Jour. Ind. Soc. Ag. Statistics
46(3), 1994 : 349–361

Unrelated Question Randomized Response Sampling Using Continuous Distributions*

Sarjinder Singh
Punjab Agricultural University, Ludhiana-141 004

SUMMARY

The present study introduces two estimators of population proportion based on unrelated question randomized response model when continuous randomizing distribution is used. The proposed estimators, obtained by method of moments, are unbiased and the relative efficiency of these estimators with respect to Franklin's [4] corresponding estimators is found to be appreciable.

Keywords : Randomized response technique; Sensitive characters; Estimation of proportion; Franklin's model.

Introduction

Warner [24] suggested an ingenious method of collecting information on sensitive characters. According to the method, for estimating the population proportion π possessing the sensitive character A, a simple random with replacement sample of n persons is drawn from the population. Each interviewee in the sample is furnished an identical randomization device where the outcome 'I possess character A' occurs with probability p while its complement 'I do not possess character A' occurs with probability (1 - p). The respondent answers 'yes' if the outcome of the randomization device tallies with his actual status otherwise he answers 'no'. Some modifications in the model have been suggested by Kuk [8], Mangat [11], Mangat and Singh [12], [13], [14], Singh and Singh [19], [20], [21], Mangat et al [17], Singh et al. [22], [23]) and Kathuria and Singh [7].

Greenburg et al. [5] provided theoretical framework for a modification to the Warner's model proposed by Horvitz et al. [6]. The proposal consisted in modifying the randomization device where the second outcome 'I do not possess character A' was replaced by the outcome 'I possess character Y' where Y was unrelated to character A. This modified model is now known as 'unrelated question model, or U-model. This model has been further investigated by Moors [18], Dowling and Shachtman [2], Lanke [9], Mangat [10] and Mangat et al. [15] [16].

---

* Paper adjudged best for Young Scientist Award during 48th annual conference of the Society held at Kerala Agricultural University, Trichur on 15 December, 1994.

Another generalization of Warner's [24] model was proposed by Franklin [3], [4]. We now discuss the Franklin's model briefly.

2. *Franklin's Model*

In Franklin's [3], [4] model, responses are obtained from each respondent of a simple random with replacement sample of size n. The response Z_{ij}, $i = 1, 2, \ldots n$; $j = 1, 2, \ldots k$ is a random number drawn from the density g_{ij} if the respondent belongs to the sensitive group A otherwise it is drawn from the density h_{ij}. The interviewer does not know the density used by the respondent for drawing the random number. The model can be specialized by having $g_{ih} = g_j$ and $h_{ij} = h_j$ for all $i = 1, 2, \ldots n$. The densities g_j and h_j respectively, have known means μ_{1j} and μ_{2j} and known variances σ^2_{1j} and σ^2_{2j}.

The choice of g_j and h_j as Bernoulli (π) with $h_j(z) = 1 - g_j(z)$ and $k = 1$ reduces this model to Warner's [24] original model. In Warner's [24] model, the densities h_j and g_j are dependent. Also if g_j and h_j denote the distributions generated by the first and second deck of cards with known proportion θ_1 and θ_2 ($\neq \theta_1$) of red cards respectively, then the Franklin's model reduces to the model recently suggested by Kuk [8]. If $\theta_1 = 1$ and $\theta_2 = (1 - p)$, the Kuk's model reduces to Mangat [11] model.

Utilizing the row averages of Z_{ij}, Franklin has defined the unbiased estimator of π as

$$\hat{\pi}_1 = \left(\sum_{j=1}^{k} \bar{Z}_j - m_2 \right) / (m_1 - m_2) \tag{2.1}$$

and the variance of $\hat{\pi}_1$ is given by

$$V(\hat{\pi}_1) = \pi(1-\pi)/n + \left\{ \pi \sum_{j=1}^{k} \sigma^2_{1j} + (1-\pi) \sum_{j=1}^{k} \sigma^2_{2j} \right\} / \{ n (m_1 - m_2)^2 \}. \tag{2.2}$$

Similarly, by concentrating on the column averages of Z_{ij} he has defined another unbiased estimator of π as

$$\hat{\pi}_2 = \sum_{j=1}^{k} w_j \hat{\pi}_j \tag{2.3}$$

where

$$\hat{\pi}_{.j} = (\bar{Z}_{.j} - \mu_{2j}) / (\mu_{1j} - \mu_{2j}). \tag{2.4}$$

