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Nonparametric Analysis of Multifactor Repeated Measurements Experiments

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SUMMARY

Occasionally we are confronted with data arising from a repeated measures experiment when the usual assumptions namely those of homogeneity, symmetry and sphericity of the analysis of variances are not satisfied. The nonparametric methods provide realistic alternative in analysis of such data. A number of hypotheses for the analysis of multifactor repeated measurements of interest are hypothesis of no main effects, the hypothesis of no interaction effects. Various formulations of these hypotheses are discussed under several combination of assumption concerning the joint distribution of the components of the observations vector. The problem is greatly simplified when it is possible to use the tools of multivariate analysis of variance instead of univariate analysis of variance. The nonparametric univariate and multivariate techniques are discussed based on ranks. The objectives of the study are to execute particular aspects of the analysis of three factor repeated measures data and its utility and practicability are also demonstrated by a numerical example.

Keywords: Direct sum, Direct product, Kronecker product, Symmetry and sphericity, Univariate analysis, Multivariate analysis.

1. Introduction

The repeated measures design is a powerful experimental procedure for studying the evolution of a response measures and which have received a great deal of attention in agricultural, biological, psychological and pre-clinical research (see, Rahman [16], Madsen [12], Lana and Lubin [10], Islam [6], [7]). Nevertheless, the need for developing theoretical nonparametric test without an explicit assumption of normality for error distribution has been recognized for quite some time in literature. Several nonparametric tests (see, Friedman [5], Kruskal and Wallis [9]; Bhapker [1] and Puri [14] are available for the analysis of such data. Koch and Sen [8] analysed a mixed model without interaction and one observation per cell using the method or ranking. Rai and

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Rao [17] have developed statistics from ranked data as a nonparametric alternative for the analysis of data from groups of experiments. Their methods made use of the assumption of normality of rank sums and is applicable only when the number of replications per treatment was four or more. Prabhakaran and Jhon [13] extended the well known Friedman's two way analysis of variance for rank data collected from groups of experiments where the ordinary analysis of variance could not validly be applied. However, for the analysis of multivariate experimental data under the assumption of multivariate normal distribution of random error, no single multivariate analysis of variance test is unequally optimal, and no definite consensus seems to have been reached as to which of several MANOVA test should be used (see, Lee [11]). Discarding the stringent assumption of normality some nonparametric tests have been developed for the multivariate problem (see, Bhapker [2], Suguria [18]).

In this paper we proposed nonparametric univariate and multivariate testing procedures based on ranks that can be applied for the tests of different effects of multifactor repeated measures data. The proposed univariate test statistics for testing the null hypothesis of different main effects and interaction effects are discussed on the basis of Friedman [5] and Chatterjee and Sen [3] which represent a multivariate version of the Kruskal and Wallis [9] test. The nonparametric multivariate test statistics for testing the different effects are discussed on the basis of David and Mckean [4]. The example is representative of a situation in which some of the standard assumptions regarding normality and variance homogeneity are not held. In this paper, certain aspects of the efficient computation of the test criteria are indicated.

2. *Nonparametric Analysis*

2.1 Univariate Analysis

Let $X_{igt}^{(p)}$ denotes the response of the ith individual in the gth group along the tth treatment at the pth occasion where $i = 1, 2, \ldots, n_g$; $g = 1, 2, \ldots, G$; $t = 1, 2, \ldots, T$ and $p = 1, 2, \ldots, P$. Since there are N subjects in all, we have the relation $N = \sum_{g=1}^{G} \sum_{t=1}^{T} n_{gt}$. The parametric analysis and different test statistics for testing the different effects of such data are given in Islam [6].

The nonparametric test statistics are constructed under the following conditions:

(a) A certain null hypothesis must be specified.

(b) If the distribution of certain quadratic form of the ranks is to be approximately a central χ^2-distribution, then the sample size must be sufficiently large.

