Available online at www.isas.org.in/jisas
JOURNAL OF THE INDIAN SOCIETY OF AGRICULTURAL STATISTICS 75(3) 2021 221–227

Improved Searls Estimation of Population Mean under Ranked Set Sampling

S.K. Yadav and Madhulika Dube

Babasaheb Bhimrao Ambedkar University, Lucknow

Received 24 March 2020; Revised 11 March 2022; Accepted 29 March 2022

SUMMARY

Working with Ranked Set Sampling (RSS) for estimation of finite population mean using auxiliary characters, the article considers a class of generalized Searls type ratio estimators that provides an improvement over the existing estimators. The sampling properties including bias and the mean squared error (MSE) of the proposed class have been studied to the approximation of order one. The optimum value of the Searls constant is obtained and the least MSE value of the introduced class of estimators is obtained for this optimum value of the Searls constant. The proposed class of estimators is compared theoretically with the competing estimators under RSS. The conditions of dominance of the suggested estimator over existing competing estimators are obtained. A numerical study has is carried out to verify the efficiencies of the introduced estimator over the existing competing estimators under RSS.

Keywords: Main variable, Auxiliary variable, Ranked Set Sampling, Bias, MSE, PRE.

2010 AMS Classification: 62D05

1. INTRODUCTION

The pivotal paper of McIntyre (1952) opened up fresh avenues in sample selection strategies. Known as the Ranked Set Sampling (RSS), the method could not gather ample attention initially. However, the method has received considerable attention in the last two decades owing to its cost effectiveness under certain conditions. Laying the theoretical foundations of Ranked Set Sampling, Takahasi and Wakimoto (1968) demonstrated that the sample mean ($\bar{y}_{rss}$) estimator is unbiased for population mean ($\bar{Y}$) of the main variable (Y) under RSS. Working in the same direction, Dell and Clutter (1972) has shown that $\bar{y}_{rss}$ under RSS is as efficient as under the simple random sampling scheme. They also demonstrated that there is no effect of ranking in the case of the mean per unit estimator of $\bar{Y}$. Extending the work and ranking auxiliary variables (X) instead of observations on Y, Stokes (1977) studied the behavior of the RSS estimator in comparison to the estimator obtained through SRSWR. Samawi and Muttlak (1996) utilized the known information on the auxiliary variable (X) and suggested the traditional ratio estimator for $\bar{Y}$ under RSS. Inspired by their work, several other authors also worked towards improvement through modifications in ratio type estimators under RSS Scheme.

Over the years, extensive work has been carried out through modifications in the usual ratio estimator for estimating population mean $\bar{Y}$ under RSS. Working in this direction, Muttlak (2003) utilized known quartiles of $\bar{Y}$, Kadilar *et al.* (2009) used auxiliary variable while power transformation on auxiliary variables was instrumental towards improved estimation of the population mean $\bar{Y}$ under RSS that was used by Mandowara and Mehta (2012). In addition to the above, Balci *et al.* (2013) suggested a modified maximum

Corresponding author: Madhulika Dube
E-mail address: madhulikadube@gmail.com

likelihood estimators of the population mean and Biradar and Santosha (2015) worked in this direction using extreme values under ranked set sampling. Many variations in improved estimation strategies in RSS have also been the focus in recent years with Khan and Shabbir (2016) suggesting to use two auxiliary variables, Abbasi and Shad (2017) utilizing robust truncation, Pelle and Perri (2018) suggesting Rao regression-type estimator and Khan and Ismail (2019) proposing an improved ratio type estimator. Very recently, Khan *et al.* (2019) and Bhushan and Kumar (2021)suggested the modified RSS scheme for elevated estimation of $\bar{Y}$.

The Searls (1964) estimation procedure suggests that if $\bar{y}_{rss}$ is sample mean of the population mean $\bar{Y}$ under the Ranked Set Sampling, then $\left(\tau \bar{y}_{RSS}\right)$ is more efficient than $\bar{y}_{rss}$, where τ is the characterizing constant to be determined in such a way that the MSE of $\left(\tau \bar{y}_{RSS}\right)$ is least. In view of the above, this paper suggests some improved estimators of the population mean $\bar{Y}$ under RSS. The paper also works out the properties of the sampling distribution of the suggested estimators to order one. A brief description of the RSS has been given in Section 2 of the paper and in Section 3 various existing estimators using RSS have been provided. The proposed estimators, their efficiency properties and an empirical study are the subject matter of Section 4 while the results have been verified by a simulation study in Section 5. A discussion and some concluding remarks have been provided at the end of the article.

