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Multivariate Product Estimators

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Summary

Some product-type estimators based on multivariate auxiliary information have been suggested. For the purpose of extension to the multivariate case a new product estimator proposed by Agrawal and Jain [1] has been used. Multivariate estimators exploiting negatively correlated ancillary information are designed along the lines followed by Olkin [4] and Singh [7]. However, apart from the customary weighted arithmetic average, two more weighted averages–geometric and harmonic-for combining the individual product estimators based on single auxiliary variables have been adopted. A wide-ranging comparison of the existing and the proposed multivariate estimators from the standpoint of bias and mean square error has been undertaken and it is found that the weighted multivariate estimators obtained through the use of 3 kinds of weighted averages would perform better than the so-called ratio-cum-product-type estimators due to Singh [7] under a large variety of conditions that would usually obtain in practice. A comparison with simple mean indicates that the weighted multivariate estimators invariably perform better under conditions that are known to hold when individual estimators, each based on a single auxiliary variable, are preferred to simple mean. Since various weighted multivariate estimators involving Murthy-type and Agrawal-Jain-type product estimators possess the same mean square error, the biases of these estimators are compared showing that the latter are less biased under well known conditions that usually apply when a uni-auxiliary variate product estimator is more efficient than a simple mean.

Introduction

In survey sampling situations the role of ancillary information has been underscored whenever we have easy and economical access to one or more auxiliary variables (X's) which are positively or negatively correlated with study variable Y. Olkin [4] has considered a multivariate ratio estimator based on multi-supplementary variables that are positively correlated with the study variable. Singh [6], following the approach due to Olkin [4], has proposed a multivariate generalization of the product estimator of Murthy [3] and besides that, Singh [7] has employed multi-auxiliary variables which are positively or negatively correlated with the study variable to construct the so-called ratio-cum-product estimators that are unweighted and multiplicative in character.

Agrawal and Jain [1] have proposed a product-type estimator which, besides being predictive in character, is found to fare better than the one due to Murthy [3] under a wide variety of conditions that usually prevail in practice. Since most of the multivariate estimators due to Singh [6], [7] involving negatively correlated auxiliary variables are generalizations of Murthy's product estimator, it would be apt to carry out an investigation of such multivariate estimators by inputting Agrawal-Jain-type product estimator. Further, in this paper, the newly proposed multivariate estimators will be structured on weighted arithmetic, geometric and harmonic means of Agrawal-Jain-type product estimators, each based on a single auxiliary variable negatively correlated with the main variable. As a natural follow-up, an exhaustive comparison embracing multivariate product estimators due to Singh [6], [7] and the newly proposed estimators has been undertaken.

For the purpose of comparison in the ensuing sections, the estimators are divided into two groups designated as G_1 and G_2 where G_1 consists of 2 unweighted estimators built along the lines of Singh [7] and G_2 of 6 weighted estimators obtained via three different weighting systems-arithmetic, geometric and harmonic-and two product-type estimators due to Murthy [3] and Agrawal and Jain [1].

2. *Multivariate Estimators, their Biases and Mean Square Errors*

Suppose that the variables $(Y ; X_1, \ldots, X_p)$, where Y is negatively correlated with $(X_1, \ldots, X_p)$, are observed for each of the n sample units selected from a population of size N according to the method of simple random sampling without replacement. Let $\bar{Y}$ and $\bar{X}_i$ be the population means and $\bar{y}$ and $\bar{x}_i$ be the sample means for the study variable Y and the anxiliary variables X_i $(i = 1, \ldots, p)$, respectively, and let $\tilde{X}_i$ and $\tilde{x}_i$ be respectively, the population and the sample harmonic means for X_i $(i = 1, \ldots, p)$, both $\bar{X}_i$ and $\tilde{X}_i$ $(i = 1, \ldots, p)$ being assumed to be known. We denote variable $Y, X_1, X_2, \ldots, X_p$ by $0, 1, 2, \ldots, p$, respectively. Further, denote the correlation coefficient between Y and X_i $(i = 1, \ldots, p)$, by ρ_{0i} and the correlation coefficient between X_i and X_j $(i \neq j = 1, \ldots, p)$ by ρ_{ij}. Furthermore, let C_0 and C_i be respectively, the coefficients of variation of Y and X_i and let $C_{0i} = \rho_{0i} C_0 C_i$ and $C_{ij} = \rho_{ij} C_i C_j$.