(c) If the null hypothesis is true, the model is no longer valid. But the "partition of χ^2-distribution technique" makes it valid if the partition may tends to "Jeopardize χ^2-approximation" for all of the components.

The proposed univariate test statistics for testing the null hypothesis of different main effects and interaction effects are discussed on the basis of Friedman [5] and Chatterjee and Sen [3] which represent a multivariate version of the Kruskal and Wallis [9] test.

Let

$$R_{igt}^{(p)} = [\text{Rank of } X_{igt}^{(p)} \text{ in the set } \{ X_{111}^{(p)} \ldots X_{n_G GT}^{(p)} \}]$$

$$R_{igt}^{(p)} = 1 + \begin{bmatrix} \text{The number of } X_{i'g't'}^{(p)} \\ \text{which is less than } X_{igt} \end{bmatrix} + 1/2 \begin{bmatrix} \text{The number of } X_{i'g't'}^{(p)} \\ \text{equal to } X_{igt}^{(p)} \end{bmatrix}$$

for $i'g't' \neq igt$ also $i, i' \neq = 1, 2, \ldots, n_g$

$$g, g' \neq = 1, 2, \ldots, G$$

$$t, t' \neq = 1, 2, \ldots, T$$

$$p, p' \neq = 1, 2, \ldots, P$$

Then the average rank

$$\overline{R}_{.gt}^{(p)} = \frac{1}{n_g} \sum_{i=1}^{n_g} R_{igt}^{(p)}$$

implies
$$\sum n_g \overline{R}_{.gt} = \frac{N(N+1)}{2}$$

The test statistic for the null hypothesis of group effect can be given as

$$L_{N\,(\text{group})} = \left(\frac{N-1}{N}\right) \sum_{g=1}^{G} n_g \left(\overline{R}_{.g..} - \frac{N+1}{2} j\right)' \sum\nolimits_N^{-1} \left(\overline{R}_{.g..} - \frac{N+1}{2} j\right)$$

j is $P \times 1$ vector of ones.

where

$$\sum\nolimits_N = \sum\nolimits_{GP \times GP} = \sum\nolimits_{G \times G} \otimes \sum\nolimits_{P \times P} \text{ which implies}$$

$\sum_{N}^{-1} = \sum_{G\times G}^{-1} \otimes \sum_{P\times P}^{-1}$; $\otimes$ denotes the Kronecker product, $\sum_{G\times G}$ is a nonsingular matrix over group G and $\sum_{P\times P}$ is a nonsingular matrix over occasion P.

Thus

$$L_{N\,(group)} = \left(\frac{N-1}{N}\right)\sum_{i=1}^{n_g} n_g \left(\bar{R}_{.g..} - \frac{N+1}{2}j\right)$$

$$\cdot \quad \left[\sum_{G\times G}^{-1} \otimes \sum_{P\times P}^{-1}\right]\left(\bar{R}_{.g..} - \frac{N+1}{2}j\right)$$

For large sample $L_{N(group)}$ has approximately a χ^2-distribution with P(G-1) degrees of freedom.

The test statistic for the null hypothesis of treatment effects can be given as

$$L_{N\,(treatment)} = \left(\frac{N-1}{2}\right)\sum_{t=1}^{T}\left[\bar{U}_{ig.}^{(p)} - 1/2\ (T+1)\right]' \left(\sum_{T\times T}^{-1} \otimes \sum_{P\times P}^{-1}\right)$$

$$\left[\bar{U}_{ig.}^{(p)} - 1/2\ (T+1)\right]$$

where

$$X_{ig.}^{(p)} = \sum_{t=1}^{T} X_{igt}^{(p)} \text{ and } \bar{U}_{ig.}^{(p)} = \bar{X}_{ig.}^{(p)} - 1/2\ (T+1)$$

and U_t is the sum of the rank $U_{ig.}^{(p)}$ and $\sum_{T\times T}$ is a nonsingular matrix over treatment T.

$$\sum_{N}^{*} = \sum_{TP\times TP} = \sum_{T\times T} \otimes \sum_{P\times P} \text{ which implies}$$

$$\sum_{N}^{*-1} = \sum_{T\times T}^{-1} \otimes \sum_{P\times P}^{-1}$$

For large sample $L_{N\,(treatment)}$ has approximately a χ^2-distribution with P(T-1) degrees of freedom.

