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Approximately Optimum Stratification for Two Study Variables using Auxiliary Information

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SUMMARY

In this paper, we have tried to develop the theoretical frame work for determination of optimum strata boundaries on an auxiliary variable (X) closely related with the study variables, appropriate for surveys involving two study variables Y_i $(i = 1, 2)$. For this purpose, the forms of regression of estimation variables on the stratification variable as also the forms of conditional variance functions $V(y_i \mid x)$ are assumed to be known. By minimizing the generalized variance of the sample means of the study variables, minimal equations have been obtained under proportional method of allocation. Due to implicit nature of these equations, a cum $\sqrt[3]{\mathbf{R}(\mathbf{x})}$ rule has been proposed for obtaining approximately optimum strata boundaries. Empirical studies have also been made on certain density functions.

Key words : Optimum stratification, Strata boundaries, Super- population, Minimal equation.

1. Introduction

In stratified random sampling, the efficiency of the estimator of population parameters depends on several factors such as choice of stratification variable, number of strata, determination of strata boundaries and allocation of sample sizes to different strata. Once it is decided about the total number of strata and the procedure of allocating sample sizes to different strata, the problem of stratification may be considered to consist of determination of strata boundaries.

The pioneering work in this field was done by Dalenius [1]. Dalenius and Gurney [2], Dalenius and Hodges [3], Taga [8], Singh and Sukhatme [7], Singh [5], and several other workers considered the problem of optimum

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stratification with respect to an auxiliary variable closely related to study variable.

Ghosh [4] considered the problem of optimum stratification with two characters, under proportional allocation, assuming stratification variables as identical to the estimation variables under consideration. Factually, it is unrealistic to presume that the distribution of the estimation variable is known in advance. Thus, in the present paper we have considered this problem for two study variables Y_i (i = 1, 2) by taking an auxiliary variable (X) as stratification variable. Here, we have assumed that the finite population under consideration is a random sample from an infinite super-population with the same population characteristics. Further, a priori knowledge of the forms of regression of Y_i on X and the forms of conditional variance functions $V(y_i \mid x)$ are also assumed.

The sets of minimal equations giving optimum strata boundaries have been developed under proportional method of allocation. These equations are implicit in nature and very difficult to solve. Hence a cum $\sqrt[3]{\mathbf{R}(\mathbf{x})}$ method of finding approximately optimum strata boundaries (AOSB) has been proposed. The paper concludes with numerical illustrations for three density functions.

2. *Variance and Covariance under Super-population Set-up*

Assuming that we are interested in estimating the population means of two study variables of a finite population of size N which is heterogeneous in nature. Then it is to be divided into an optimum number of non-overlapping strata, say, L and a stratified simple random sample of size n is to be drawn from it by taking independent samples of sizes $n_1, n_2, \ldots, n_L$, respectively, from the L strata so that $\sum_{h=1}^{L} n_h = n$.

If the finite population correction can be neglected then the variance expressions of the unbiased estimators $\bar{y}_{ist}$ (i = 1, 2), under proportional allocation, can be given by

$$V(\bar{y}_{ist}) = \frac{1}{n} \sum_{h=1}^{L} W_h \sigma_{hy_i}^2 \qquad (i = 1, 2) \tag{2.1}$$

where W_h is the h-th stratum weight and $\sigma_{hy_i}^2$ denote the population variances for the characters Y_i (i = 1, 2) in the h-th stratum.

Further, if we assume that the study variables Y_i (i = 1, 2) are correlated then the covariance term, under proportional allocation, is given by

$$\mathrm{Cov}\,(\bar{y}_{1st}, \bar{y}_{2st})_P = \frac{1}{n} \sum_{h=1}^{L} W_h \sigma_{hy_1 y_2} \tag{2.2}$$

where $\sigma_{hy_1y_2}$ denotes the population covariance between Y_1 and Y_2 in the h-th stratum.

Let us now assume that the finite population under consideration is a random sample from an infinite super population with the same population characteristics. Further, assuming that the study variables are linearly related with the auxiliary variable, so that the regression of Y_i (i = 1, 2) on X be given by the linear model

$$Y_i = C_i(X) + e_i \tag{2.3}$$

where $C_i(X)$ is a real valued function of X, e_i is error component such that $E(e_i \mid X) = 0$, $E(e_i e_i' \mid X, X') = 0$, for $x \neq x'$ and $V(e_i \mid X) = \eta_i(x) > 0$, $i = 1, 2$, for all $x \in (a, b)$ where $(b - a) < \infty$. It may also be noted that $E(e_i C_j) = 0$, but $E(C_1 C_2) \neq 0$.