The two unweighted multivariate product estimators constituting the group G_1 are given by

$$\bar{y}_{P1} = \bar{y} \prod_{i=1}^{p} \frac{\bar{x}_i}{\bar{X}_i}$$

$$\bar{y}_{P2} = \bar{y} \prod_{i=1}^{p} \frac{\tilde{x}_i}{\tilde{X}_i}$$

While the six weighted multivariate product estimators forming the group G_2 are

$$\bar{y}'_{P1} = \bar{y} \sum_{i=1}^{p} \frac{w_i \bar{x}_i}{\bar{X}_i} \qquad \bar{y}'_{P2} = \bar{y} \prod_{i=1}^{p} \left(\frac{\bar{x}_i}{\bar{X}_i}\right)^{w_i}$$

$$\bar{y}'_{P3} = \bar{y} \left(\sum_{i=1}^{p} \frac{w_i \bar{X}_i}{\bar{x}_i}\right)^{-1} \qquad \bar{y}'_{P4} = \bar{y} \sum_{i=1}^{p} \frac{w_i \tilde{x}_i}{\tilde{X}_i}$$

$$\bar{y}'_{P5} = \bar{y} \prod_{i=1}^{p} \left(\frac{\tilde{x}_i}{\tilde{X}_i}\right)^{w_i} \qquad \bar{y}'_{P6} = \bar{y} \left(\sum_{i=1}^{p} \frac{w_i \tilde{X}_i}{\tilde{x}_i}\right)^{-1}$$

where w_i's ($i = 1, \ldots, p$) are weights such that $\sum_{i=1}^{p} w_i = 1$. It may be noted that the estimators with and without a prime allude to G_2 and G_1, respectively.

The biases of the various estimators, to the first degree of approximation, that is, to order n^{-1}, G_1 and G_2 are expressible as

$$B(\bar{y}_{P1}) = \theta \bar{Y} \left[\sum_{i=1}^{p} C_{0i} + \sum_{i<j}\sum C_{ij}\right]$$

$$B(\bar{y}_{P2}) = \theta \bar{Y} \left[\sum_{i=1}^{p} C_i^2 + \sum_{i=1}^{p} C_{0i} + \sum_{i<j}\sum C_{ij}\right]$$

$$B(\bar{y}'_{P1}) = \theta \bar{Y} \sum_{i=1}^{p} w_i C_{0i}$$

$$B(\bar{y}'_{P2}) = \theta \bar{Y} \left[\sum_{i=1}^{p} w_i (w_i - 1) C_i^2 + \sum_{i=1}^{p} w_i C_{0i} + \sum_{i<j}\sum w_i w_j C_{ij} \right]$$

$$B(\bar{y}'_{P3}) = \theta \bar{Y} \left[\sum_{i=1}^{p} w_i (w_i - 1) C_{1/2}^2 + \sum_{i=1}^{p} w_i C_{0i} + 2 \sum_{i<j}\sum w_i w_j C_{ij} \right]$$

$$B(\bar{y}'_{P4}) = \theta Y \left[\sum_{i=1}^{p} w_i C_i^2 + \sum_{i=1}^{p} w_i C_{0i} \right]$$

$$B(\bar{y}'_{P5}) = \theta \bar{Y} \left[\sum_{i=1}^{p} w_i (w_i + 1) C_{1/2}^2 + \sum_{i=1}^{p} w_i C_{0i} + \sum_{i<j}\sum w_i w_j C_{ij} \right]$$

$$B(\bar{y}'_{P6}) = \theta \bar{Y} \left[\sum_{i=1}^{p} w_i^2 C_i^2 + \sum_{i=1}^{p} w_i C_{0i} + 2 \sum_{i<j}\sum w_i w_j C_{ij} \right]$$

and the mean square errors, to order n^{-1}, are

$$M(\bar{y}_{P1}) = M(\bar{y}_{P2}) = \theta \bar{Y}^2 \left(C_0^2 + \sum_{i=1}^{p} C_i^2 + 2 \sum_{i=1}^{p} C_{0i} + 2 \sum_{i<j}\sum C_{ij} \right)$$

$$M(\bar{y}_{Pk}) = \underset{\sim}{w}\, A\, \underset{\sim}{w}', \quad (k = 1, \ldots, 6) \qquad (2.1)$$

where $\underset{\sim}{w} = (w_1, \ldots, w_p)$, $A = (a_{ij})$ is a semi-positive definite matrix, $a_{ij} = \theta \bar{Y}^2 \left[C_0^2 + C_{0i} + C_{0j} + C_{ij} \right]$ for $(i, j = 1, \ldots, p)$ and $\theta = \dfrac{N-n}{Nn}$.