To test the null hypothesis of occasion effects, the test statistic can be given as

$$L_{N\,(\text{occasion})} = \sum_{i=1}^{n_g} W_{n_g}$$

where

$$W_{n_g} = T_{n_g}{}' C_1{}' \left[C_1 \sum_{n_g} C_1{}' \right]^{-1} C_1 T_{n_g}$$

let $\qquad U_{igt}^{(p)} = C_1 X_{igt}^{(p)}$

where C_1 is any $(P\text{-}1) \times P$ matrix whose rows are linearly independent constant.

$$T_{n_g} = \frac{1}{n_g} \sum_{i=1}^{n_g} S_{igt}^{(p)} \text{ and } \sum_{n_g} = \frac{1}{n_g^2} \sum_{i=1}^{n_g} S_{igt}^{(p)}\, S_{igt}^{(p')}$$

and

$$S_{igt}^{(p)} = \sum_{p=1}^{P} \left\{ \sin\left(X_{igt}^{(p)} - X_{igt}^{(p')} \right)\right\} \left[\text{Rank of} \left| X_{igt}^{(p)} - X_{igt}^{(p')} \right| \text{ in} \right.$$
$$\left. \left\{ \left| X_{igt}^{(p)} - X_{igt}^{(p')} \right| \cdots \left| X_{n_g gt}^{(p)} - X_{n_g gt}^{(p')} \right| \right\} \right]$$

The test statistic $L_{N\,(\text{occasion})}$ has approximately a χ^2-distribution with GT(P-1) degrees of freedom.

The test statistic for the null hypothesis of no group $\times$ treatment interaction effects can be given as

$$L_{N\,(\text{group} \times \text{treatment})} = \left(\frac{N-1}{N} \right) \sum_{g=1}^{G} \sum_{t=1}^{T} \left[\overline{X}_{.gt} - \frac{1}{4} (G+1)(T+1) \right]'$$
$$\left[\sum_{G \times G}^{-1} \otimes \sum_{T \times T}^{-1} \right] \left[\overline{X}_{.gt} - \frac{1}{4} (G+1)(T+1) \right]$$

where

$$X_{gt}^{(p)} = \sum_{g=1}^{G} \sum_{t=1}^{T} \overline{X}_{.gt}^{(p)}$$

For large sample, $L_{N\,(\text{group} \times \text{treatment})}$ has approximately a χ^2- distribution with P(G-1) (T-1) degrees of freedom.

To test the null hypothesis of no group × occasion interaction effects, the test statistic can be given as

$$L_{N\,(\text{group} \times \text{occasion})} = (N-1) \sum_{i=1}^{n_g} \sum_{t=1}^{T} n_g \, \tilde{U}'_{GP} \, C'_1$$

$$\left[C_1 \left(\sum_{G \times G} \otimes \sum_{P \times P} \right) C'_1 \right]^{-1} C_1 \, \tilde{U}_{GP}$$

Here

$$\tilde{U}_{GP}^{(pp')} = [\text{Rank of} \left(X_{igt}^{(p)} - X_{igt}^{(p')} \right) \text{in all} \, (X_{igt} - X_{i'g't'})]$$

and $\tilde{U}_{GP} = \text{sum of } U_{GP}^{(pp')}$

For large sample,

$L_{N\,(\text{group} \times \text{occasion})}$ has approximately a χ^2-distribution with T(G-1)(P-1) degrees of freedom.

