPMU22 = YPXPMU22 + (YP(I)-YMP)**2*(XP(I)-
      XMP)**2
43 CONTINUE
      YPMU40 = YPMU40/(ANP-1)
      XPMU04 = XPMU04/(ANP-1)
      YPXPMU22 = YPXPMU22/(ANP-1)
      RHOXYP = SIGXYP/SQRT(SIGYP2*SIGXP2)
```

```
      NS = 200
      ANS = NS
      DO 5555 NR = 180, 191, 10
      ANR = NR
      VARP =(1/ANS-1/ANP)*(YPMU40-SIGYP2**2)
      1 + (1/ANR-1/ANS)*(YPMU40+(SIGYP2/
      SIGXP2)**2*XPMU04
      1-2*(SIGYP2/SIGXP2)*YPXPMU22)
      NITRP1 = 100
      NITRP2 = 100
      VARVARE = 0.0
      NITRP = 0.0
      DO 9999 III=1, NITRP1
      ISEED = 13031963
      CALL RNSET (ISEED)
      CALL RNSRI(NS,NP,IR)
      DO 12 I=1,NS
      YF(I) = YP(IR(I))
12 XF(I) = XP(IR(I))
      SUMXF = 0.0
      DO 61 I =1, NS
61 SUMXF = SUMXF + XF(I)
      XFM = SUMXF/ANS
      VARXF = 0.0
      DO 62 I=1, NS
62 VARXF = VARXF +(XF(I)-XSM)**2
      VARXF = VARXF/(ANS-1)
      DO 27777 JJJ = 1, NITRP2
      ISEED = 13031963
      CALL RNSET (ISEED)
      CALL RNSRI(NR, NS, IP)
      DO 14 I=1, NR
      YS(I) = YF(IP(I))
14 XS(I) = XF(IP(I))
      SUMYS = 0.0
      SUMXS = 0.0
      DO 16 I=1, NR
      SUMYS = SUMYS + YS(I)
16 SUMXS = SUMXS + XS(I)
      YSM = SUMYS/ANR
      XSM = SUMXS/ANR
      VARYS = 0.0
      VARXS = 0.0
      DO 17 I =1, NR
      VARYS = VARYS + (YS(I)-YSM)**2
17 VARXS = VARXS + (XS(I)-XSM)**2
      VARYS = VARYS/(ANR-1)
      VARXS = VARXS/(ANR-1)
      VARE = VARYS*VARXF/VARXS
      VARVARE = VARVARE + (VARE-SIGYP2)**2
      NITRP = NITRP+1
27777 CONTINUE
9999 CONTINUE
      VARVARE = VARVARE/DBLE(NITRP)
      RAT = VARVARE/VARP
!*****************
      NITR = 100
      VARV11 = 0.0
      VARV21 = 0.0
      VARV31 = 0.0
      VARV41 = 0.0
      VARV12 = 0.0
      VARV22 = 0.0
      VARV32 = 0.0
      VARV42 = 0.0
      RBV11 = 0.0
      RBV21 = 0.0
      RBV31 = 0.0
      RBV41 = 0.0
      RBV12 = 0.0
      RBV22 = 0.0
      RBV32 = 0.0
      RBV42 = 0.0
      ANITR = 0.0
      DO 7777 IIII=1,NITR
      ISEED = 13031963
      CALL RNSET (ISEED)
      CALL RNSRI(NS,NP,IM)
      DO 31 I=1, NS
31 IR(I)=IM(I)
      DO 223 I =1, NS
      YF(I) = YP(IR(I))
223 XF(I) = XP(IR(I))
      NITRR = 100
      DO 6666 KKKK = 1, NITRR
      ISEED = 13031963
      CALL RNSET (ISEED)
      CALL RNSRI(NR,NS,IN)
      DO 214 I=1, NR
214 IT(I) = IN(I)
      DO 34 I=1, NR
      ID(I) = IM(IT(I))
      YS(I) = YF(IN(I))
34 XS(I) = XF(IN(I))
      K = 0
      DO 23 J =1, NR
      DO 21 I =1, NS
      IF(ID(J).EQ.IR(I))THEN
      K = K+1
      YS(K) = YP(IR(I))
```

```
      XS(K) = XP(IR(I))
      IIR(K) = IR(I)
      IR(I) = 0
      ELSE
      GO TO 21
      ENDIF
21 CONTINUE
23 CONTINUE
      K = NR
      NK = 0
      JJ=0
      DO 24 I=1, NS
      IF(IR(I).GT.0.01)THEN
      K = K+1
      NK = NK+1
      JJ = JJ+1