If the joint density function of (X, Y_1, Y_2) in the super-population is $f_s(x, y_1, y_2)$ and the marginal density function of X is f(x), then under the model (2.3), it can be easily seen that

$$W_h = \int_{x_{h-1}}^{x_h} f(x)\,dx$$

$$\mu_{hy_i} = \mu_{hc_i} = \frac{1}{W_h} \int_{x_{h-1}}^{x_h} c_i(x) f(x)\,dx$$

$$\sigma^2_{hc_i} = \frac{1}{W_h} \int_{x_{h-1}}^{x_h} c_i^2(x) f(x)\,dx - (\mu_{hc_i})^2 \tag{2.4}$$

$$\sigma_{hc_1c_2} = \frac{1}{W_h} \int_{x_{h-1}}^{x_h} c_1(x)\, c_2(x) f(x)\,dx - \mu_{hc_1}\mu_{hc_2}$$

$$\sigma^2_{hy_i} = \sigma^2_{hc_i} + \mu_{h\eta_i}$$

and $$\sigma_{hy_1y_2} = \sigma_{hc_1c_2} \qquad (i = 1, 2)$$

where (x_{h-1}, x_h) are the boundaries of the h-th stratum, $\mu_{h\eta_i}$ are the expected values of the functions $\eta_i(x)$, the conditional variance, in the h-th stratum.

The variance and covariance expressions under the super- population set-up for the case of proportional allocation are, therefore, given by

$$\sigma_i^2 = V(\bar{y}_{ist})_P = \frac{1}{n}\sum_{h=1}^{L} W_h (\sigma^2_{hc_i} + \mu_{h\eta_i}) \qquad (i = 1, 2) \tag{2.5}$$

and $$\sigma_{12} = Cov(\bar{y}_{1st}, \bar{y}_{2st})_P = \frac{1}{n}\sum_{h=1}^{L} W_h \sigma_{hc_1c_2} \tag{2.6}$$

3. *Minimal Equations*

Let us assume that the stratification variable X is continuous with p.d.f. f(x), $a \le x \le b$, and the points of demarcation forming L strata are $x_1, x_2, \ldots, x_{L-1}$. Let us denote the set of optimum points of stratification as $\{x_h\}$, then corresponding to these strata boundaries the generalized variance G, the determinant of variance-covariance matrix, which is a function of points of stratification, is minimum. These points $\{x_h\}$ are the solutions of the minimal equations which are obtained by equating to zero the partial derivatives of

$$G = \begin{vmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{vmatrix}$$

$$= \sigma_1^2 \sigma_2^2 - (\sigma_{12})^2 \tag{3.1}$$

with respect to $\{x_h\}$. Hence we get

$$\sigma_1^2 \frac{\partial \sigma_2^2}{\partial x_h} + \sigma_2^2 \frac{\partial \sigma_1^2}{\partial x_h} - 2\sigma_{12}\frac{\partial \sigma_{12}}{\partial x_h} = 0, \qquad h = 1, 2, \ldots, L-1 \tag{3.2}$$

By putting the respective terms of σ_i^2 and σ_{12} from (2.5) and (2.6) in (3.2), and then taking their partial derivatives, we obtain

$$\sigma_1^2\left[W_h\frac{\partial\sigma_{hc_2}^2}{\partial x_h}+\sigma_{hc_2}^2\frac{\partial W_h}{\partial x_h}+W_j\frac{\partial\sigma_{jc_2}^2}{\partial x_h}+\sigma_{jc_2}^2\frac{\partial W_j}{\partial x_h}\right]$$
$$+\sigma_2^2\left[W_h\frac{\partial\sigma_{hc_1}^2}{\partial x_h}+\sigma_{hc_1}^2\frac{\partial W_h}{\partial x_h}+W_j\frac{\partial\sigma_{jc_1}^2}{\partial x_h}+\sigma_{jc_1}^2\frac{\partial W_j}{\partial x_h}\right]$$
$$-2\sigma_{12}\left[W_h\frac{\partial\sigma_{hc_1c_2}}{\partial x_h}+\sigma_{hc_1c_2}\frac{\partial W_h}{\partial x_h}+W_j\frac{\partial\sigma_{jc_1c_2}}{\partial x_h}+\sigma_{jc_1c_2}\frac{\partial W_j}{\partial x_h}\right]=0 \quad (3.3)$$