To test the null hypothesis of no treatment × occasion interaction effects, the test statistic can be given as

$$L_{N\,(\text{treatment} \times \text{occasion})} = (N-1) \sum_{t=1}^{T} \sum_{p=1}^{P} \tilde{U}_{TP} \, C'_1 \sum^{(***)} C'_1]^{-1} \, C_1 \, \tilde{U}_{TP}$$

Here

$$\tilde{U}_{TP}^{(pp')} = [\text{Rank of} \left(X_{igt}^{(p)} - X_{igt}^{(p')} \right) \text{in all} \left(X_{igt}^{(p)} - X_{i'g't'}^{(p')} \right)]$$

and $\sum^{(***)} = \sum_{TP \times TP} = \left[\sum_{T \times T} \otimes \sum_{P \times P} \right]$

then $L_{N\,(\text{treatment} \times \text{occasion})}$

$$= (N-1) \sum_{t=1}^{T} \sum_{p=1}^{P} \tilde{U}'_{TP} \, C'_1 \left(\sum_{T \times T}^{-1} \otimes \sum_{P \times P}^{-1} C'_1 \right) C_1 \, \tilde{U}_{TP}$$

For large sample,

$L_{N\,(\text{treatment} \times \text{occasion})}$ has approximately a χ^2-distribution with G(T-1) (P-1) degrees of freedom.

The test statistics for the null hypothesis of no group × treatment × occasion interaction effects can be given as

$L_{N\ (\text{group} \times \text{treatment} \times \text{occasion})}$

$$= (N-1) \sum_{g=1}^{G} n_g \tilde{U}^{*'} C'_1 [C_1 \sum^{***} C'_1]^{-1} C_1 \tilde{U}^*$$

where

$$\sum^{***} = \sum_{G \times G} \otimes \sum_{T \times T} \otimes \sum_{P \times P}$$

then $L_{N\ (\text{group} \times \text{treatment} \times \text{occasion})}$

$$= \sum_{g=1}^{G} n_g \tilde{U}^{*'} \left[C_1 \left(\sum_{G \times G} \otimes \sum_{T \times T} \otimes \sum_{P \times P} \right) \right]^{-1} C_1 \tilde{U}^*$$

where

$$\tilde{U}^* = \left\{ U_{igt}^{(1)} \ldots U_{igt}^{(p)} \right\} \text{with } U_{igt}^{(P)} = \sum_{p=1}^{P} U_{igt}^{(p)}$$

For large sample,

$L_{N\ (\text{group} \times \text{treatment} \times \text{occasion})}$ has approximately a χ^2-distribution with (G–1) (T–1) (P–1) degrees of freedom.

2.2 Multivariate Analysis

Considering the general null hypothesis for multivariate model

$H_{o\ (\text{general})} : C_{r \times \{G \otimes T\}} B_{\{G \otimes T\} \times P} H_{P \times 1} = 0$ against

$H_{A\ (\text{general})} : C_{r \times \{G \otimes T\}} B_{\{G \otimes T\} \times P} H_{P \times 1} \neq 0$

where

$C_{r \times \{G \otimes T\}}$: matrix of rank r; where $r \leq \{G \otimes T\}$ and

$H_{P \times 1}$: matrix of rank 1; where $1 \leq p$; $\otimes$ denotes direct product.

Assuming the variance-covariance matrix $\sum = \left(I_{N \times N} \otimes \sum_{P \times P} \right)$ where $\sum_{P \times P}$ denotes the common variance covariance matrix and $\otimes$ Kronecker product. If $\sum_{P \times P}$ is not known and all column of X have the same scale, then Davis and Mckean [4] suggested a nonparametric methods and the general test statistic are as follows :

$$\cap = \text{Trace}\,[C(\hat{\beta}_o\ \hat{\beta}'_1)H]'\ (CS_N C')^{-1}\ [C(\hat{\beta}_o\ \hat{\beta}'_1)H]\ (H'\textstyle\sum H)^{-1} \quad (2.2.1)$$

which has the χ^2-distribution with $(r \times 1)$ degrees of freedom.