Taking where $w_j = D_j / D$, where $D = |\mu_{1j} - \mu_{2j}|$ and $\sum_{j=1}^{k} D_j$ in (2.3) the variance of the estimator $\hat{\pi}_2$ is given by

$$V(\hat{\pi}_2) = \pi(1-\pi)/n + \left\{ \pi \sum_{j=1}^{k} \sigma_{1j}^2 + (1-\pi) \sum_{j=1}^{k} \sigma_{2j}^2 \right\} / \{nD^2\}. \tag{2.5}$$

The Franklin's model is quite general and adds new dimensions in the formation of randomized device and its use. Popularity of this model motivated the author to think for extending this approach for the situation where the use of unrelated questions can be made in the RR procedure.

3. *The Proposed Randomization Device*

In this paper, we have modified the procedure suggested by Franklin [4] by using the known proportion of unrelated character π_y in the population and by suitably choosing known parameters of the proposed randomization device. According to this proposed randomized response technique, a simple random sample of n respondents is drawn with replacement from the population.

In the proposed randomized response model, if a respondent:

(i) belongs to A but not to Y, he/she is instructed to use the density g_{1ij},

(ii) does not belong to both A and Y he/she is instructed to use the density g_{2ij}.

(iii) belongs to both A and Y, he/she is instructed to use the density g_{3ij}.

(iv) belongs to Y but not to A, he/she is instructed to use the density g_{4ij}.

A total of $k \geq 1$ trails per respondent are conducted. This whole procedure is completed by the respondents unobserved by the interviewer. the interviewer knows the exact form of g_{rij}, $r = 1, 2, 3, 4$. The respondent, however, tells only the random number obtained from the density used by him/her without giving any indication about the density use and hence the interviewer does not know for certain from which of the four densities it has come. We denote the value given by the respondent as Z_{ij}. From a total of such kn observations Z_{ij} ($i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots k$) inference can be made about π. The conditional density of a k-tuple $Z_{i1}, Z_{i2}, \ldots, Z_{ik}$ representing a random observation of the ith interviewee given π is

$$\pi (1-\pi_y) \prod_{j=1}^{k} g_{1ij} (Z_{ij}) + (1-\pi)(1-\pi_y) \prod_{j=1}^{k} g_{2ij} (Z_{ij})$$

$$+ \pi \pi_y \prod_{j=1}^{k} g_{3ij} (Z_{ij}) + \pi_y (1-\pi) \prod_{j=1}^{k} g_{4ij} (Z_{ij}). \quad (3.1)$$

Thus $(Z_1, Z_2, \ldots, Z_k)$ can be thought of as mixture of the random variables.

$$P_1 (A_{i1}, A_{i2}, \ldots, A\, ik) + P_2 (B_{i1}, B_{i2}, \ldots, B_{ik})$$

$$+ P_3 (C_{i1}, C_{i2}, \ldots, C_{ik}) + P_4 (D_{i1}, D_{i2}, \ldots, D_{ik}) \quad (3.2)$$

such that $\Sigma P_i = 1$ and P_i $(i = 1, 2, 3, 4)$ are multinomials while $A_{ij} \sim g_{1ij}$, $B_{ij} \sim g_{2ij}$, $C_{ij} \sim g_{3ij}$ and $D_{ij} \sim g_{4ij}$. This model can be specialized by having $g_{rij} = g_{rj}$ for all i and $r = 1, 2, 3, 4$. Thus all the respondents will be observing the same distribution and this allows us to perceive the k-tuple responses (i.i.d) random variables with form

$$P_1 (A_1, A_2, \ldots, A_k) + P_2 (B_1, B_2, \ldots, B_k)$$

$$+ P_3 (C_1, C_2, \ldots, C_k) + P_4 (D_1, D_2, \ldots, D_k) \quad (3.3)$$

from the density

$$\pi (1-\pi_y) \prod_{j=1}^{k} g_{1j} (Z_j) + (1-\pi)(1-\pi_y) \prod_{j=1}^{k} g_{2j} (Z_j)$$

$$+ \pi \pi_y \prod_{j=1}^{k} g_{3j} (Z_j) + \pi_y (1-\pi) \prod_{j=1}^{k} g_{4j} (Z_j). \quad (3.4)$$

Any density may be used for g_{rj} (e.g., weibull, normal, uniform or any discrete distribution). In the present model, we will consider all the four as continuous densities with known means and known standard deviations. In other words. $g_{rj} \sim D(\mu_{rj}, \sigma^2_{rj})$, $r = 1, 2, 3, 4$. For simplicity, let us take $\mu_{3j} = \mu_{1j}$ and $\mu_{4j} = \mu_{2j}$. In what follows, we discuss two estimators of π obtained by using the method of moments.