2. RANKED SET SAMPLING

In RSS scheme, m^2 units are selected from the population under consideration, using a simple random sampling scheme. These m^2 units are then distributed in m sets of distinct units, each of size, m . Then ranking is done in each group or set on the basis of the auxiliary characteristic under considerationby visual inspection or any rough method not requiring actual quantification. Finally, the i^{th} ranked unit is selected from i^{th} set to get actual measurement for $i = 1, 2, ..., m$. In such a way a ranked set sample of size m is obtained. Further, if we wish to get more samples, this procedure is repeated r times giving rm identified units. The structures of the randomly sampled units for the auxiliary values before ranking and quantification and after ranking and quantification are discussed in Bhushan and Kumar (2021) and are respectively given as

Before Ranking and Quantification	
Set	
1	$X_{11}X_{12} \ldots X_{1m}$
2	$X_{21}X_{22} \ldots X_{2m}$
⋮	⋮
⋮	⋮
m	$X_{m1}X_{m2} \ldots X_{mm}$

After Ranking and Quantification	
Set	
1	$\left[\boldsymbol{X}_{1(1)}\right]\boldsymbol{X}_{1(2)} \ldots \boldsymbol{X}_{1(\boldsymbol{m})}$
2	$\boldsymbol{X}_{2(1)}\left[\boldsymbol{X}_{2(2)}\right] \ldots \boldsymbol{X}_{2(\boldsymbol{m})}$
⋮	⋮
⋮	⋮
m	$\boldsymbol{X}_{\boldsymbol{m}(1)}\boldsymbol{X}_{\boldsymbol{m}(2)} \ldots \left[\boldsymbol{X}_{\boldsymbol{m}(\boldsymbol{m})}\right]$

A similar structure may be used for Y values as well.

Suppose a bivariate population having probability density function $f(x, y)$ has N distinct and identifiable units.It is assumed that the population means of the variables X and Y are μ_x and μ_y , population variances are σ_x^2 and σ_y^2 , population covariance between X and Y is σ_{xy} and the population correlation coefficient between X and Y is ρ_{xy} . Suppose m independent random vectors $\left[(X_{11}, Y_{11}), (X_{12}, Y_{12}) \ldots (X_{1m}, Y_{1m})\right]$, $[(X_{21}, Y_{21}), (X_{22}, Y_{22}) \ldots (X_{2m}, Y_{2m})], \ldots [(X_{m1}, Y_{m1}),$ $(X_{m2}, Y_{m2}), \ldots (X_{mm}, Y_{mm})]$ are drawn from this population.

In order to construct a ranked set sample, let $\left(X_{1(1)}, Y_{1[1]}\right)$, $\left(X_{2(2)}, Y_{2[2]}\right) \ldots \left(X_{i(i)}, Y_{i[i]}\right) \ldots \left(X_{m(m)}, Y_{m[m]}\right)$ be the order statistics of $X_{i1}, X_{i2} \ldots X_{im}$ and the judgement order of $Y_{i1}, Y_{i2} \ldots, Y_{im}$; $i = 1, 2, \ldots m$. Therefore, $\left(X_{1(1)}, Y_{1[1]}\right)$, $\left(X_{2(2)}, Y_{2[2]}\right) \ldots \left(X_{i(i)}, Y_{i[i]}\right) \ldots \left(X_{m(m)}, Y_{m[m]}\right)$ is the ranked set sample constructed in such a way that $X_{i(i)}$ is the i^{th} order statistics in the i^{th} sample of the auxiliary variable X and $Y_{i[i]}$ is the i^{th} judgment order in the i^{th} sample of the main variable Y. The notations have been simplified and $\left(X_{i(i)}, Y_{i[i]}\right)$ is denoted as $\left(X_{(i)}, Y_{[i]}\right)$ in rest of the paper. It may be noted that the parenthesis () and the bracket [] that have been used as the subscripts for X and Y represent that the ranking to be perfect and imperfect respectively.