Here β_o is a $1 \times G$ row vector which contains the intercept parameter of the model, β_1 is a $G \times P$ matrix containing partial regression coefficient.

Now C can be partitioned as

$$C_{r\times(G\otimes T)} = [C^1_{r_1 \times G}\,(+)\,C^2_{r_2 \times T}];\ r = r_1 + r_2;\ r > 1\ (+)\text{ denotes the direct sum}$$

where

r_1 is the rank of $C^1_{r_1 \times G}$ matrix and

r_2 is the rank of $C^2_{r_2 \times T}$ matrix

Now

$$[C^1_{r_1 \times G}\,(+)\,C^2_{r_2 \times T}]_{(r_1+r_2)\times(G\otimes T)}$$

are used in (2.2.1) instead of $C_{r\times(G\otimes T)}$, then

$$\begin{aligned}\cap = \text{Trace}\ &\{[C^1_{r_1 \times G}\,(+)\,C^2_{r_2 \times T}]\,(\hat{\beta}_o\ \hat{\beta}'_1)\,H\}'\ \{[(C^1_{r_1 \times G}\,(+)\,C^2_{r_2 \times T}]\\ &S_N\,[C^1_{r_1 \times G}\,(+)\,C^2_{r_2 \times T}]'\}^{-1}\ \{[(C^1_{r_1 \times G}\,(+)\,C^2_{r_2 \times T})\,(\hat{\beta}_o\ \hat{\beta}'_1)\,H]\}\\ &[H'(I_{N\times N}\,(X)\textstyle\sum_{P\times P})\,H]^{-1}\end{aligned} \quad (2.2.2)$$

which has the χ^2-distribution with $(r \times 1)$ degrees of freedom.

To test the null hypothesis of group effects, the test statistic $\pi_{(group)}$ can be calculated by selecting

$$C^1_{(T-1)\times T} = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 & -1 \end{bmatrix}_{(T-1)\times T} \quad \text{and } H_{P\times 1} = \begin{bmatrix} \frac{1}{P} \\ \frac{1}{P} \\ \cdots \\ \cdots \\ \cdots \\ \frac{1}{P} \end{bmatrix}$$

then $\cap_{(group)}$ has approximately a χ^2-distribution with (G-1) degrees of freedom.

To test the null hypothesis of treatment effects, the test statistic $\cap_{(treatment)}$ can be calculated by selecting

$$C^2_{(-1)x} = \begin{bmatrix} 1 & -1 & 0 & \dots & 0 & 0 \\ 0 & 1 & -1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & -1 \end{bmatrix}_{(-1)x} \quad \text{and } H_{P\times 1} = \begin{bmatrix} \frac{1}{P} \\ \frac{1}{P} \\ \dots \\ \dots \\ \dots \\ \frac{1}{P} \end{bmatrix}$$

then $\cap_{(treatment)}$ has approximately a χ^2-distribution with (T-1) degrees of freedom.

To test the null hypothesis of no group $\times$ treatment interaction effects, the test statistic

$\cap_{(group \times treatment)}$ can be calculated by selecting

$$[\, C^1_{r_1 \times G} \,(+)\, C^2_{r_2 \times T} \,] = \left[\begin{array}{c|c} C_{(G-1)\times G} & O_{r_2 \times T} \\ \hline O_{r_1 \times G} & C_{(T-1)\times T} \end{array}\right]_{(r_1 + r_2) \times (G \otimes T)}$$

and $$H_{P\times 1} = \begin{bmatrix} \frac{1}{P} \\ \dots \\ \dots \\ \dots \\ \frac{1}{P} \end{bmatrix}_{P\times 1}$$

Then $\cap_{(group \times treatment)}$ has approximately χ^2-distribution with (G-1) (T-1) degrees of freedom.