The values of the partial derivative terms involved in (3.3) can be easily obtained on the lines of Singh and Sukhatme [7]. By inserting these values in (3.3) and simplifying we shall get the required minimal equation as

$$\sigma_1^2(c_2(x_h)-\mu_{hc_2})^2+\sigma_2^2(c_1(x_h)-\mu_{hc_1})^2-2\sigma_{12}(c_1(x_h)-\mu_{hc_1})(c_2(x_h)-\mu_{hc_2})$$
$$=\sigma_1^2(c_2(x_h)-\mu_{jc_2})^2+\sigma_2^2(c_1(x_h)-\mu_{jc_1})^2-2\sigma_{12}(c_1(x_h)-\mu_{jc_1})(c_2(x_h)-\mu_{jc_2})$$

where $j = h+1, h = 1, 2, \ldots, L-1$ (3.4)

On solving the equations (3.4), we can get the optimum points of stratification $\{x_h\}$. Obviously, the minimal equations (3.4) are the functions of parameter values which are themselves functions of points of strata boundaries, thereby creating difficulty in finding out the exact solutions of the minimal equations. Hence we shall obtain approximate solutions to this system of equations.

4. Approximate Solution of the Minimal Equations

For obtaining the approximate solutions to the minimal equations (3.4), we have to expand both sides of minimal equation about the point x_h, the common boundary point of the h-th and i-th strata. The series expansions for W_h, μ_{hc_1}, μ_{hc_2}, $\sigma_{hc_1}^2$, $\sigma_{hc_2}^2$ and $\sigma_{hc_1c_2}$ can be obtained by using Taylor's theorem about both the upper and lower boundaries of the h-th stratum, on the lines of Singh and Sukhatme [7]. Thus the different terms involved in the left hand side of the minimal equations (3.4) can be obtained as

$$(c_i(x_h)-\mu_{hc_i})^2=\frac{K_h^2}{4}[c_i'^2-\frac{(f'c_i'^2+2fc_i'c''_i)}{3f}K_h$$
$$+\frac{(6ff''c_i'^2+10ff'c'_ic''_i+6f^2c'_ic''_i-5f'^2c_i'^2+4f^2c''^2_i)}{36f^2}K_h^2+0(K_h^3)]\ (i=1,2)$$

and the cross product term is given by

$$(c_1(x_h) - \mu_{hc_1})(c_2(x_h) - \mu_{hc_2})$$

$$= \frac{K_h^2}{4}\left[c'_1 c'_2 - \frac{[f' c'_1 c'_2 + f(c''_1 c'_2 + c'_1 c''_2)]}{3f} K_h + 0(K_h^2)\right]$$

where $K_h = x_h - x_{h-1}$

Further, the system of equations (3.4) also involves the terms σ_i^2 $(i = 1, 2)$ and σ_{12}. The expansions of these terms are given in the form of following lemmas and corollary. The proofs of these results can be easily obtained by proceeding on the lines of Singh [6].

Lemma 4.1 : In case of proportional allocation, variance $V(\bar{y}_{ist})_P$ can be expressed as

$$nV(\bar{y}_{ist})_P = \mu_{\eta_i} + \frac{1}{12}\sum_{h=1}^{L} K_h^2 \int_{x_{h-1}}^{x_h} c_i'^2(t) f(t)\, dt\, [1 + 0(K_h^2)] \tag{4.1}$$

where $\mu_{\eta_i} = \int_a^b \eta_i(t) f(t) dt \qquad (i = 1, 2)$

Lemma 4.2 : In case of proportional allocation, the covariance between $\bar{y}_{1st}$ and $\bar{y}_{2st}$ can be expressed as

$$n\,\mathrm{Cov}(\bar{y}_{1st}, \bar{y}_{2st})_P = \frac{1}{12}\sum_{h=1}^{L} K_h^2 \int_{x_{h-1}}^{x_h} c'_1(t)\, c'_2(t)\, f(t)\, dt\, [1 + 0(K_h^2)] \tag{4.2}$$