Following the notations used in Takahasi and Wakimoto (1968), let the sample means of Y and X under Ranked Set Sampling be $\bar{y}_{RSS} = \frac{1}{rm}\sum_{i=1}^{m}\sum_{j=1}^{r} y_{[i]j}$ and $\bar{x}_{RSS} = \frac{1}{rm}\sum_{i=1}^{m}\sum_{j=1}^{r} x_{(i)j}$

and the sampling variances of $\bar{y}_{rss}$ and $\bar{x}_{RSS}$ and the covariance between $\bar{y}_{rss}$ and $\bar{x}_{RSS}$ under RSS are respectively

$$V\left(\bar{y}_{RSS}\right) = \frac{\sigma_y^2}{rm} - \frac{1}{rm^2}\sum_{i=1}^{m}\tau_{y[i]}^2$$

$$V\left(\bar{x}_{RSS}\right) = \frac{\sigma_x^2}{rm} - \frac{1}{rm^2}\sum_{i=1}^{m}\tau_{x(i)}^2 .$$

and $Cov\left(\bar{y}_{RSS}, \bar{x}_{RSS}\right) = \frac{\sigma_{xy}}{rm} - \frac{1}{rm^2}\sum_{i=1}^{m}\tau_{yx(i)}$, where

$\tau_{y[i]} = \left[\mu_{y[i]} - \bar{Y}\right]$, $\tau_{x(i)} = \left[\mu_{x(i)} - \bar{X}\right]$,

$\tau_{yx(i)} = \left[\mu_{y[i]} - \bar{Y}\right]\left[\mu_{x(i)} - \bar{X}\right]$ and $\mu_{y[i]} = E\left[Y_{[i]}\right]$,

$\mu_{x(i)} = E\left[X_{(i)}\right]$, $\mu_{yx(i)} = E\left[Y_{[i]}X_{(i)}\right]$.

Note that, $\mu_{y[i]}$ and $\mu_{x(i)}$ are the values which depend on the order statistics of some specific distribution. For more details in this regard see; e.g. Arnold *et al.* (1993).

3. A BRIEF REVIEW OF EXISTING ESTIMATORS UNDER RSS

The following Table 1 represents some of the modified ratio estimators for estimating $\bar{Y}$ using known auxiliary variable under RSS scheme along with their MSE and the constants.

where, $\theta = \frac{1}{rm}$, $C_y^2 = \frac{S_y^2}{\bar{Y}^2}$, $C_x^2 = \frac{S_x^2}{\bar{X}^2}$,

$W_y^2 = \frac{1}{rm^2}\frac{1}{\bar{Y}^2}\sum_{i=1}^{m}\tau_{y[i]}^2$, $W_x^2 = \frac{1}{rm^2}\frac{1}{\bar{X}^2}\sum_{i=1}^{m}\tau_{x(i)}^2$ and

$W_{yx} = \frac{1}{rm^2}\frac{1}{\bar{Y}\bar{X}}\sum_{i=1}^{m}\tau_{yx(i)}$. $\beta_1(x)$ and $\beta_2(x)$ are respectively the coefficients of skewness and kurtosis of the variable X

4. PROPOSED ESTIMATORS

Motivated by the works of Searls (1964) and Khan and Ismail (2019), for the more efficient estimation of the population mean $\bar{Y}$, the following family of estimators utilizing an auxiliary variable X under RSS is suggested as;

$$t_{pj}^{rss} = \kappa_j\left[\bar{y}_{rss}\left(\frac{a\bar{X}+b}{a\bar{x}_{rss}+b}\right)\right];\ j = 1,2,3,4 \qquad (1)$$

where, $\kappa_j; j = 1,2,3,4$ are the characterizing Searls constants which are obtained in such a way that the MSE of the proposed estimator is minimum. The other terms used in the expression, i.e., $a(\neq 0)$ and b may either be constants or the parameters of the auxiliary variable X. In view of this, some members of the suggested class of estimators are presented in Table 2.

