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A Series of Group Divisible Designs from Self-complementary BIB Designs

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SUMMARY

A method of construction of regular group divisible designs from self complementary BIB designs is given. The procedure has been illustrated with the help of an example.

Key Words : Self-complementary BIB design, PBIB design, GD design.

1. Introduction

Group divisible designs form a very important class of two associate partially balanced incomplete block (PBIB) designs. Constructional aspects of these designs have been the focus of attention for quite some time. The earlier results are reported in Raghavarao [6] and Clatworthy [2]. Clatworthy [2] also tabulated these designs for $r, k \le 10$. Later results were found in references given in Sinha and Kageyama [9] and Sinha [10]. Sinha [10] provided a list of new group divisible designs not found in the table of Clatworthy [2]. Here, a construction of self-complementary Balanced Incomplete Block (BIB) designs and a class of regular group divisible designs therefrom is given. The method of construction is similar to that of John [4].

2. The Construction

A design is said to be self-complementary if in the design half of the blocks are obtained as complementary of the remaining half. These designs were also discussed by Preece [5].

Theorem 2.1

The existence of a BIB design with parameters

$$v = 2k + 1, b, r, k, \lambda \text{ with } b = 3r - 2\lambda \tag{2.1}$$

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implies the existence of a self-complementary BIB design with parameters

$$v' = 2(k+1),\ b' = 2b,\ r' = b, k' = k+1,\ \lambda' = r \qquad (2.2)$$

and hence a Group Divisible Design with parameters

$$v_1 = 2(k+1),\ b_1 = 2(b-1),\ r_1 = b-1,\ k_1 = k+1,$$

$$\lambda_1 = r-1,\ \lambda_2 = r,\ m = 2,\ n = k+1 \qquad (2.3)$$

Proof : Let D_1 denotes the blocks of a BIB design with parameters (2.1). Then to each block of BIBD we add an invariant treatment '∞' once only to get an arrangement D_1^*. Let D_2 denotes blocks of BIBD which are complement of the blocks of the design with parameters (2.1). Now consider the design $D = D_1^* \cup D_2$. Since $\lambda_{i\infty} = r$, and $\lambda_{ii'} = b - 2r + 2\lambda = r, i \neq i' = 1, 2, \ldots, v,$ by (2.1) the design D represents the self complementary BIBD with parameters (2.2). The other parameters follow obviously.

By deleting one complete replicate from the self complementary resolvable BIBD with parameters (2.2) we get a Group Divisible Design (GDD) with parameters (2.3). Q.E.D.

It may be noted that the treatments in the same block of a deleted replicate form a group.

Remark : The GDD with parameters (2.3) is regular if $r > 2\lambda + 1$.

Cor. 2.1.1.

The series of BIBD with parameters

$$v = 2cs+1, \qquad b = 2c\ (2cs+1)$$

$$r = 2c^2 s,\ k = cs, \qquad \lambda = c\,(cs-1) \qquad (2.4)$$

(cf. Saha [7]) yields a self-complementary BIBD with parameters $v = 2(cs+1),\ b = 4c(2cs+1),\ r = 2c(2cs+1),\ k = cs+1,\ \lambda = 2c^2 s$

(2.5)

Now, by deleting one replicate from the above series of BIBD we get a group divisible design with parameters

$$v_1 = 2(cs+1), \qquad b_1 = 4c(2cs+1) - 2$$
$$r_1 = 2c(2cs+1) - 1, \qquad k_1 = cs+1$$
$$\lambda_1 = 2c^2 s - 1, \qquad \lambda_2 = 2c^2 s \qquad (2.6)$$

It is clear that $r_1 - \lambda_1 > 0$ and $r_1 k_1 - v_1 \lambda_2 > 0$ so that the design with parameters given in (2.6) represents a regular GD design. Since $\lambda_2 = \lambda_1 + 1$, the design happens to be most balanced GD design of type 1 (cf. Cheng [1]).

It is to be noted that for $v = 4t+3$ the BIBD of John [4], with parameters

$$v = b = 4t+3, \; r = k = 2t+1, \; \lambda = t \tag{2.7}$$

can be obtained from half of the blocks of (2.4) with $c = 1$.

And consequently using the method just outlined, we obtain the following series of SRGD designs :

$$v_1^* = 4(t+1), b_1^* = 4(2t+1), r_1^* = 2(2t+1), k_1^* = 2(t+1)$$

$$\lambda_1^* = 2t, \; \lambda_2^* = \lambda_1^* + 1 \tag{2.7}$$

Since in many situations, PBIB designs with more than 10 replications are useful, the parameters of GDD with replications in the range $10 \le r \le 20$ are exhibited in Table 1 (cf Theorem 2.1).

Cor. 2.1.2.

The BIBD with parameters

$$v = b = 4h^2 - 1, \; r = k = 2h^2 - 1, \; \lambda = h^2 - 1 \tag{2.8}$$

obtained from Shrikhande and Singh [8] provides a self complementary BIB Design and hence a GD Design with parameters :

$$v_1 = 4h^2, b_1 = 4(2h^2 - 1), r_1 = 2(2h^2 - 1), k_1 = 2h^2, \; \lambda_1 = 2(h^2 - 1),$$

$$\lambda_2 = \lambda_1 + 1$$

which is semi regular.

It may be mentioned in this context that John [4] also gives a method of construction of a SRGD design from self complementary BIBD for the following two series :

$$v = s^N + s^{N-1}, \; m = s+1, \; k = v/s \text{ and } v = 6t+6, \; m = 3, \; k = 2t+2$$

We now illustrate the method of Cor. 2.1.2. with the help of an example.

Example : Let us consider a Balanced Incomplete Block Design with parameters

$$v = b = 15,\ r = k = 7,\ \lambda = 3$$

which corresponds to h = 2 in (2.8). This design can be obtained by cyclic substitution in the initial block

(a, b, c, e, f, i, k)

(see Fisher and Yates' Table [3]). Now by applying Cor. 2.1.2. of Theorem 2.1, we get an SRGD with parameters

$$v_1 = 16,\ b_1 = 28,\ r_1 = 14,\ k_1 = 8,\ \lambda_1 = 6,\ \lambda_2 = 7$$

Table 1. The parameters of GDD with replication in the range $10 \leq r \leq 20$.

Sl. No.	v_1	b_1	r_1	k_1	λ_1	λ_2
1	12	20	10	6	4	5
2	8	26	13	4	5	6
3	16	28	14	8	6	7
4	10	34	17	5	7	8
5	20	36	18	10	8	9
6	8	40	20	4	8	9

The designs 1, 3 and 5 are SRGD and can also be obtained from John [4]. The other designs viz. 2, 4 and 6 are regular.

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