3. *Case of Two Auxiliary Variables*

To have a clear grasp of the relative performance of the various multivariate estimators mooted in the preceding section, consider a special case when $p = 2$. Besides, in many practical situations, two auxiliary variables are frequently used.

3.1 *A Comparison of Mean Square Errors with Optimum Weights*

For the case of two auxiliary variables, mean square errors of the estimators, to order n^{-1}, given in the section 2 will reduce to

$$M(\bar{y}_{P1}) = M(\bar{y}_{P2}) = \theta \bar{Y}^2 \left[C_0^2 + C_1^2 + C_2^2 + 2\rho_{01} C_0 C_1 \right.$$
$$\left. 2\rho_{02} C_0 C_2 + 2\rho_{12} C_1 C_2\right]$$

and
$$M(\bar{y}'_{Pk}) = \theta \bar{Y}^2 \left[w_1^2 C_1^2 + w_2^2 C_2^2 + C_0^2 + 2w_1 \rho_{01} C_0 C_1\right.$$
$$\left. + 2w_2\rho_{02} C_0 C_2 + 2w_1 w_2 \rho_{12} C_1 C_2\right]$$
$$= M', \text{ say}, (k = 1, \ldots, 6)$$

where the optimum weight w_1 is given by

$$w_1 = \frac{C_2^2 - \rho_{01} C_0 C_1 + \rho_{02} C_0 C_2 - \rho_{12} C_1 C_2}{C_1^2 + C_2^2 - 2\rho_{12} C_1 C_2}$$
$$= 1 - w_2. \qquad (3.1)$$

Then one can write

$$M' - M = \theta \bar{Y}^2 \left[-w_1^2 \left(C_1^2 + C_2^2 - 2\rho_{12} C_1 C_2\right) - C_1^2 - 2\rho_{01} C_0 C_1 \right.$$
$$\left. - 2\rho_{12} C_1 C_2\right] \qquad (3.2)$$

where w_1 is the optimum weight given in (3.1). Hence, in order that a G_2-estimator be more efficient than a G_1-estimator we have the following necessary and sufficient condition obtainable from (3.2):

$$C_2^2 \left(\rho_{02} C_0 + C_2\right)^2 + C_1^2 \left(\rho_{01} C_0 + C_1\right)^2 + C_1^2 C_2^2 \left(1 - \rho_{12}^2\right)$$
$$- 2C_1 C_2 \left(\rho_{01} C_0 + \rho_{12} C_2\right) \left(\rho_{02} C_0 + \rho_{12} C_1\right) \geq 0, \qquad (3.3)$$

which would be satisfied in a number of cases cited below:

Case 1. The condition (3.3) is satisfied if

$$\text{Min}\left(-\rho_{01} \frac{C_0}{C_1}, -\rho_{02} \frac{C_0}{C_2}\right) \leq \rho_{12} \leq \text{Max}\left(-\rho_{01} \frac{C_0}{C_1}, -\rho_{02} \frac{C_0}{C_2}\right).$$

Case 2. When $C_1 = C_2 = C$, say, any G_2 - estimator will be more efficient than a G_1 - estimator if

$$\rho_{12} > \frac{C_0^2}{C^2} - 1.$$

Case 3. When $\rho_{01} = \rho_{02}$, a G_2 – estimator will be more efficient than a G_1 – estimator if

$$\rho_{12} > \frac{C_0^2 + \left(C_2 - C_1\right)^2}{C_1\, C_2} - 1.$$

Case 4. When $C_0 = C_1 = C_2$, then (3.3) is satisfied if $\rho_{12} > 0$.

It may be noted that, if in addition to either $C_1 = C_2$ or $\rho_{01} = \rho_{02}$, we have $\rho_{12} = 1$, the condition (3.3) will be invariably satisfied.