Using all these results in the minimal equation and retaining the terms upto order $0(m^3)$, $m = \underset{(a,b)}{\mathrm{Sup}}(K_h)$, the expanded form of the equations (3.4) can be obtained as

$$\frac{K_h^2}{4}\left[(\mu_{\eta_1} fc_2'^2 + \mu_{\eta_2} f c_1'^2) - \frac{K_h}{3}\frac{d}{d x_h}(\mu_{\eta_1} fc_2'^2 + \mu_{\eta_2} f c_1'^2) + 0(K_h^{\ 2})\right]$$

$$= \frac{K_j^2}{4}\left[(\mu_{\eta_1} fc_2'^2 + \mu_{\eta_2} f c_1'^2) - \frac{K_j}{3}\frac{d}{d x_h}(\mu_{\eta_1} fc_2'^2 + \mu_{\eta_2} f c_1'^2) + 0(K_j^{\ 2})\right] \tag{4.3}$$

hich can again be simplified as

$$(\mu_{\eta_1} f c_2'^2 + \mu_{\eta_2} f c_1'^2)^{1/3} \left[K_h^2 \int_{x_{h-1}}^{x_h} l(t) f(t) dt \, [1 + 0(K_h^2)] \right]^{2/3}$$

$$= (\mu_{\eta_1} f c_2'^2 + \mu_{\eta_2} f c_1'^2)^{1/3} \left[K_j^2 \int_{x_h}^{x_{h+1}} l(t) f(t) dt \, [1 + 0(K_j^2)] \right]^{2/3} \quad (4.4)$$

where $l(t) = \mu_{\eta_1} c_2'^2(t) + \mu_{\eta_2} c_1'^2(t), \; j = h+1, \; h = 1, 2, ..., L-1$

Therefore, if we have large numbers of strata so that the strata widths K_h are small and higher powers of K_h in the expression can be neglected, the system of minimal equations (3.4) or equivalently the system of equations (4.3) can be approximately put as

$$\left[K_h^2 \int_{x_{h-1}}^{x_h} l(t) f(t) dt \right]^{2/3} = \left[K_j^2 \int_{x_h}^{x_{h+1}} l(t) f(t) dt \right]^{2/3} \quad (4.5)$$

which gives

$$K_h^2 \int_{x_{h-1}}^{x_h} l(t) f(t) dt = \text{Constant}, \; h = 1, 2, \ldots, L \quad (4.6)$$

where terms of order $0(m^4)$, $m = \sup_{(a,b)} (K_h)$, have been neglected on both the sides of the equation. Further, if we have a function $Q_1(x_{h-1}, x_h)$ which is such that

$$K_h^2 \int_{x_{h-1}}^{x_h} l(t) f(t) dt = Q(x_{h-1}, x_h) \, [1 + 0(K_h^2)] \quad (4.7)$$

then the system of minimal equation (3.4) can, to the same degree of approximation as involved in (4.6), be approximated by

$$Q(x_{h-1}, x_h) = \text{Constant}, \; h = 1, 2, \ldots, L \quad (4.8)$$

Various methods of finding approximate solutions to the minimal equations (3.4) can be established through the system of equations (4.6). Singh and Sukhatme [7] developed different forms of $Q(x_{h-1}, x_h)$ corresponding to univariate case, under Neyman allocation. Proceeding on the same lines, one such form of the function $Q(x_{h-1}, x_h)$ can be obtained as

$$\left[\int_{x_{h-1}}^{x_h} \sqrt[3]{l(t)\, f(t)}\, dt\right]^3 = \text{Constant (say, C)} \qquad (4.9)$$

where $$C = \frac{1}{L^3}\left[\int_a^b \sqrt[3]{l(t)\, f(t)}\, dt\right]^3$$

Thus we get the following $\sqrt[3]{R(x)}$ rule of finding AOSB in case of two study variables.

Cum $\sqrt[3]{R(x)}$ Rule :

If the function $R(x) = l(x)\, f(x)$, where $l(x) = \mu_{\eta_1} c_2'^2(x) + \mu_{\eta_2} c_1'^2(x)$, is bounded and its first two derivatives exist for all x in (a, b), then for a given value of L taking equal intervals on the cumulative cube root of R(x) will give AOSB $\{x_h\}$.

