SOME INVESTIGATIONS ON RESPONSE TO FERTILIZER AND DETERMINATION OF OPTIMUM DOSE USING SOIL TEST VALUES

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SUMMARY

Different regression models describing the relationship between the response to fertilizer and soil characteristics and soil test values are examined with a view to estimate the optimum dose of fertilizer for a given set of soil test values. Two methods are used. In the first method, the responses to different levels of phophorus obtained at each site were used to fit regression models using the soil test values. Then for a given set of soil test values, the responses to different levels of phosphorus were determined and a quadratic response curve was fitted to obtain the optimum dose.

In the second method, orthogonal polynomials were fitted to describe the yield dose relationship. The coefficients of polynomial then regressed to soil test values. Then for a given set of soil test values estimates of coefficients of orthogonal polynomial were obtained and the optimum dose of fertiliser worked out.

The methods are illustrated by using the phosphorus response data of 24 trials on farmers' fields conducted during 1972-73 in Sangrur district (Punjab) on wheat.

INTRODUCTION

For efficient fertilizer use, it is necessary to have information on the optimum doses of fertilizers under different agroclimatic conditions. The optimum dose of fertilizers for a given area is obtained by using experimental data having graded dressings of fertilizers by fitting a suitable response model and taking into account the cost of fertilizer and price of produce. This approach, however, does not take into consideration the available nutrient status of soil. For the same cultural practices and weather conditions, the optimum doses of fertilizer applied to different fields may vary with the variation in soil characteristics. As the available soil status is not same for all the fields in a given area, it is obvious that the optimum doses will differ from field to field.

Abraham (1973) made some studies on using soil test data for making fertilizer recommendations. Colwell (1967) investigated the relationship between yield of crop, fertilizer application rate and soil test values.

In this paper, different regression models describing the relationship between the response to fertilizer and soil characteristics and soil test values are examined with a view to estimate the optimum dose of fertilizer for a given set of soil test values.

Method Used

It is widely recognised that relationship between soil test data and yield responses to fertilizer cannot be established adequately by the use of one independent variable alone, since many other factors influence crop yield. Accordingly, an attempt has been made by using both soil variables suc has available nutrient and pH status and non-soil variables such as applied fertilizers in the regression model. Two different methods, using three multiple regression models in each have been attempted.

Method—I

The general procedure adopted is to fit multiple regressions separately to responses to each level of fertilizer on soil test values. Using the regression functions, response to various levels was estimated for a given vector x_0 (soil test values). The estimated responses were then used to fit a response curve and from the response curve the optimum dose of fertilizei taking into account the cost of fertilizer and price of grain produce was worked out.

Let R_{ji} be the response to the j-th level of fertilizer at the i-th site.

$$R_{ji}=Y_{ji}-Y_{oj} \qquad i=1, 2 \ldots N$$

$$j=1, 2, 3$$

Where Y_{ji} denotes the yield of the j-th level of fertilizer at the i-th site aud Y_{oi} is the corresponding yield of the control plot.

Let x_1, $x_2 \ldots x_k \ldots x_p$ be soil variables. The three regression models can be expressed as

Model—I $R_j = a_{oj} + a_{1j}X_1 + \ldots + a_{kj}X_k + \ldots + a_{pj}X_p$

Model—II $R_j = b_{oj} + b_{1j}X_1 + \ldots + b_{kj}X_k + b_{pj}X_p + b_jX_k^2$

Model—III $R_j = c_{oj} + c_{1j}X_1 + \ldots + c_{kj}X_k + c_{pj}X_p + c_jX_k^{1/2}$

Where R_j is the yield response to the j-th level of fertilizer (applied P_2O_5). Models II and III include square and square root terms of x_k (available P_2O_5) respectively as the main interest is to estimate optimum dose of phosphate.

Using the well known technique of Least Squares (Williams 1959), the partial regression coefficients for the above three models are obtained. The responses to different levels of phosphorus for a given set of soil test values were estimated from regression equations.

A variety of yield functions have been used in the past to describe the relationship between the crop response and rate of fertilizer application and suitability of these functions has been examined by Heady et. al. (1955), Abraham and Rao (1966) and several others. These workers favoured simple polynomial models (quadratic polynomials in the natural and square root scales).