Thus, a G_2 – estimator performs better than a G_1 – estimator under a variety of conditions that usually obtain in practice. Besides, it should be underscored that, a G_2 – estimator performs better than the usual uni-auxiliary variate product estimator unconditionally, while, a G_1 – estimator does so only under a condition, viz.,

$$\rho_{12} \le -\rho_{02}\frac{C_0}{C_1} - \frac{C_2}{2C_1}$$

In this context it would be desirable to compare both G_1 and G_2 estimators with the sample mean $\bar{y}$ whose variance is given by

$$V(\bar{y}) = \theta\,\bar{Y}^2 C_0^2.,$$

which yields

$$M' - V(\bar{y}) = \theta\,\bar{Y}^2\left[-w_1^2\,(C_1^2 + C_2^2 - 2\rho_{12}C_1C_2) + C_2^2 + 2\rho_{02}C_0C_2\right] \qquad (3.4)$$

where w_1 is given in (3.1). It is then obvious from (3.4) that a G_2 – estimator will perform better than $\bar{y}$ if

$$\left.\begin{aligned} \rho_{02} &\le -\frac{C_2}{2C_0} \\ \text{or analogously,}\quad \rho_{01} &\le -\frac{C_1}{2C_0} \end{aligned}\right\} \qquad (3.5)$$

since it does not matter if we interchange the subscripts 1 and 2. The alternative conditions given in (3.5) are the usual ones paving the way for the use of an auxiliary variate in the form of a product estimator. However, a G_1-estimator would not necessarily be superior to $\bar{y}$ even if the two supplementary variables enlisted for the product method of estimation separately satisfy each of the relevant conditions given in (3.5).

It is also clear from (3.2) that a necessary and sufficient condition for a G_2 - estimator to be more efficient than a G_1 - estimator is

$$w_1^2 \geq \frac{\left(C_1^2 + 2\rho_{01} C_0 C_1 + 2\rho_{12} C_1 C_2\right)}{C_1^2 + C_2^2 - 2\rho_{12} C_1 C_2}; \tag{3.6}$$

otherwise the latter will be more efficient than the former. As regards (3.6), it will always hold if

$$C_1^2 + 2\rho_{01} C_0 C_1 + 2\rho_{12} C_1 C_2 \geq 0$$

or

$$\rho_{01} \geq -\frac{1}{2}\frac{C_1}{C_0} - \rho_{12}\frac{C_2}{C_0}.$$

It is clear that, given $\rho_{12} \geq 0$, a G_2 - estimator is more efficient than a G_1 - estimator if $\rho_{01} \geq -C_1/2C_0$ which implies that, one can even afford to involve, in G_2 estimator, an auxilliary variate though the same does not help uni-auxiliary variate product estimator to perform better than simple mean. However, if we additionally include a second auxiliary variable satisfying the usual condition $\rho_{02} \leq -C_2/2C_0$, then this estimator will be more efficient than both a G_1 - estimator and $\bar{y}$.

To summarize the results of this subsection, one can say that a G_2 - estimator would perform better than a G_1 - estimator under wide-ranging conditions that are likely to obtain in practice. An empirical study in section 4 points to what has been said in the preceding line.

3.2 *A Comparison of Biases*

In subsection 3.1, a large variety of practical conditions are spelt out and under which a G_2 - estimator is more efficient than a G_1 - estimator and $\bar{y}$. Since all the G_2 - estimators listed in section 2 have the same mean square error, we compare these estimators from the standpoint of bias. Relative performance of the G_1 - estimators from the point of view of bias has also been undertaken.

For the purpose of comparison of biases of various estimators, we have, besides $\sum_{i=1}^{p} w_i = 1$, assumed non-negative w_i's (i = 1, 2). The assumption of non-negativity of weights is, in fact, not restrictive because it is found through investigations that the presence of negative weights ordinarily points to ineffectiveness (or irrelevance) of the corresponding auxiliary variables as judged by reduction in mean square error of $\bar{y}'_{Pk}$. Hence, it is felt that there is ample justification in assuming w_i's to be non-negative after weeding out ineffective (or redundant) auxiliary variables which otherwise would tend to complicate the estimator whose performance, at best, is marginally improved by including such X variables. The need to weed out ineffective auxiliary variables has been stressed in regression analysis [see Sarndal et al. [5], p. 276].