Table 1. Various estimators, their MSE's and constants under RSS

S. No.	Author(s)	Estimator	MSE	Constant
1.	Samawi and Muttalak (1996)	$t_1^{rss} = \bar{y}_{rss}\left(\frac{\bar{X}}{\bar{x}_{rss}}\right)$	$\bar{Y}^2\left[\theta\left\{C_y^2 + C_x^2 - 2\rho_{yx}C_yC_x\right\} - \left\{W_y^2 + W_x^2 - 2W_{yx}\right\}\right]$	---
2.	Mehta and Mandowara (2016)	$t_2^{rss} = \bar{y}_{rss}\left(\frac{\bar{X}+C_x}{\bar{x}_{rss}+C_x}\right)$	$\bar{Y}^2\left[\begin{array}{l}\theta\left\{C_y^2 + \lambda_2^2C_x^2 - 2\lambda_2\rho_{yx}C_yC_x\right\}\\ -\left\{W_y^2 + \lambda_2^2W_x^2 - 2\lambda_2W_{yx}\right\}\end{array}\right]$	$\lambda_2 = \frac{\bar{X}}{\bar{X}+C_x}$
3.	Khan and Ismail (2019)	$t_3^{rss} = \bar{y}_{rss}\left(\frac{\bar{X}+\beta_1(x)}{\bar{x}_{rss}+\beta_1(x)}\right)$	$\bar{Y}^2\left[\begin{array}{l}\theta\left\{C_y^2 + \lambda_3^2C_x^2 - 2\lambda_3\rho_{yx}C_yC_x\right\}\\ -\left\{W_y^2 + \lambda_3^2W_x^2 - 2\lambda_3W_{yx}\right\}\end{array}\right]$	$\lambda_3 = \frac{\bar{X}}{\bar{X}+\beta_1(x)}$
4.	Khan and Ismail (2019)	$t_4^{rss} = \bar{y}_{rss}\left(\frac{\beta_2(x)\bar{X}+\beta_1(x)}{\beta_2(x)\bar{x}_{rss}+\beta_1(x)}\right)$	$\bar{Y}^2\left[\begin{array}{l}\theta\left\{C_y^2 + \lambda_4^2C_x^2 - 2\lambda_4\rho_{yx}C_yC_x\right\}\\ -\left\{W_y^2 + \lambda_4^2W_x^2 - 2\lambda_4W_{yx}\right\}\end{array}\right]$	$\lambda_4 = \frac{\beta_2(x)\bar{X}}{\beta_2(x)\bar{X}+\beta_1(x)}$
5.	Khan and Ismail (2019)	$t_5^{rss} = \bar{y}_{rss}\left(\frac{C_x\bar{X}+\beta_1(x)}{C_x\bar{x}_{rss}+\beta_1(x)}\right)$	$\bar{Y}^2\left[\begin{array}{l}\theta\left\{C_y^2 + \lambda_5^2C_x^2 - 2\lambda_5\rho_{yx}C_yC_x\right\}\\ -\left\{W_y^2 + \lambda_5^2W_x^2 - 2\lambda_5W_{yx}\right\}\end{array}\right]$	$\lambda_5 = \frac{C_x\bar{X}}{C_x\bar{X}+\beta_1(x)}$

Table 2. Members of the suggested family of estimators

S. No.	Estimator	a	b
1.	$t_{p1}^{rss} = \kappa_1 \bar{y}_{rss}\left(\frac{\bar{X}+C_x}{\bar{x}_{rss}+C_x}\right)$	1	C_x
2.	$t_{p2}^{rss} = \kappa_2 \bar{y}_{rss}\left(\frac{\bar{X}+\beta_1(x)}{\bar{x}_{rss}+\beta_1(x)}\right)$	1	$\beta_1(x)$
3.	$t_{p3}^{rss} = \kappa_3 \bar{y}_{rss}\left(\frac{\beta_2(x)\bar{X}+\beta_1(x)}{\beta_2(x)\bar{x}_{rss}+\beta_1(x)}\right)$	$\beta_2(x)$	$\beta_1(x)$
4.	$t_{p4}^{rss} = \kappa_4 \bar{y}_{rss}\left(\frac{C_x\bar{X}+\beta_1(x)}{C_x\bar{x}_{rss}+\beta_1(x)}\right)$	C_x	$\beta_1(x)$

In order to study the sampling properties of the proposed estimators, the following standard approximations have been used

