A NOTE ON CONSTRUCTION OF LATIN CUBES OF THE SECOND AND THIRD ORDER

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The concept of latin and hyper graecolatin cubes was first introduced by Kishen (1942). The method of construction of such cubes has been given by Kishen (1949) and Saxena (1960). The present paper gives a simple method of constructing latin cubes of the second and third orders of any side k.

Let I_n be the identity matrix of order n; O_{pq} a $(p\times q)$ null matrix and E_{pq} a $(p\times q)$ matrix with all elements unity. $V\times W$ is the matrix obtained by taking the kronecker product of the matrices V and W. C_r is the cyclic latin square of side r in r integers 0, 1,..., $r-1$. The $k\times k$ matrix A is the arrangement of k^2 integers 0, 1, 2,..., k^2-1, in any order.

Theorem 2.1.

If
$$B=\begin{bmatrix} O_{k-1,\,1} & I_{k-1} \\ I_1 & O_{1,\,k-1} \end{bmatrix}$$

$$U_{rs}=B^{k-s}\ AB^{k-r+s}$$

$$G_{ft}=B^{k-f+t}\ AB^{k+2t-f}$$

$$Z_{r\cdot}=[U_{r0}\mid U_{r1}\mid \dots \mid U_{r,\,k-1}]$$

and

$$X_{\cdot t}=[G_{0t}\mid G_{1t}\mid \ \dots\ \mid G_{k-1,\,t}]$$

then for every value of r and t; and r, $t=0, 1, \dots, k-1$

(*i*) $Z_{r\cdot}$ and $X_{\cdot t}$ are $k\times k\times k$ latin cubes of second order of side k

(*ii*) $E_{k1} \times Z_{r\cdot}$ and $E_{k1} \times X_{\cdot t}$ are $k \times k \times k \times k$ or 4 fold latin cubes of second order

and

(*iii*) $E_{k1} \times Z_{r\cdot} + C_k \times k^2 \, E_{kk}$

and

$E_{k1} \times X_{\cdot t} + C_k \times k^2 \, E_{kk}$

are the $k \times k \times k \times k$ latin cubes of the third order of side k.

Example 2.1

Take $k=4$ and $A = \begin{bmatrix} 0 & 4 & 8 & 12 \\ 1 & 5 & 9 & 13 \\ 2 & 6 & 10 & 14 \\ 3 & 7 & 11 & 15 \end{bmatrix}$

Here $B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

and

$$C_4 = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{bmatrix}$$

then

$$Z_{0\cdot} = \left[\begin{array}{cccc|cccc|cccc|cccc} 0 & 4 & 8 & 12 & 15 & 3 & 7 & 11 & 10 & 14 & 2 & 6 & 5 & 9 & 13 & 1 \\ 1 & 5 & 9 & 13 & 12 & 0 & 4 & 8 & 11 & 15 & 3 & 7 & 6 & 10 & 14 & 2 \\ 2 & 6 & 10 & 14 & 13 & 1 & 5 & 9 & 8 & 12 & 0 & 4 & 7 & 11 & 15 & 3 \\ 3 & 7 & 11 & 15 & 14 & 2 & 6 & 10 & 9 & 13 & 1 & 5 & 4 & 8 & 12 & 0 \end{array}\right]$$

is the $4 \times 4 \times 4$ latin cube of order 2.

and $E_{41}\times Z_{0}.+C_{4}\times 16E_{44}=$

0	4	8	12	31	19	23	27	42	46	34	38	53	57	61	49
1	5	9	13	28	16	20	24	43	47	35	39	54	58	62	50
2	6	10	14	29	17	21	25	40	44	32	36	55	59	63	51
3	7	11	15	30	18	22	26	41	45	33	37	52	56	60	48
16	20	24	28	47	35	39	43	58	62	50	54	5	9	13	1
17	21	25	29	44	32	36	40	59	63	51	55	6	10	14	2
18	22	26	30	45	33	37	41	56	60	48	52	7	11	15	3
19	23	27	31	46	34	38	42	57	61	49	53	4	8	12	0
32	36	40	44	63	51	55	59	10	14	2	6	21	25	29	17
33	37	41	45	60	48	52	56	11	15	3	7	22	26	30	18
34	38	42	46	61	49	53	57	8	12	0	4	23	27	31	19
35	39	43	47	62	50	54	58	9	13	1	5	20	24	28	16
48	52	56	60	15	3	7	11	26	30	18	22	37	41	45	33
49	53	57	61	12	0	4	8	27	31	19	23	38	42	46	34
50	54	58	62	13	1	5	9	24	28	16	20	39	43	47	35
51	55	59	63	14	2	6	10	25	29	17	21	36	40	44	32

is the $4\times4\times4\times4$ latin cube of third order of side 4.

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