To test the null hypothesis of occasion effect the test statistic $\cap_{(occasion)}$ can be calculated by selecting C and H as

$$[C^1 \otimes C^2]_{1 \times \{G \otimes T)} = \left[\frac{1}{\{G \otimes T\}}, \frac{1}{\{G \otimes T\}}, \ldots, \frac{1}{\{G \otimes T\}} \right]_{1 \times \{G \otimes T\}}$$

and

$$H_{P \times (P-1)} = \begin{bmatrix} 1 & -1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & -1 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & 1 & -1 \end{bmatrix}_{P \times (P-1)}$$

then $\cap_{(occasion)}$ has approximately a χ^2-distribution with P-1 degrees of freedom.

To test the null hypothesis of no group $\times$ occasion interaction effects, the test statistic

$\cap_{(group \times occasion)}$ can be calculated by selecting

$$C^1_{(G-1) \times G} = \begin{bmatrix} 1 & -1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & -1 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & 1 & -1 \end{bmatrix}_{(G-1) \times G}$$

and

$$H_{P \times (P-1)} = \begin{bmatrix} 1 & -1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & -1 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & 1 & -1 \end{bmatrix}_{P \times (P-1)}$$

Then $\cap_{(group \times occasion)}$ has approximately a χ^2-distribution with (G-1) (T-1) degrees of freedom.

To test the null hypothesis of no treatment $\times$ occasion interaction effects, the test statistic

$\cap_{(treatment \times occasion)}$ can be calculated by selecting

$$C^2_{(G-1)\times G} = \begin{bmatrix} 1 & -1 & 0 & \dots & 0 & 0 \\ 0 & 1 & -1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & -1 \end{bmatrix}_{(G-1)\times G}$$

and

$$H_{P\times(P-1)} = \begin{bmatrix} 1 & -1 & 0 & \dots & 0 & 0 \\ 0 & 1 & -1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & -1 \end{bmatrix}_{P\times(P-1)}$$

Then $\cap_{(\text{treatment} \times \text{occasion})}$ has approximately a χ^2-distribution with (T–1) (P–1) degrees of freedom.

To test the null hypothesis of no group × treatment × occasion interaction effects, the test statistic

$\cap_{(\text{group} \times \text{treatment} \times \text{occasion})}$ can be calculated by selecting

$C^1_{(G-1)\times G}$ (+) $C^2_{(T-1)\times T}$ and H as in the test of group × treatment interaction effects.

Then the $\cap_{(\text{group} \times \text{treatment} \times \text{occasion})}$ has approximately a χ^2-distribution with (P–1)(G–1) (T–1) degrees of freedom.

3. Illustrative Example

3.1 Data from Green Belt Project, 1993

The data was collected from an experiment of the "Green Belt Project, 1993" Jahangirnagar University, allowing three groups of "Mehogni" trees to grow at similar rates on three different fertilizers (no fertilizer, DAP 100 gm, N:P:K 100 gm). The groups (pH) region are characterized into agroclimate condition (tropical to sub-tropical) having acid soil. The soil of the experimental area was silty loams with a pH value 5.4, 5.5 and 5.6 respectively. The land was prepared well and planted the Mehogani trees at a depth of 15 to 16 cm with a plant to plant spacing 70 cm. 5 trees were randomly assigned to each fertilizer in each group. The time period of the experiment was 20th November, 1993 to 19th July, 1994. The height of the trees were measured every two months and were recorded in cm. Thus G = 3, T = 3, P = 5, N = 45. The nonparametric univariate and multivariate tests for the hypothesis of different effects are given in Table 3.1.

