

A SEQUENTIAL METHOD FOR CONSTRUCTION OF A CLASS OF B.I.B. DESIGNS

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SUMMARY

A method for construction of B. I. B. designs with the parameters $v = s^2$, $b = s(s + 1)$, $r = s + 1$, $k = s$ and $\lambda = 1$ when s is a prime has been proposed. The technique is sequential in nature.

Keywords : Sequential, construction of B.I.B.D.

Introduction

An alternative technique for construction of B.I.B. Designs with parameters $v = s^2$, $b = s(s + 1)$, $r = s + 1$, $k = s$ and $\lambda = 1$, where 's' is prime, is presented. This method is sequential in the sense that it uses one set of blocks to get a second set and does not require obtaining of mutually orthogonal latin squares or Euclidean finite geometry initially as required for the method of this construction due to Fisher (1940) and Bose (1939).

2. Proposed Method

The method contains the steps given below :

- (a) The s^2 numbers from 0 to $s - 1$ are first written in form of an $s \times s$ square where s is a prime number.
- (b) The rows of this square are taken as one set of 's' blocks.
- (c) The 's' diagonal elements of the square beginning with 0 and written in ascending order are taken to form an initial block. It is developed to get s blocks by the modified method of development as discussed subsequently in section 2.1.

- (d) These s blocks obtained in (c) above again form an $s \times s$ square of the s^2 numbers. Again, the diagonal numbers beginning with 0 are taken to form another initial block and by developing it as before another set of s blocks is obtained.
- (e) Again, taking the diagonal numbers of the square formed of the blocks in (d) another initial block is obtained. By developing this block as earlier another set of s blocks is obtained.
- (f) This procedure is to continue till the first initial block as indicated in (c) reappears.
- (g) These blocks form the B.I.B. design of the series $v = s^2$, $b = s(s + 1)$, $r = s + 1$, $k = s$, $\lambda = 1$, where s is prime number.
- (h) The procedure can be discontinued at any intermediate stage to get any required square lattice design.

2.1 Method of Development

A modified method of development of initial blocks is as under :

Any of the initial blocks indicated above has s numbers in it in ascending order beginning with 0. To get the next block from any one of the initial blocks, unity is to be added to each of the numbers in it and this procedure is to be continued to get the other blocks. The number in the p th position ($p = 1, 2, \dots, s$) in each of these blocks obtained by developing is to be reduced mod ps , if it is greater than or equal to ps and $(p - 1)s$ is to be added to the residue. This procedure ensures that the numbers appearing in the p th position in any block can be only any one of the numbers from $(p - 1)s$ to $ps - 1$. For example if $s = 5$ and initial block taken is

0	6	12	18	24
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then the other blocks obtained by developing it according to the above method are

1	7	13	19	20
2	8	14	15	21
3	9	10	16	22
4	5	11	17	23

This method is illustrated below by taking $s = 5$. For $s = 5$, $v = 25$, $b = 30$, $r = 6$, $k = 5$ and $\lambda = 1$. First set of 5 blocks :

0	1	2	3	4
5	6	7	8	9
10	11	12	13	14
15	16	17	18	19
20	21	22	23	24

Initial block for the second set of s blocks as formed of the diagonal elements of first set of blocks is 0, 6, 12, 18, 24 and on developing it by using the method as stated above we get the second set of blocks

0	6	12	18	24
1	7	13	19	20
2	8	14	15	21
3	9	10	16	22
4	5	11	17	23

Initial block for the third set of blocks as formed of the diagonal elements of second set of blocks is 0, 7, 14, 16, 23 and again developing it in the same way we get the third set of blocks as

0	7	14	16	23
1	8	10	17	24
2	9	11	18	20
3	5	12	19	21
4	6	13	15	22

Similarly taking the diagonal elements of preceding sets of blocks as further initial blocks and developing them we get the remaining fourth, fifth and sixth sets of blocks as given below :

Fourth set of blocks :

0	8	11	19	22
1	9	12	15	23
2	5	13	16	24
3	6	14	17	20
4	7	10	18	21

Fifth set of blocks is

0	9	13	17	21
1	5	14	18	22
2	6	10	19	23
3	7	11	15	24
4	8	12	16	20

and Sixth set of blocks is

0	5	10	15	20
1	6	11	16	21
2	7	12	17	22
3	8	13	18	23
4	9	14	19	24

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