Accordingly, using the values of estimated responses, a quadratic response curve of the following type was fitted with four graded levels of phosphorus (*viz.* $l=0, 1, 2, 3$)

$$R = bl + cl^2$$

The estimates of b and c (obtained by the method of Least Squares) and optimum dose of the nutrient are given by :

$$\hat{b} = \frac{1}{266}(267\,R_1 + 182\,R_2 - 105\,R_3)$$

$$\hat{c} = \frac{1}{38}(-11\,R_1 - 8\,R_2 + 9\,R_3)$$

and

$$l_{opt}\,(l_o) = \left(\frac{q/r - \hat{b}}{2\hat{c}}\right)$$

Where r is the price per unit of produce and q, the cost of one unit of fertilizer.

Estimation of Variance of Optimum Dose

Let the three regression equations corresponding to the three levels for Model-I be denoted as :

$$R_1 = X\beta + e_1$$
$$R_2 = X\gamma + e_2$$
$$R_3 = X\delta + e_3$$

Where R is a response vector, X is a observational matrix of order $N \times p$ and β, γ, δ are vectors of order $p \times 1$ of parameters of the model and e's the error vectors. It can be shown that

$$e_1' e_2 = (R_1 - X\beta)' \quad (R_2 - x\gamma) = R_1' R_2 - R_1' X\gamma$$

Similarly, $$e_1' e_3 = R_1' R_3 - R_1' X\delta$$

$$e_2' e_3 = R_2' R_3 - R_2' X\delta$$

Let W_{ii}' =estimate of variance of e_i ($i = 1, 2, 3$)

$$= \frac{e_i' e_i}{d.f.} = \frac{e_i' e_i}{N-p-1}$$

W_{ij}' =estimate of covariance between e_i and e_j ($i \neq j$)

$$= \frac{e_i' e_j}{d.f} = \frac{e_i' e_j}{N-p-1} \qquad (i \neq j = 1, 2, 3)$$

Using the estimate of variance and covariance of error terms, variances of $\hat{b}$ and $\hat{c}$ and their covariance are obtained as follows :

$$V(\hat{b}) = \frac{K_1}{70756}(47089\, W_{11}' + 33124\, W_{22}' + 11025\, W_{33}'$$
$$+ 78988\, W_{12}' - 45570\, W_{13}' - 38220\, W_{23}')$$
$$= K_1 W_{11}$$

Where $$K_1 = \frac{1}{N} + \sum_{ij} c^{ij}(X_i - \bar{X}_i)(X_j - \bar{X}_j)$$

c^{ij} denotes the $(i, j)^{th}$ element of $(X'X)^{-1}$

and

$$V(\hat{c}) = \frac{k_1}{1444}(121 W_{11}' + 64 W_{22}'$$
$$+ 81\, W_{33}' + 176 W_{12}' - 198 W_{43}' - 144 W_{23}')$$
$$= K_1 W_{22}$$

$$\text{and Cov}(\hat{b}, \hat{c}) \left(= \frac{k_1}{10108} - 2387W'_{11} - 1456W'_{22} - 945W'_{33} - 3738W'_{12} + 3108W'_{13} + 2478W'_{33}\right)$$

$$= K_1 W_{12}$$

The asymplotic variance of optimum dose l_o is given by

$$V(l_o) = \frac{k_1}{4\hat{c}^2}(W_{11} + 4l_o W_{12} + 4l_o^2 W_{22})$$

$$= \frac{K_1 W}{4\hat{c}^2}$$

where $W = W_{11} + 4l_o W_{12} + 4l_o^2 W_{22}$

Following the above procedure, the variance of the optimum dose under Models II and III can be obtained.

Method—II

The first step is to fit an orthogonal polynomial to describe the yield does relationship at each site. Secondly with each of the parameters of the fitted polynomial a multiple regression with soil test values as independent variables is fitted. These multiple regression equations replace the site coefficients obtained in the yield curve. The optimum dose of fertilizer for a given soil test values can be determined from the multi-variate yield response curve so obtained.

The yield dose relationship can be represented as :

$$Y_{ij} = C_{oi}\xi_{oj} + C_{1i}\xi_{1j} + C_{2i}\xi_{2j} + C_{3i}\xi_{3j}$$

Where Y_{ij} is the yield for the i-th site with fertilizer rate j, C_{ki} $(k=0,\ 1, 2, 3)$ are regression coefficients of the orthogonal polynomials of zero, first, second and third degree.

In fitting the orthogonal polynomials both the square root and natural scales of fertilizer rates were considered.