Theorem 1 : An unbiased estimator of π based on the row averages of Z_{ij} is given by

$$\hat{\pi}_{s1} = \left(\sum_{j=1}^{k} Z_{.j} - m_2 \right) \Big/ (m_1 - m_2), \quad (3.5)$$

where $$m_1 = \sum_{j=1}^{k} \mu_{1j} = \sum_{j=1}^{k} \mu_{3j} \text{ and } m_2 = \sum_{j=1}^{k} \mu_{2j} = \sum_{j=1}^{k} \mu_{4j}$$

Proof: Suppose

$$\bar{Z}_{i.} = \sum_{j=1}^{k} Z_{ij} / k, \text{ for } i = 1, 2, 3, \ldots, n. \tag{3.6}$$

represents the ith row average and $\bar{Z}_{.j} = n^{-1} \sum_{i=1}^{n} Z_{ij}$, for j = 1, 2, . . . k, represents the jth column average. Then we clearly have

$$E(\bar{Z}_{i.}) = k^{-1} \sum_{j=1}^{k} E(Z_{ij}) = k^{-1} \sum_{j=1}^{k} E(Z_j), \tag{3.7}$$

since the k-tuple responses are i.i.d. Thus we get

$$\begin{aligned} E(Z_j) &= \pi (1 - \pi_\pi) \mu_{1j} + (1 - \pi)(1 - \pi_y) \mu_{2j} + \pi \pi_y \mu_{3j} + \pi_y (1 - \pi) \mu_{4j} \\ &= \pi \mu_{1j} + (1 - \pi) \mu_{2j}, \text{ for } j = 1, 2, \ldots, k, \end{aligned} \tag{3.8}$$

since $\mu_{3j} = \mu_{1j}$ and $\mu_{4j} = \mu_{2j}$.

This gives

$$\begin{aligned} E(\bar{Z}_{i.}) &= k^{-1} [\pi \sum_{j=1}^{k} \mu_{1j} + (1 - \pi) \sum_{j=1}^{k} \mu_{2j}] \\ &= k^{-1} [\pi m_1 + (1 - \pi) m_2] \end{aligned} \tag{3.9}$$

From these, the standard method of moments (MM) approach yields

$$\hat{\pi}_{s1} = (k \hat{Z} - m_2) / (m_1 - m_2) = (\sum_{j=1}^{k} \bar{Z}_{.j} - m_2) / (m_1 - m_2) \tag{3.10}$$

since the k-tuple responses are i.i.d. The relation (3.8)

$$E(\hat{\pi}_{s1}) = \pi. \tag{3.11}$$

This proves the theorem.

Before obtaining the variance of the estimator $\hat{\pi}_{s1}$, we prove the following lemmas.

Lemma 1 : The variance of the j-th response Z_j from a certain single respondent is given by

$$V(Z_j) = \pi(1-\pi)(u_{1j}-\mu_{2j})^2 + \pi\,\sigma^2_{1j} + (1-\pi)\,\sigma^2_{2j}$$
$$+\,\pi_y\,\{\pi(\sigma^2_{3j} - {}^2_{1j}) + (1-\pi)(\sigma^2_{4j} - \sigma^2_{2j})\}. \qquad (3.12)$$

where j = 1, 2, 3, . . . , k.