5. *Limiting Form of the Generalised Variance*

Here we shall obtain a limit expression of the generalized variance G, as given by (3.1), through which we shall be able to infer as to how the value of G changes with an increase/decrease in the number of strata.

Multiplying both the sides of (3.1) by n^2, where n is the sample size, we have

$$n^2 G = (n\sigma_1^2)(n\sigma_2^2) - (n\sigma_{12})^2 \qquad (5.1)$$

Now inserting the approximate values of $n\sigma_i^2$ $(i = 1, 2)$ and $n\sigma_{12}$ from (4.1) and (4.2), respectively, in (5.1) and thereafter solving and neglecting the terms of $0(m^4)$, $m = \sup\limits_{(a, b)} (K_h)$ we shall get

$$n^2 G = \mu_{\eta_1}\mu_{\eta_2} + \frac{\mu_{\eta_1}}{12}\sum_{h=1}^{L} K_h^2 \int_{x_{h-1}}^{x_h} c_2'^2(t)\, f(t)\, dt + \frac{\mu_{\eta_2}}{12}\sum_{h=1}^{L} K_h^2 \int_{x_{h-1}}^{x_h} c_1'^2(t)\, f(t)\, dt \qquad (5.2)$$

Now, by using the result (3.8) of Singh and Sukhatme [7], the equation (5.2) can be put as

$$n^2 G = \mu_{\eta_1} \mu_{\eta_2} + \frac{\mu_{\eta_1}}{12} \sum_{h=1}^{L} \left[\int_{x_{h-1}}^{x_h} \sqrt[3]{c'^2_2(t) f(t)}\, dt \right]^3$$

$$+ \frac{\mu_{\eta_2}}{12} \sum_{h=1}^{L} \left[\int_{x_{h-1}}^{x_h} \sqrt[3]{c'^2_1(t) f(t)}\, dt \right]^3$$

or
$$n^2 G = \mu_{\eta_1} \mu_{\eta_2} + \frac{1}{12} \sum_{h=1}^{L} K_h^2 \int_{x_{h-1}}^{x_h} l(t) f(t)\, dt \tag{5.3}$$

which can further be obtained as

$$n^2 G = \mu_{\eta_1} \mu_{\eta_2} + \frac{1}{12} \sum_{h=1}^{L} \left[\int_{x_{h-1}}^{x_h} \sqrt[3]{l(t) f(t)}\, dt \right]^3 \tag{5.4}$$

In case the strata boundaries are determined by making use of the proposed cum $\sqrt[3]{R(x)}$ rule, then for h=1, 2, . . ., L

$$\int_{x_{h-1}}^{x_h} \sqrt[3]{l(t) f(t)}\, dt = \frac{1}{L} \int_a^b \sqrt[3]{l(x) f(x)}\, dx \tag{5.5}$$

Using (5.5) in (5.4), we get

$$G = \frac{1}{n^2} \left[\mu_{\eta_1} \mu_{\eta_2} + \frac{\beta}{12 L^2} \right] \tag{5.6}$$

where
$$\beta = \left[\int_a^b \sqrt[3]{l(x) f(x)}\, dx \right]^3$$

Now taking limit as $L \to \infty$ on both the sides of (5.6), we get

$$\lim_{L \to \infty} G = \frac{\mu_{\eta_1} \mu_{\eta_2}}{n^2} \tag{5.7}$$

From the limiting expression (5.7) it is obvious that as the number of strata L increases the generalized variance goes on decreasing and it approaches to $\mu_{\eta_1} \mu_{\eta_2}/n^2$.