      IRS(JJ)=IR(I)
      XSNR(JJ) = XP(IR(I))
      ENDIF
24 CONTINUE
      JJJ = 0
      DO 28 I = 1, NS
      IF(I.LE.NR)THEN
      XSS(I)=XS(I)
      YSS(I)=YS(I)
      RAN(I)=IIR(I)
      ELSE
      IF(I.GT.NR)THEN
      JJJ=JJJ+1
      XSS(I)=XSNR(JJJ)
      YSS(I)=999999
      RAN(I)=IRS(JJJ)
      ENDIF
      ENDIF
28 CONTINUE
!******* files are merged ******
      AM11SS = 0.0
      AM20SS = 0.0
      AM02SS = 0.0
      AM40SS = 0.0
      DO 36 I = 1, NR
      DO 36 J =1, NR
      IF(I.NE.J) THEN
      AM11SS = AM11SS + (YSS(I)-YSS(J))*(XSS(I)-XSS(J))
      AM20SS = AM20SS +(YSS(I)-YSS(J))**2
      AM02SS = AM02SS +(XSS(I)-XSS(J))**2
      AM40SS = AM40SS +(YSS(I)-YSS(J))**4
      ELSE
      CONTINUE
      ENDIF
36 CONTINUE
      AM11SS = AM11SS/(2*ANR*(ANR-1))
      AM20SS = AM20SS/(2*ANR*(ANR-1))
      AM02SS = AM02SS/(2*ANR*(ANR-1))
      AM40SS = AM40SS/(2*ANR*(ANR-1))
      BHAT = AM20SS/AM02SS
      DO 38 I = 1, NR
      DO 38 J = 1, NR
      IF(I.NE.J) THEN
      D(I,J)=(YSS(I)-YSS(J))**2-BHAT*(XSS(I)-XSS(J))**2
      ELSE
      CONTINUE
      ENDIF
38 CONTINUE
      SUMDIDJ2 = 0.0
      DO 39 I=1, NR
      DO 39 J=1, NR
      IF(I.NE.J)THEN
      SUMDIDJ2 = SUMDIDJ2 + D(I,J)*D(I,J)
      ELSE
      CONTINUE
      ENDIF
39 CONTINUE
      SIGMDIJ2 = SUMDIDJ2/(2*ANR*(ANR-1))
      V1HAT = (1/ANR-1/ANS)*SIGMDIJ2
      1 +(1/ANS-1/ANP)*(AM40SS-(4*ANP-1)*AM20SS**2/
      ANP)
      1 -2* ((ANP-1)/ANP)*(1/ANR-1/ANS)
      1 *(2*AM20SS**2-BHAT*AM11SS**2)
      SIGDX2 = 0.0
      DO 40 I = 1, NR
      DO 40 J = 1, NR
      IF(I.NE.J)THEN
      SIGDX2 = SIGDX2 + D(I,J)*(XSS(I)-XSS(J))**2
      ELSE
      CONTINUE
      ENDIF
40 CONTINUE
      SIGDX2 = SIGDX2/(2.*ANR*(ANR-1))
      AM02FF = 0.0
      AM04FF = 0.0
      DO 44 I = 1, NS
      DO 44 J= 1, NS
      IF(I.NE.J)THEN
      AM02FF = AM02FF +(XSS(I)-XSS(J))**2
      AM04FF = AM04FF +(XSS(I)-XSS(J))**4
      ELSE
      CONTINUE
      ENDIF
44 CONTINUE
```

```
      AM02FF = AM02FF/(2*ANS*(ANS-1))
      AM04FF = AM04FF/(2*ANS*(ANS-1))
      V2HAT=(1/ANR-1/ANP)*SIGMDIJ2
      1+ (1/ANS-1/ANP)*BHAT**2*(AM04FF-(4*ANP-
      1)*AM02FF**2/ANP)
      1+2*(1/ANS-1/ANP)*BHAT*SIGDX2
      1-2*((ANP-1)/ANP)*(1/ANR-1/ANS)
      1*(2*BHAT**2*AM02FF**2-BHAT*AM11SS**2)
      V3HAT = (AM02FF/AM02SS)**2*(1/ANR-1/
      ANP)*SIGMDIJ2
      1+2*(AM02FF/AM02SS)*(1/ANS-1/
      ANP)*BHAT*SIGDX2
      1+(1/ANS-1/ANP)*BHAT**2*(AM04FF-((4*ANP-1)/
      ANP)*AM02FF**2)
      1-2*((ANP-1)/ANP)*(1/ANR-1/
      ANS)*(2*BHAT**2*AM02FF**2
      1-BHAT*AM11SS**2)*(AM02FF/AM02SS)**2
      DO 46 I = 1, NS
      DO 46 J = 1, NS
      IF(I.NE.J) THEN
      AM02F(I,J)=(2*ANS*(ANS-1)*AM02FF-(XSS(I)-
      XSS(J))**2)/
      1 (2*ANS*(ANS-1)-2)
      ELSE