Proof: Since the observations $Z_1, Z_2, \ldots, Z_k$ from a certain single respondent are not independent and need not even be identically distributed, we find that the joint density of the responses $(Z_j, Z_t)$of the two trails from a single respondent is given by

$$f(Z_j, Z_t) = \pi(1-\pi_y)\,g_{1j}(Z_j)\,g_{1t}(Z_t) + (1-\pi)(1-\pi_y)\,g_{2j}(Z_j)\,g_{2t}(Z_t)$$
$$+\,\pi\,\pi_y\,g_{3j}(Z_j)\,g_{3t}(Z_t) + \pi_y(1-\pi)g_{4j}(Z_j)\,g_{4t}(Z_t). \qquad (3.13)$$

We have

$$V(Z_j) = E(Z_j^2) - \{E(Z_j)\}^2 \qquad (3.14)$$

Now from (3.8)

$$E(Z_j) = \pi\,\mu_{1j} + (1-\pi)\mu_{2j} \qquad (3.15)$$

Also

$$E(Z_j^2) = \int_j\int_t Z_j^2\,f(Z_j, Z_t)\,dZ_j\,dZ_t$$
$$= \pi(1-\pi_y)(\sigma^2_{1j}+\mu^2_{1j}) + (1-\pi)(1-\pi_y)(\sigma^2_{2j}+\mu^2_{2j})$$
$$+\,\pi\,\pi_y(\sigma^2_{3j}+\mu^2_{1j}) + \pi_y(1-\pi)(\sigma^2_{4j}+\mu^2_{2j})$$
$$= \pi\,\mu^2_{1j} + (1-\pi)\,\mu^2_{2j} + \pi\,\sigma^2_{1j} + (1-\pi)\,\sigma^2_{2j}$$
$$+\,\pi_y\,\{\pi(\sigma^2_{3j} - \sigma^2_{1j}) + (1-\pi)(\sigma^2_{4j} - \sigma^2_{2j})\} \qquad (3.16)$$

Making substitutions from (3.15) and (3.16) in (3.14), we get (3.12). This proves the lemma.

Lemma 2 : The covariance between the j-th and t-th responses Z_j and Z_t from a certain single respondent is given by

$$Cov(Z_j, Z_t) = \pi(1-\pi)(\mu_{1j}-\mu_{2j})(\mu_{1t}-\mu_{2t}). \qquad (3.17)$$

Proof: We have

$$Cov(Z_j, Z_t) = E(Z_j Z_t) - E(Z_j)\,E(Z_t). \qquad (3.18)$$

Also

$$E(Z_j, Z_t) = \int_j \int_t Z_j Z_t f(Z_j, Z_t)\, dZ_j\, dZ_t$$

$$= \pi(1-\pi_y) \int_j \int_t Z_j Z_t g_{1j} Z_j g_{1t}(Z_t)\, dZ_j\, dZ_t$$

$$+ (1-\pi)(1-\pi_y) \int_j \int_t Z_j Z_t g_{2j}(Z_j) g_{2t}(Z_t)\, dZ_j\, d Z_t$$

$$+ \pi \pi_y \int_j \int_t Z_j Z_t g_{3j}(Z_j) g_{3t}(Z_t)\, dZ_j\, d Z_t$$

$$+ \pi_y (1-\pi) \int_j \int_t Z_j Z_t g_{4j}(Z_j) g_{4t}(Z_t)\, dZ_j\, dZ_t$$

$$= \pi(1-\pi_y)\mu_{1j}\mu_{1t} + (1-\pi)(1-\pi_y)\mu_{2j}\mu_{2t}$$

$$+ \pi\pi_y \mu_{1j}\mu_{1t} + \pi_y(1-\pi)\mu_{2j}\mu_{2t}$$

$$= \pi\mu_{1j}\mu_{1t} + (1-\pi)\mu_{2j}\mu_{2t} \qquad (3.19)$$

Thus from the relations (3.15), (3.18) and (3.19), we get (3.17) and it proves the lemma.

Theorem 2 : The variance of the unbiased estimator $\hat{\pi}_{s1}$ of the population parameter is given by

$$V(\hat{\pi}_{S1}) = \pi(1-\pi)/n + \{\pi \sum_{j=1}^{k} \sigma_{1j}^2 + (1-\pi)\sum_{j=1}^{k} \sigma_{2j}^2\}/\{n(m_1-m_2)^2\}$$

$$+ \pi_y [\pi \{\sum_{j=1}^{k} \sigma_{3j}^2 - \sum_{j=1}^{j} \sigma_{1j}^2\} + (1-\pi)\{\sum_{j=1}^{k} \sigma_{4j}^2 - \sum_{j=1}^{k} \sigma_2 j^2\}]/\{n(m_1-m_2)^2\}$$

$$(3.20)$$

Proof : We have

$$V(\hat{\pi}_{S1}) = V[(k\bar{Z} - m_2)/(m_1-m_2)]$$

$$= k^2 V(\bar{Z})/(m_1-m_2)^2$$

$$= V(Z_1 + Z_2 + \ldots + Z_k)/\{n(m_1-m_2)^2\}$$

$$= [\sum_{j=1}^{k} V(Z_j) + 2\sum_{j<t} \mathrm{Cov}(Z_j, Z_t)]/\{n(m_1-m_2)^2\}. \qquad (3.21)$$