In the foregoing context, we may add that the weights are non-negative and uniform for the case when $C_i = C$ and $\rho_{0i} = \rho$ (i = 1, 2)

The biases of various G_2 - estimators, to order n^{-1}, for the case of two auxiliary variables can be obtained as

$$B(\bar{y}'_{P1}) = \theta \bar{Y} \left[w_1 \rho_{01} C_0 C_1 + w_2 \rho_{02} C_0 C_2 \right] = B'_1, \text{ say,}$$

$$B(\bar{y}'_{P2}) = \theta \bar{Y} \Big[w_1 (w_1 - 1) \frac{C_1^2}{2} + w_2 (w_2 - 1) \frac{C_2^2}{2} + w_1 \rho_{01} C_0 C_1 + w_2 \rho_{02} C_0 C_2 + w_1 w_2 \rho_{12} C_1 C_2 \Big] = B'_2, \text{ say,}$$

$$B(\bar{y}'_{P3}) = \theta \bar{Y} \Big[w_1 (w_1 - 1) C_1^2 + w_2 (w_2 - 1) C_2^2 + w_1 \rho_{01} C_0 C_1 + w_2 \rho_{02} C_0 C_2 + 2 w_1 w_2 \rho_{12} C_1 C_2 \Big] = B'_3, \text{ say,}$$

$$B(\bar{y}'_{P4}) = \theta \bar{Y} \Big[w_1 C_1^2 + w_2 C_2^2 + w_1 \rho_{01} C_0 C_1 + w_2 \rho_{02} C_0 C_2 \Big] = B'_4, \text{ say,}$$

$$B(\bar{y}'_{P5}) = \theta \bar{Y} \Big[w_1 (w_1 + 1) \frac{C_1^2}{2} + w_2 (w_2 + 1) \frac{C_2^2}{2} + w_1 \rho_{01} C_0 C_1 + w_2 \rho_{02} C_0 C_2 + w_1 w_2 \rho_{12} C_1 C_2 \Big] = B'_5, \text{ say,}$$

$$B(\bar{y}'_{P6}) = \theta \bar{Y} \Big[w_1^2 C_1^2 + w_2^2 C_2^2 + w_1 \rho_{01} C_0 C_1 + w_2 \rho_{02} C_0 C_2 + 2 w_1 w_2 \rho_{12} C_1 C_2 \Big] = B'_6, \text{ say.}$$

As regards G_1 - estimators, the biases, to order n^{-1}, are

$$B(\bar{y}_{P1}) = \theta \bar{Y} \big[\rho_{01} C_0 C_1 + \rho_{02} C_0 C_2 + \rho_{12} C_1 C_2 \big]$$

$$B(\bar{y}_{P2}) = \theta \bar{Y} \big[C_1^2 + C_2^2 + \rho_{01} C_0 C_1 + \rho_{02} C_0 C_2 + \rho_{12} C_1 C_2 \big].$$

Using the above expressions for biases, it can be checked that, for the first three G_2 - estimators based on Murthy's product estimators, the inequality

$$|B'_1| \le |B'_2| \le |B'_3|$$

will invariably hold since $B'_1 \le 0$, while the last three G_2-estimators involving Agrawal-Jain-type product estimator will satisfy the following relations

$$|B'_4| \le |B'_1|, \quad |B'_5| \le |B'_1| \quad \text{and} \quad |B'_6| \le |B'_1|$$

provided the usual conditions

$$\rho_{0i} \le -\frac{C_0}{2C_i} \quad (i = 1, 2) \tag{3.7}$$

hold. It should be stressed that, the conditions in (3.7) are the prerequisites for the use of an auxiliary variate in the product method of estimation, for otherwise a simple mean (using no auxiliary variate) would be better.

Thus, viewed against the background of the conditions in (3.7), any G_2 - estimator involving weighted arithmetic or geometric or harmonic mean of Agrawal-Jain-type product estimators is less biased than any other G_2 - estimator based on Murthy's product estimator.

Regarding the biases B'_4, B'_5 and B'_6, the inequality

$$B'_6 \leq B'_5 \leq B'_4 \leq 0$$

or equivalently,

$$|B'_4| \leq |B'_5| \leq |B'_6|$$

will hold if $B'_4 \leq 0$, while the inequality

$$B'_4 \geq B'_5 \geq B'_6 \geq 0$$

or equivalently,

$$|B'_6| \leq |B'_5| \leq |B'_4|$$

will hold if $B'_6 \geq 0$.