      CONTINUE
      ENDIF
46 CONTINUE
      DO 47 I =1, NR
      DO 47 J =1, NR
      IF(I.NE.J)THEN
      AM02S(I,J)=(2*ANR*(ANR-1)*AM02SS-(XSS(I)-
      XSS(J))**2)/
      1(2*ANR*(ANR-1)-2)
      AM20S(I,J)=(2*ANR*(ANR-1)*AM20SS-(YSS(I)-
      YSS(J))**2)/
      1(2*ANR*(ANR-1)-2)
      ELSE
      CONTINUE
      ENDIF
47 CONTINUE
      DO 48 I=1, NS
      DO 48 J =1, NS
      IF(I.NE.J) THEN
      IF((I.LE.NR).AND.(J.LE.NR))THEN
      VJACK(I,J) = AM20S(I,J)*AM02F(I,J)/AM02S(I,J)
      ELSE
      IF((I.GT.NR).AND.(J.GT.NR))THEN
      VJACK(I,J)=AM20SS*AM02F(I,J)/AM02SS
      ENDIF
      ENDIF
      ENDIF
48 CONTINUE
      VJP1 = 0.0
      VJP2 = 0.0
      DO 49 I = 1, NS
      DO 49 J = 1, NS
      IF(I.NE.J) THEN
      IF((I.LE.NR).AND.(J.LE.NR))THEN
      VJP1 = VJP1 +(VJACK(I,J)-AM20SS*AM02FF/
      AM02SS)**2
      ELSE
      IF ((I.GT.NR).AND.(J.GT.NR))THEN
      VJP2 = VJP2 +(VJACK(I,J)-AM20SS*AM02FF/
      AM02SS)**2
      ENDIF
      ENDIF
      ENDIF
49 CONTINUE
      VJP = VJP1+VJP2
      V4HAT = (ANS*(ANS-1)-1)*VJP/((ANR-1)-1)
      VARV11 = VARV11 + (V1HAT-VARP)**2
      VARV21 = VARV21 + (V2HAT-VARP)**2
      VARV31 = VARV31 + (V3HAT-VARP)**2
      VARV41 = VARV41 + (V4HAT-VARP)**2
      VARV12 = VARV12 + (V1HAT-VARVARE)**2
      VARV22 = VARV22 + (V2HAT-VARVARE)**2
      VARV32 = VARV32 + (V3HAT-VARVARE)**2
      VARV42 = VARV42 + (V4HAT-VARVARE)**2
      RBV11 = RBV11 + V1HAT
      RBV21 = RBV21 + V2HAT
      RBV31 = RBV31 + V3HAT
      RBV41 = RBV41 + V4HAT
      RBV12 = RBV12 + V1HAT
      RBV22 = RBV22 + V2HAT
      RBV32 = RBV32 + V3HAT
      RBV42 = RBV42 + V4HAT
      ANITR = ANITR+1
6666 CONTINUE
7777 CONTINUE
      RE12 = VARV11*100/VARV21
      RE13 = VARV11*100/VARV31
      RE14 = VARV11*100/VARV41
      RE22 = VARV12*100/VARV22
      RE23 = VARV12*100/VARV32
      RE24 = VARV12*100/VARV42
      RBV11 = (RBV11/ANITR-VARP)*100/VARP
      RBV21 = (RBV21/ANITR-VARP)*100/VARP
      RBV31 = (RBV31/ANITR-VARP)*100/VARP
      RBV41 = (RBV41/ANITR-VARP)*100/VARP
      RBV12 = (RBV12/ANITR-VARVARE)*100/VARVARE
      RBV22 = (RBV22/ANITR-VARVARE)*100/VARVARE
```

```
RBV32 = (RBV32/ANITR-VARVARE)*100/VARVARE
RBV42 = (RBV42/ANITR-VARVARE)*100/VARVARE
IF(
(RE12.GT.100).AND.(RE13.GT.100).AND.(RE14.GT.100))
THEN
WRITE(*,128)G,R,NP,NS,NR,RE12,RE13,RE14,RE22,
RE23,RE24,
1 RBV11,RBV21,RBV31,RBV41,RBV12,RBV22,RBV32,
RBV42,RHOXYP,RAT
WRITE(42,128)G,R,NP,NS,NR,RE12,RE13,RE14,RE22,
RE23,RE24,
1
RBV11,RBV21,RBV31,RBV41,RBV12,RBV22,RBV32,RBV42,
RHOXYP,RAT
128 FORMAT(2X,F6.2,2X,F7.2,2X,I5,2X,I3,2X,I3,2X,F9.2,
2X,F9.2,2X,
    1 F9.2,2X,F9.2,2X,F9.2,2X,F9.2,3X,F9.5,3X,F9.4,3X,
    F9.4,3X,F9.4,
    1 F9.4,3X,F9.4,3X,F9.4,3X,F9.4,2X,F9.4,2X,F9.4)
    ENDIF
5555 CONTINUE
8888 CONTINUE
STOP
END
```