6. Empirical Study

The effectiveness of the proposed method of finding the set $\{x_h\}$ of AOSB has been demonstrated empirically. For this purpose the following three density functions of the stratification variable X have been considered as follows :

1. Uniform Distribution : $f(x) = 1, 1 \leq x \leq 2$
2. Right Triangular Distribution : $f(x) = 2(2-x), 1 \leq x \leq 2$
3. Exponential Distribution : $f(x) = e^{-x+1}, 1 \leq x \leq \infty$

Among the three densities considered here, the range of the exponential distribution is infinite. But, as stated in Section 2, we have assumed that the range of the stratification variable should be finite. And, therefore, in case of exponential distribution we have truncated it at x = 6, so that the area on the right of this point becomes even less than 0.01. For the sake of simplicity, the slopes of the regression lines Y_1 on X, and Y_2 on X have been taken as 45 degree and 63.5 degree, respectively, so that the regression lines take the form $Y_1 = \alpha_1 + X + e_1$ and $Y_2 = \alpha_2 + 2X + e_2$, respectively. The conditional variances of the error terms, i.e. $V(e_1 | x)$ and $V(e_2 | x)$ are assumed to be of the forms $A_1 x^{g_1}$ and $A_2 x^{g_2}$ respectively, where $A_1, A_2 > 0$ and g_1 and g_2 being the constants. The values for $g_1, g_2 = 0, 1, 2$ were considered. The values of the constants A_1 and A_2 were determined for each value of g_1 and g_2 the correlation coefficients ρ_1 and ρ_2 between the study variables Y_i (i = 1, 2) with the stratification variable X, respectively, by using the following formulae

$$A_1 = \frac{\beta_1^2 \sigma_x^2 (1-\rho_1^2)}{\rho_1^2 E(x^{g_1})} \tag{6.1}$$

$$A_2 = \frac{\beta_2^2 \sigma_x^2 (1-\rho_2^2)}{\rho_2^2 E(x^{g_2})} \tag{6.2}$$

where σ_x^2 is the variance for the stratification variable X.

For the purpose of numerical illustration we have assumed $\rho_1^2 = 0.80$ and $\rho_2^2 = 0.50$.

The AOSB were obtained by using proposed cum $\sqrt[3]{R(x)}$ rule for the two study variables for all the three densities considered, through appropriate computer programme. The variances and covariances were calculated by using

Table 1. AOSB and per cent RE with respect to no stratification (uniform distribution)

L	AOSB							$n^2 G$	% R.E.
1	1	2						.0417154	–
2	1	1.49953	2					.0155671	267.97
3	1	1.33280	1.66561	2				.0107819	386.90
4	1	1.24949	1.49898	1.74847	2			.0091278	457.01
5	1	1.19948	1.39897	1.59845	1.79793	2		.0083498	499.60
6	1	1.16618	1.33235	1.49853	1.66471	1.83089	2	.0079245	526.41

Table 2. AOSB and per cent RE with respect to no stratification (right triangular distribution)

L	AOSB							$n^2 G$	% R.E.
1	1	2						.0185131	-
2	1	1.40492	2					.0074194	249.52
3	1	1.26179	1.56029	2				.0050893	363.76
4	1	1.19368	1.40447	1.64476	2			.0042363	437.01
5	1	1.15368	1.31735	1.49553	1.69852	2		.0038313	483.21
6	1	1.12737	1.26134	1.40402	1.55929	1.73617	2	.0036084	513.05

Table 3. AOSB and per cent RE with respect to no stratification (exponential distribution)

L	AOSB							$n^2 G$	% R.E.
1	1	6						4.222815	-
2	1	2.56028	6					1.842004	229.25
3	1	1.94569	3.33292	6				1.253156	336.97
4	1	1.67971	2.55935	3.80872	6			1.028900	410.42
5	1	1.53078	2.17593	2.99829	4.13486	6		0.921131	458.44
6	1	1.43549	1.94501	2.55853	3.33067	4.37304	6	0.861415	490.22

the expressions (2.5) and (2.6), respectively. Then the value of the generalized variance G was obtained and percent relative efficiency (RE) of optimum stratification as compared to no stratification (L=1) were calculated thereof. The AOSB and RE figures are given in Tables 1 to 3.

The percent RE ranges between 267.97 to 526.41 in case of uniform distribution, 249.52 to 513.05 in case of right triangular distribution and it lies between 229.25 to 490.22 for exponential distribution. These figures show a considerable gain in efficiency of estimators when the proposed method of determining AOSB is used. It is also observed that the percent RE increases as the number of strata (L) increases. However, the rate of increase in RE decreases as L increases. Further, it was found that the AOSB as well as RE values were almost same for all pairs of g_1 and g_2 (g_1, g_2 = 0, 1, 2). Therefore, only one table has been provided for each distribution under consideration.

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