In case of the natural scale with equi-spaced levels the orthogonal polynomials are :

$$\xi_0 = 1$$

$$\xi_1 = (l - \bar{l})$$

$$\xi_2 = (l - \bar{l})^2 - \frac{5}{4}$$

$$\xi_3 = (l - \bar{l})^3 - \frac{41}{20}(l - \bar{l})$$

With the square root scale the polynomials are :

$$\xi_0=1, \qquad \xi_1=(l^{1/2}-T), \qquad \xi_2=(l^{1/2}-T)^2-\frac{5}{4}$$

$$\xi_3=(l^{1/2}-T)^3-\frac{41}{20}(l^{1/2}-T)$$

Where $T=\dfrac{(0+1+2^{1/2}+3^{1/2})}{4}$

and l is the graded level of fertilizer.

The coefficients of the polynomials were fitted to soil test values for each site using the following three simultaneous regression equations,

$$\text{Model I} \quad C_k=C_{ok}+\sum_{1}^{p}C_{pk}X_p$$

$$\text{Model II} \quad C_k=C_{ok}+\sum_{1}^{p}C_{pk}X_p+C_{(p+1)k}X_{k'}^2$$

$$\text{Model III} \quad C_k=C_{ok}+\sum_{1}^{p}C_{pk}X_p+C_{(p+1)k}\sqrt{X_{k'}}$$

Where $X_{k'}$ denotes the additional term for available phosphorus. The parameters of the above three models are obtained by using the least square technique.

For determining the optimum dose the estimates of C_k ($k=0, 1, 2, 3$) are obtained for a given set (X_o) of soil test values. Substituting these estimates in the orthogonal polynomials and considering the cost price ratio the optimum dose for natural scale is given below :

$$l_{opt}=\left(\bar{l}-\frac{C^2}{3C_3}\right)\pm\frac{\left[C_2^2-3C_3\left(C_1-\frac{41}{20}C_3-\frac{q}{r}\right)\right]^{\frac{1}{2}}}{3C_3}$$

Similarly for the square root scale the optimum dose is

$$l_{opt}=\left[T-\frac{\left(C_2-\frac{q}{r}\right)}{3C_3}\right]\pm\frac{\left[\left(C_1-\frac{q}{r}\right)^2-3C_3\left(C_1-\frac{41}{20}C_3-\frac{2Tq}{r}\right)\right]^{\frac{1}{2}}}{3C_3}$$

One of the roots will give the optimum dose of fertilizer.

VARIANCE OF OPTIMUM DOSE

Using the natural scale, Model-I can be represented as

$$C_0=X\alpha+e_0$$
$$C_1=X\beta+e_1$$
$$C_2=X\gamma+e_2$$
$$C_3=X\delta+e_3$$

The estimates of variances and covariances of e_i and e_{ij} $(i\neq j)$ denoted by W'_{ii} and W'_{ij} $(i\neq j)$ respectively can be defined in the same way as given in method I.

Taking logarithm of both sides of expression for $l_{(opt)}$ and differentiating it partially and then squaring we get variance

$$V(l_{opt})=k_i l_o^2\left[\left(\frac{3C_3}{2A_3\sqrt{A_2}}\right)^2 W'_{11}+\frac{1}{A_2}W'_{22}+\frac{A_5}{2A_3C_3\sqrt{A_2}}W'_{33}\right.$$
$$\left.-\frac{3C_3}{A_2A_3}W'_{12}+\frac{3A_5}{2A_2A_3^2}W'_{13}-\frac{A_5}{A_2A_3C_3}W'_{23}\right]$$

Where
$$A_1=3\left(C_1-\frac{41}{20}C_3-\frac{q}{r}\right)$$
$$A_2=\left(C_2^2-C_3A_1\right)$$
$$A_3=(C_2+\sqrt{A_2})$$
$$A_4=\left(\frac{123}{20}C_3-A_1\right)$$
$$A_5=(2A_3\sqrt{A_2}-A_4C_3)$$

and
$$K_1=\frac{1}{N}+\sum_{ij}X^{ij}(X_i-\bar{X}_i)(X_j-\bar{X}_j)$$

Where X^{ij} denotes the $(i, j)^{th}$ element of matrix $(X', X)^{-1}$

Similarly using square root scale, we get

$$V(l_{opt})=k_o l_o^2\left[\left(\frac{B_8}{2}\right)^2 W'_{11}+\left(\frac{B_6}{B_4}\right)^2 W'_{22}+B_7^2\,W'_{33}\right.$$
$$\left.-\left(\frac{B_6B_8}{B_4}\right)W'_{12}+\left(\frac{6-B_5B_8}{2B_4\sqrt{B_3}}\right)W'_{13}+\frac{2B_6B_7}{B_4}W'_{23}\right]$$