Table 3.1. Nonparametric Univariate and Multivariate Tests

Source of Variation	Test Statistics	d.f.	Sig (%)
Univariate			
Group	15.0285	10	.0095218
Treatment	65.21928	10	.00011978
Group × treatment	5.90192	20	.1222975
Occasion (month)	112.3587	36	.00009091
Group × occasion	3.3994	24	.78987
Treatment × occasion	75.66592	24	.008512
Group × treatment × occasion	2.544693	16	.682579
Multivariate			
Group	3.9825	2	.1352
Treatment	30.39442	2	.0009985
Group × treatment	0.69578	4	.676685
Occasion (month)	45.28956	4	.0008788
Group × occasion	1.11295	8	.722993
Treatment × occasion	6.24459	8	.0197285
Group × treatment × occasion	5.278533	16	.3922119

The Table 3.1 indicates that treatments and occasions effects are significant in both the nonparametric univariate and multivariate approaches. All interaction effects except treatment × occasion interaction are insignificant. The occasion effect is highly significant i.e., a significant variation of growth of plant over the time.

REFERENCES

[1] Bhapkar, V.P., 1961. A nonparametric test for the problems of several sample. *Ann. Math. Statist.*, **32**, 1108-1117.

[2] Bhapkar, V.P., 1965. Some nonparametric tests for the multivariate several sample location problems. *Proc. Internal. Symp. Mult. Analysis*, Academic Press, New York, 28-42.

[3] Chatterjee, S.K. and Sen, P.K., 1966. *Nonparametric tests for the multisample location problems.* S.N. Roy Memorial Volume, Edited by R.C. Bose.

[4] David, J.B. and Mckean, J.W., 1993. Rank-based methods for multivariate linear models. *J. Am. Statist. Assoc.*, **88**, 245-251.

[5] Friedman, M., 1937. The use of ranks to avoid the assumption of normality implicit in the analysis of variance. *J. Am. Statist. Assoc.*, **32**, 675-701.

[6] Islam, M.N., 1992. Analysis of repeated measurements experiment. M. Phil. Thesis, Jahangirnagar University.

[7] Islam, M.N., 1997. Parametric and nonparametric analysis of multifactor repeated measurements experiments. Ph.D. Thesis, Jahangirnagar University.

[8] Koch, G.G. and Sen, P.K. 1968. Some aspects of the statistical analysis of the mixed model. *Biometrics*, **33**, 27-47.

[9] Kruskal, W.H. and Wallis, W.A., 1952. Use of ranks in one criterion variance analysis. *J. Am. Statist. Assoc.*, **46**, 583-621.

[10] Lana, R.E. and Lubin, A., 1963. The effect of correlation on the repeated measures design. *Education and Psychological Measurements*, **23**, 729-739.

[11] Lee, Y.S., 1971. Asymptotic formula for the distribution of a multivariate test statistics : power comparisons of certain multivariate tests. *Biometrica*, **58**, 647-653.

[12] Madsen, K.S., 1977. A growth curve model for studies in morophomatrics. *Biometrics*, **33**, 659-669.

[13] Prabhakaran, P.V. and Rani Jhon, 1994. Rank analysis of variance in groups of experiments. *Jour. Ind. Soc. Agril. Statist.*, **46(2)**, 201-209.

[14] Puri, M.L., 1964. Asymptotic efficiencies of a class of C samples. *Ann. Math. Statist.*, **35**, 101-121.

[15] Quade, D., 1966. On analysis of variance for the K sample problem. *Ann. Math. Statist.*, **37**, 1747-1756.

[16] Rahman, M.M., 1989. Comparison of methods for the analysis of repeated measurements experiments., Ph.D. thesis. Aberdeen University, U.K.

[17] Rai, S.C. and Rao. P.P., 1980. Use of ranks in groups of experiments. *Jour. Ind. Soc. Agril. Statist.*, **32(2)**, 25-33.

[18] Suguria, N., 1965. Multi-sample and multivariate nonparametric test based on U-statistics and their asymptotic efficiencies. *Osaka. J. Math.*, **2**, 385-426.