Using the relations (3.12) and (3.17) in (3.21), we get (3.20) and it proves the theorem.

We now consider the other method of moments estimator $\hat{\pi}_{s2}$ of π. This estimator $\hat{\pi}_{S2}$ is based on the column averages. For this we have the following theorem.

Theorem 3 : An unbiased estimator of π based on the column averages of Z_{ij} is given by

$$\hat{\pi}_{S2} = \sum_{j=1}^{k} w_j \hat{\pi}_{Sj} \tag{3.22}$$

where

$$\hat{\pi}^*_{Sj} = (\bar{Z}_{.j} - \mu_{2j})/(\mu_{1j} - \mu_{2j}) \tag{3.23}$$

and

$$\sum_{j=1}^{k} w_j = 1. \tag{3.24}$$

Proof: Suppose

$$\bar{Z}_{.j} = n^{-1} \sum_{i=1}^{n} Z_{ij}, j = 1, 2, \ldots k, \tag{3.25}$$

represents the jth column average. Clearly, we have for j = 1, 2, . . . , k,

$$\begin{aligned} E(\bar{Z}_{.j}) &= n^{-1} \sum_{i=1}^{n} E(Z_{ij}) \\ &= E(Z_j) \\ &= \pi(1-\pi_y)\mu_{1j} + (1-\pi)(1-\pi_y)\mu_{2j} + \pi\pi_y\mu_{1j} + \pi_y(1-\pi)\mu_{2j} \\ &= \pi\mu_{1j} + (1-\pi)\mu_{2j}, \end{aligned} \tag{3.26}$$

since Z_{ij} are i.i.d. in i, $\mu_{3j} = \mu_{1j}$ and $\mu_{4j} = \mu_{2j}$.

Hence another MM estimator $\hat{\pi}^*_{Sj}$ can be formed from the average of the th column of the observations by setting $\bar{Z}_{.j} = E(\bar{Z}_{.j})$. On solving for π one gets.

$$\hat{\pi}^*_{Sj} = (\bar{Z}_{.j} - \mu_{2j})/(\mu_{1j} - \mu_{2j}), \qquad j = 1, 2, \ldots, k. \tag{3.27}$$

Thus, a total of k such estimators (not independent of one anther) for j = 1, 2, ..., k can be formed each of which is an unbiased estimator of π.

Therefore, any weighted average of our k estimators $\hat{\pi}^*_{S1}, \hat{\pi}^*_{S2}, \ldots, \hat{\pi}^*_{Sk}$ will also be an unbiased estimator for π. In other words, we get an unbiased estimator of π as

$$\hat{\pi}_{S2} = \sum_{j=1}^{k} w_j \hat{\pi}^*_{Sj} \tag{3.28}$$

Hence the proof of the theorem.

If we choose weights w_j, $j = 1, 2, 3, \ldots, k$, as suggested by Franklin [4], where

$$w_j = \frac{|\mu_{1j} - \mu_{2j}|}{D} \text{ and } D = \sum_{j=1}^{k} |\mu_{1j} - \mu_{2j}|. \tag{3.29}$$

we have the following theorem.