Where
$$B_1=\left[C_2-\frac{q}{r}\right]$$
$$B_2=3\left[C_1-\frac{41}{20}C_3-2T\frac{q}{r}\right]$$
$$B_3=B_1^2\ B_2C_3$$
$$B_4=-B_1+\sqrt{B_3}$$
$$B_5=\frac{123}{20}C_3-B_2$$
$$B_6=\frac{B_1}{\sqrt{B_3}}-1$$
$$B_7=B_5/2B_4\sqrt{B_3}-\frac{1}{C_3}$$
$$B_8=3C_3/B_4\sqrt{B_3}$$

Results and Discussions

The responses to four graded levels of phosphorus viz. 0, 30, 60 and 90 kg. P_2O_2/ha. (over a basal dose of 120 kg.N+60 kg K_2O/ha) of wheat obtained from 24 field trials conducted on farmers' fields in Sangrur district (Punjab) in 1972-73 have been utilized as an illustration. The soil test values used as independent variables in regression models are pH (X_1), organic carbon percentage (X_2), available phosphorus in kg/ha (X_3), available potassium in kg/ha (X_4) and conductivity in m. mhos/cm (X_5).

The variables included in the regression models, their mean values together with their standard deviations are given below in Table 1.

TABLE 1

Mean value and standard deviations of soil test values.

	Soil variables						
	pH (X_1)	*Organic carbon (%)* (X_2)	*Available* KO_5 *(Kg/ha)* (X_3)	*Available* K_2O *(Kg/ha)* (X_4)	*Conductivity (m.mhos/cm)* (X_5)	X_6*	X_6**
Mean	8.91	0.491	18.74	429.08	0.32	440.36	4.18
S.D.	0.340	0.1301	9.45	202.88	0.0921	429.88	1.121

*For Model-II, **For Model-III.

Method-I

The multiple correlation coefficients for response to three levels of P_2O_5 (30, 60, 90 kg/ha.) for the models are summarised in Table 2.

TABLE 2

Muitiple Correlation Coefficients

Response at Kg. P_2O_5/ha	MODEL I	II	III
R_1 (30)	.5771	.5880	.6056
R_2 (60)	.6289	.7042	.7338
R_3 (90)	.6633	.6684	.6677

From the value of multiple correlation coefficients, Models II and III appear to be superior to Model I, particularly at the lower levels of fertilizer applcation.

Estimate of responses, regression coefficients, optimum doses and their S.E's are presented in Table 3. The estimates of responses were calculated for a given set (x_0)of soil test values *viz.*, pH (8.91), organic carbon percentage (0.491) available phosphorus in kg/ha (18.74), available potassium in kg/ha (429.08) and conductivity in m.mhos/cm (0.32) and for $q/r=60$.

TABLE 3

Estimated Responses, Optimum Doses of Phosphorus and S.E. (Kg/ha).

	MODEL I	II	III
R_1	414.41	381.35	371.24
R_2	769.57	897.51	896.75
R_3	897.82	152.32	862.05
$\hat{b}$	17.07	19.63	19.24
$\hat{c}$	−.0770	−.0108	−.0102
Optimum dose	109.95	90.27	92.52
S.E.	25.80	5.61	18.81

Method-II

Multiple correlation coefficients for Models I, II and III (k=0, 1, 2, 3) are given in Table 4.

TABLE 4

Multiple Correlation Coefficients

	MODEL		
	I	*II*	*III*
k=0	.5276	.5550	.5640
k=1	.6558	.6685	.6683
k=2	.3756	.4721	.5471
k=3	.4323	.5874	.6147

Taking the same soil test values x_o as in method I and q/r=60 optimum doses obtained using the natural and square root scales for the three models are given in Table 5.

TABLE 5

Optimum Dose of Phosphorus with S.E's (Kg/ha)

	MODEL		
	I	*II*	*III*
Natural scale	88.36 (6.32)	90.78 (4.60)	89.86 (8.34)
Square root scale	92.73 (4.48)	92.61 (4.93)	94.22 (8.78)

(The figures in bracket denote S.E's.)

It can be seen that the optimum doses of P_2O_5 obtained by the three models are almost of the same order within each scale. However, with square root scale, the optimum dose of fertilizer obtained is slightly higher than that for natural scale.

The results of the study showed that the amount of variation explained by the five independent variables in the regression analysis generally ranges from 35 to 50 per cent with different models. As the crop response to fertilizer is governed by several factors, soil and non-soil, such as applied fertilizer, cultural practices and climate, it will be useful to include other contributory variables in the regression analysis.

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