It is implicit in the above discussion that

$$B'_5 \leq 0 \Rightarrow |B'_6| \geq |B'_5|$$

and $$B'_5 \geq 0 \Rightarrow |B'_4| \geq |B'_5|.$$

However, if $B'_4 \geq 0$ then a necessary and sufficient condition for

$$|B'_4| \geq |B'_5| \text{ and } |B'_4| \geq |B'_6|$$

to hold is

$$w_1 \left[(w_1 + 1)\ C_1^2 + 2\rho_{01}\, C_0\, C_1 + w_2 \rho_{12}\, C_1\, C_2 \right] + w_2 \left[(w_2 + 1)\ C_2^2 + 2\rho_{02}\, C_0\, C_2 + w_1\, \rho_{12}\, C_1\, C_2 \right] \geq 0.$$

Further, if, aside from $B'_4 \geq 0$, we have $B'_5 \geq 0$ and $B'_6 \leq 0$ then a necessary and sufficient condition for

$$|B'_4| \geq |B'_5| \geq |B'_6|$$

to hold is

$$w_1\left[(3w_1+1)\frac{C_1^2}{2}+2\rho_{01}C_0C_1+\frac{3}{2}w_2\rho_{12}C_1C_2\right]$$

$$+w_2\left[(3w_2+1)\frac{C_2^2}{2}+2\rho_{02}C_0C_2+\frac{3}{2}w_1\rho_{12}C_1C_2\right]\geq 0.$$

It is also of interest to point out that, if G_1 – estimators are more efficient than $\bar{y}$, the newly proposed G_1 – estimator $\bar{y}_{P2}$ is less biased than $\bar{y}_{P1}$ due to Singh [7].

4. *An Empirical Study*

For comparing the various estimators from the standpoint of bias and mean square error, we refer to an investigation carried out by the Biometry Research Unit of the Indian Statistical Institute with regard to multivariate investigation of blood chemistry. The details of this investigation are given by Das [2]. To illustrative the performance of various estimators, consider the findings of this investigation using 'eosinophil' (one of the 32 variables) as study variable and 'age' and 'height' as the supplementary variables negatively correlated with the study variable. Singh [6] has also referred to the same investigation for illustrating the performance of the product estimator $\bar{y}'_{P1}$ proposed by him.

For the purpose of computing the biases and the mean square errors of the various estimators considered in this paper, the following quantities are utilised.

$$C_0 = 0.6088 \qquad \rho_{01} = -0.2505$$

$$C_1 = 0.2825 \qquad \rho_{02} = -0.1752$$

$$C_2 = 0.0335 \text{ and } \qquad \rho_{12} = 0.0099$$

which yield the following optimum weights

$$w_1 = 0.5021 \text{ and } \quad w_2 = 0.4979.$$

The biases and mean square errors of various product estimators are presented in Table 1.

Table 1.

	Estimator	Bias/$\theta\bar{Y}$	MSE/$\theta\bar{Y}^2$
1.	Simple mean ($\bar{y}$)	0	0.3706
2.	Murth-ytype using X_1	-0.0431	0.3643
3.	Murthy-type using X_2	-0.0036	0.3646
4.	Agrawal-Jaintype using X_1	0.0367	0.3643
5.	Agrawal-Jain-type using X_2	-0.0025	0.3646
6.	Singh's unweighted using X_1 & X_2 ($\bar{y}_{P1}$)	-0.0466	0.3584
7.	Proposed unweighted using X_1 & X_2 ($\bar{y}_{P2}$)	0.0344	0.3584
8.	Singh's weighted using X_1 & X_2 ($\bar{y}'_{P1}$)	-0.0234	0.3443
9.	Proposed weighted using X_1 & X_2 ($\bar{y}'_{P2}$)	-0.0335	0.3443
10.	Proposed weighted using X_1 & X_2 ($\bar{y}'_{P3}$)	-0.0436	0.3443
11.	Proposed weighted using X_1 & X_2 ($\bar{y}'_{P4}$)	0.0172	0.3443
12.	Proposed weighted using X_1 & X_2 ($\bar{y}'_{P5}$)	0.0071	0.3443
13.	Proposed weighted using X_1 & X_2 ($\bar{y}'_{P6}$)	-0.0030	0.3443

The above Table points to the fact that G_2 - estimators are more efficient than simple mean, uni-auxiliary variate product estimators due to Murthy [3] and Agrawal-Jain [1] and G_1 - estimators. Within the bouquet of G_2 - estimators (having the same mean square error), the weighted harmonic mean of the Agrawal-Jain-Type uni-auxiliary variate product estimators is least biased. It may also be noted that out of the two G_1 - estimators, the one using the Agrawal-Jain-type product estimator has a smaller bias.

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