Theorem 4 : The variance of the estimator $\hat{\pi}_{S2}$ of π with weights w_j defined in (3.29) is given by

$$V(\hat{\pi}_{s2}) = \frac{\pi(1-\pi)}{n} + \frac{\pi \sum_{j=1}^{k} \sigma_{ij}^2 + (1-\pi)\sum_{j=1}^{k} \sigma_{2j}^2}{nD^2}$$

$$+ \pi y \frac{\pi\left(\sum_{j=1}^{k} \sigma_{3j}^2 - \sum_{j=1}^{k} \sigma_{1j}^2\right) + (1-\pi)\left(\sum_{j=1}^{k} \sigma_{4j}^2 - \sum_{j=1}^{k} \sigma_{2j}^2\right)}{nD^2}.$$

Proof : The variance of the estimator $\hat{\pi}^*_{sj}$ at (3.23) is given by

$$V(\hat{\pi}^*_{Sj}) = V(\bar{Z}_{.j}) / \{(\mu_{1j} - \mu_{2j})\}$$

$$= V(Z_j) / \{n(\mu_{1j} - \mu_{2j})^2\}, \tag{3.31}$$

since Z_{ij} are i.i.d. for each $i = 1, 2, \ldots, n$.

Using (3.12) in (3.31), we get

$$V(\hat{\pi}^*_{Sj}) = \pi(1-\pi)/n + \{\pi \sum_{j=1}^{k} \sigma_{1j}^2 + (1-\pi)\sum_{j=1}^{k} \sigma_{2j}^2\} / \{n(\mu_{1j} - \mu_{2j})^2\}$$

$$+ \pi_y [\pi \{\sum_{j=1}^{k} \sigma_{3j}^2 - \sum_{j=1}^{k} \sigma_{1j}^2\} + (1-\pi)\{\sum_{j=1}^{k} \sigma_{4j}^2 - \sum_{j=1}^{k} \sigma_{2j}^2\}] / \{n(\mu_{1j} - \mu_{2j})^2 \tag{3.32}$$

Also

$$\text{Cov}\,(\hat{\pi}^*_{Sj}, \hat{\pi}^*_{St}) = \text{Cov}\,(\bar{Z}_{.j}, \bar{Z}_{.t})/(\mu_{1j}-\mu_{2j})(\mu_{1t}-\mu_{2t})$$

$$= \sum_{m=1}^{n} \text{Cov}\,(Z_{mj}, Z_{mt})/\{n^2(\mu_{1j}-\mu_{2j})(\mu_{1t}-\mu_{2t})\}$$

But since the responses are i.i.d., we have by (3.17)

$$\text{Cov}\,(\hat{\pi}^*_{Sj}, \hat{\pi}^*_{St}) = n\,\text{Cov}\,(Z_j, Z_t)/\{n^2(\mu_{1j}-\mu_{2j})(\mu_{1t}-\mu_{2t})\}$$

$$= \pi(1-\pi)/n \tag{3.34}$$

Relation (3.22) shows that

$$V(\hat{\pi}_{s2}) = \sum_{j=1}^{k} w_j^2\, V(\hat{\pi}^*_{sj}) + 2\sum_{j<t}^{k} w_j w_t\, \text{Cov}\,(\hat{\pi}^*_{sj}, \hat{\pi}^*_{st}). \tag{3.35}$$

Using relations (3.32), (3.34) and (3.35) we get (3.30) and it proves the theorem.

This particular choice of weights w_j is made for making the algebraic comparison of the efficiency of the proposed estimator with Franklin [4] estimator easier. One can, however, have any other choice of w_j subjected to the restriction (3.24).

4. *Discussion of Proposed Model*

(i) As reported by Franklin [4], the two MM estimators $\hat{\pi}_{s1}$ and $\hat{\pi}_{s2}$ will be identical if terms $(\mu_{1j}-\mu_{2j})$ for all j have the same sign. In any other case, $\hat{\pi}_{s2}$ will have a smaller variance than the variance of $\hat{\pi}_{s1}$. This aspect of keeping the signs of all $(\mu_{1j}-\mu_{2j})$ the same is a particularly important design consideration in using the estimator. Again if (m_1-m_2) could be equal to or near to zero then the variance of the estimator $\hat{\pi}_{s1}$ could be extremely large. On the other hand, the variations of the estimator $\hat{\pi}_{s2}$ can be decreased by making at least one absolute difference D large.

(ii) The proposed estimator $\hat{\pi}_{s1}$ is found to be more efficient than the Franklin's [4] estimator $\hat{\pi}_1$ if we choose the densities g_{rj} $(r = 1, 2, 3, 4)$ such that $\sigma_{3j} < \sigma_{1j}$ and $\sigma_{4j} < \sigma_{2j}$ for all $j = 1, 2, \ldots, k$. It may be noted that the estimators $\hat{\pi}_{s1}$ and $\hat{\pi}_1$ are obtained by concentrating only on the row averages. Similarly, the estimator $\hat{\pi}_{s2}$ obtained from the column averages is more efficient than the Franklin's estimator $\hat{\pi}_2$ if $\sigma_{3j} < \sigma_{1j}$ and $\sigma_{4j} < \sigma_{2j}$ for all $j = 1, 2, \ldots, k$. Since σ_{rj} $(r = 1, 2, 3, 4.)$

are known quantities and hence may be suitably chosen so as to make the proposed estimators more efficient.

The proposed unrelated question model involves the use of four densities while the Franklin [4] model uses only two densities. It will, therefore, be more difficult to guess the density used by the respondent in the proposed procedure as compared to the Franklin [4] model. The proposed model will, therefore, provide more confidentiality to the respondents and will result in enhanced cooperation from them.

(iii) The proposed randomized response technique is quite general. In Franklin's model, if a respondent belonging to sensitive group A is instructed to use the density g_{ij} and to report his/her value as the random number drawn divided by known constant Δ. If the respondent belongs to non-sensitive group not-A, he/she is instructed to use the density h_{ij} and to report his/her value as the random number drawn divided by known constant Δ. Also, if the densities are dependent, that is, $h_j(Z) = 1 - g_j(Z)$, the Franklin's model reduces to the model recently studied by Singh and Singh [20]

We had seen that the model proposed by Franklin [4] covers the model suggested by Warner [24], Kuk [8], Singh [20] and Mangat [11] as particular cases. However, it is not so easy to see that the model suggested by Mangat and singh [12] is also a special case of it. But, if in the proposed randomized response model, if each respondent possessing both the character A and Y is instructed to use the density g_{ij} but if he/she does not possess character Y then he/she is instructed to use the Franklin's model with dependent densities. Then the proposed model reduces to the model suggested by Mangat and Singh [12].

If $\sigma_{3j} = \sigma_{1j}$ and $\sigma_{4j} = \sigma_{2j}$, for all $j = 1, 2, \ldots, k$, then the proposed model reduces to the Franklin's. It also reduces to the Franklin model if either $\pi_y = 0$ or $\pi_y = 1$.

(iv) In the theory presented in this paper, we have assumed that $\mu_{3j} = \mu_{1j}$ and $\mu_{4j} = \mu_{2j}$, but one need not make this assumption. The theory in the general case, though tedious, is straight forward.

(v) A further generalization of the proposed unrelated question model by following Singh and Singh [19], [21] decision randomization device is straightforward and hence ignored to save the space.

5. *Numerical Illustration*

For the purpose of a numerical illustration, the response generated is either from a continuous distribution $\mu_{1j} = \mu_{3j} = 40$ (for the sensitive category) or

$\mu_{2j} = \mu_{4j} = 50$ (for the non-sensitive category) with $\sigma_{1j} = \sigma_{2j} = 7$ and k = 1 or 3. With these values, the two methods of moments estimators $\hat{\pi}_1$ and $\hat{\pi}_2$ are equivalent. In addition to these values, we have also assumed in the proposed model $\sigma_{3j} = \sigma_{4j} = 5$ such that both of the proposed estimators $\hat{\pi}_{s1}$ and $\hat{\pi}_{s2}$ become equivalent. In this way, the comparison with Franklin's model becomes easier. The relative efficiency of the proposed model over the Franklin's model is given in Table 1 for the different values of π and π_y.

Table 1. Per cent relative efficiency of the proposed unrelated question model over the Franklin's model.

π	.1		.3		.5		.7		.9	
π_y	k = 1	k = 3	k = 1	k = 3	k = 1	k = 3	k = 1	k = 3	k = 1	k = 3
.1	104.3	103.3	103.6	102.2	103.4	102.2	103.6	102.2	104.3	103.3
.3	114.2	110.5	111.5	106.9	110.8	106.2	111.5	106.9	114.2	110.5
.5	126.1	118.8	120.7	112.0	119.4	110.7	102.7	112.0	126.1	118.8
.7	140.8	128.4	131.6	177.6	129.4	115.7	131.6	117.6	140.8	128.4
.9	159.3	139.7	144.6	123.9	141.2	121.1	144.6	123.9	159.3	139.7

REFERENCES

[1] Bansal, M.L., Singh, Sarjinder and Singh, R., 1994. Multi character Survey using randomized response technique. *Commun. Statist. Theory Meth.* **23**, 6, 1705-1715.

[2] Dowling, T.A. and Shachtman, R.H., 1975. On the relative efficiency of randomized response models. *Jour. Amer. Statist. Assoc.,* **70**, 84-87.

[3] Franklin, L.A., 1989a. Randomized response sampling from dichotomous population with continuous randomization. *Survey Methodology,* **15(2)**, 225-235.

[4] Franklin, L.A., 1989b. A comparison of estimators for randomized response sampling with continuous distributions from a dichotomous population. *Commun. Satist. Theory Meth.,* **18(2)**, 489-505.

[5] Greenberg, B.G., Abul-Ela, A.L., Simmons, W.R. and Horvitz, D.G., 1969. The unrelated question randomized response model : Theoretical framework. *Jour. Amer. Statist. Assoc.,* **64**, 520-539.

[6] Horvitz, D.G., Shah, B.V. and Simmons, W. R., 1969. The unrelated question randomized response model. Proceedings of social statistics section. *American Statistical Association, Washington,* D.C., 65-72.

[7] Kathuria, O.P. and Singh, Rajendra., 1992. Supplementary information quantitative randomized response model. *Jour. Indian Soc. Agril. Statist.* **44**(1), 45-54.

[8] Kuk, A.Y.C., 1990. Asking sensitive questions indirectly. *Biometrika,* **77**(2), 436-438.

[9] Lanke, J., 1975. On the choice of the unrelated question in simmons' version of randomized response. *Jour. Amer. Statist. Assoc.*, **70**, 80-83.

[10] Mangat, N.S., 1992. Two stage randomized response sampling Procedure using unrelated question. *Jour. Indian Soc. Agric. Statist.*, **44**(1), 82-87.

[11] Mangat, N.S., 1994. An improved randomized response strategy. *Jour. Royal Statist. Soc., B,* **56**(1), 93-95.

[12] Mangat, N.S. and Singh, Ravindra., 1990. An alternative randomized response procedure. *Biometrika,* **77**(2), 439-442.

[13] Mangat, N.S. and Singh, Ravindra., 1991a. An alternative randomized response approach. *Statistical, anno LI,* **N3**, 328-332.

[14] Mangat, N.S. and Singh Ravindra., 1991b. An alternative randomized response procedure for sampling without replacement. *Jour. Indian. Satist. Assoc.*, **29**, 127-31.

[15] Mangat, N.S., Singh, Ravindra and singh, Sarjinder., 1992. An improved unrelated question randomized response strategy. *Calcutta Satist. Assoc. Bull.* **42**, 277-281.

[16] Mangat, N.S., Singh, Ravindra, Singh, Sarjinder and Singh, B., 1993. On Moors randomized response model. *Biometrical Journal*, **35**(6), 727-755.

[17] Mangat, N.S., Singh, Sarjinder and Singh, R., 1993. On the use of a modified randomization device in randomized response inquiries. *Metron,* **51**, 211-216.

[18] Moors, J.J.A., 1971. Optimization of the unrelated question randomized response model. *Jour. Amer. Statist. Assoc.* **66**, 627-629.

[19] Singh, Sarjinder and Singh, Ravindra., 1992. Improved Franklin's model for randomized response sampling. *Jour. Indian Statist. Assoc.* **30**, 109-122.

[20] Singh, Sarjinder and Singh, Ravindra., 1992. An alternative estimator for randomized response sampling. *Jour. Indian Soc. Agril. Satist.*, **44**, 148-154.

[21] Singh, Sarjinder and Singh, Ravindra., 1993. Generalized Franklin's model for randomized response sampling. commun. *Statist. Theory Meth.* **22**(2), 741-755.

[22] Singh, Sarjinder, Mangat, N.S., Singh, Ravindra., 1994. On estimation of mean/total of stigmatized quantitative variable. Statistica (In press).

[23] Singh, Sarjinder, Singh, R., Mangat, N.S. and Tracy, D.S., 1994. An alternative device for randomized responses. Statistica (In press).

[24] Warner, S.L., 1965. Randomized response : A survey technique for eliminating evasive answer bias. *Jour. Amer. Statist. Assoc.*, **60**, 63-69.