$\bar{y}_{rss} = \bar{Y}(1+e_0)$, $\bar{x}_{rss} = \bar{X}(1+e_1)$,

where, $E(e_0) = E(e_1) = 0$

We also have,

$$E\left(e_0^2\right) = \frac{V\left(\bar{y}_{rss}\right)}{\bar{Y}^2} = \theta C_y^2 - W_y^2,$$

$$E\left(e_1^2\right) = \frac{V\left(\bar{x}_{rss}\right)}{\bar{X}^2} = \theta C_x^2 - W_x^2 \text{ and } E(e_0e_1) = \theta\rho_{yx}C_yC_x - W_{yx}$$

Using above approximations, the suggested estimator may be written as,

$$t_{pj}^{rss} = \kappa_j \bar{Y}(1+e_0)(1+\lambda_j e_1)^{-1}, \text{ where } \lambda_j = \frac{a\bar{X}}{a\bar{X}+b}$$

Expanding the term $(1+\lambda_j e_1)^{-1}$ up to the approximation of order one and simplifying the right hand side of the estimator, we get,

$$t_{pj}^{rss} = \kappa_j \bar{Y}\left(1+e_0-\lambda_j e_1-\lambda_j e_0 e_1+\lambda_j^2 e_1^2\right)$$

Subtracting $\bar{Y}$ from the above equation on both sides, we have,

$$t_{pj}^{rss} - \bar{Y} = \kappa_j \bar{Y}\left(1+e_0-\lambda_j e_1-\lambda_j e_0 e_1+\lambda_j^2 e_1^2\right) - \bar{Y} \quad (2)$$

so that the bias of the suggested estimator t_{pj}^{rss} may be obtained as

$$B\left(t_{pj}^{rss}\right) = \bar{Y}\left[\kappa_j\left\{1-\lambda_j\left(\theta\rho_{yx}C_yC_x - W_{yx}\right)+\lambda_j^2\left(\theta C_x^2 - W_x^2\right)\right\}-1\right]$$

Squaring and taking expectation of (2) yields the MSE of the suggested estimator t_{pj}^{rss} as follows:

$$MSE\left(t_{pj}^{rss}\right) = \bar{Y}^2\left[1+\kappa_j^2\left\{1+\left(\theta C_y^2 - W_y^2\right)+3\lambda_j^2\left(\theta C_x^2 - W_x^2\right)-4\lambda_j\left(\theta\rho_{yx}C_yC_x - W_{yx}\right)\right\}-2\kappa_j\left\{1+\lambda_j^2\left(\theta C_x^2 - W_x^2\right)-\lambda_j\left(\theta\rho_{yx}C_yC_x - W_{yx}\right)\right\}\right]$$

so that, $MSE\left(t_{pj}^{rss}\right) = \bar{Y}^2\left[1+\kappa_j^2 B_j - 2\kappa_j A_j\right]$ (3)

where, $A_j = \left\{1+\lambda_j^2\left(\theta C_x^2 - W_x^2\right)-\lambda_j\left(\theta\rho_{yx}C_yC_x - W_{yx}\right)\right\}$

and $B_j = \left\{1+\left(\theta C_y^2 - W_y^2\right)+3\lambda_j^2\left(\theta C_x^2 - W_x^2\right)-4\lambda_j\left(\theta\rho_{yx}C_yC_x - W_{yx}\right)\right\}$

Table 3. Efficiency conditions of the proposed estimators over competing estimators

S. No.	Efficiency comparison	Efficiency condition
1.	$MSE\left(t_1^{rss}\right) - MSE_{min}\left(t_{pj}^{rss}\right) > 0$	$\left[\theta\left\{C_y^2 + C_x^2 - 2\rho_{yx}C_yC_x\right\} - \left\{W_y^2 + W_x^2 - 2W_{yx}\right\}\right] - \left[1 - \frac{A_j^2}{B_j}\right] > 0$
2.	$MSE\left(t_2^{rss}\right) - MSE_{min}\left(t_{pj}^{rss}\right) > 0$	$\left[\theta\left\{C_y^2 + \lambda_2^2C_x^2 - 2\lambda_2\rho_{yx}C_yC_x\right\} - \left\{W_y^2 + \lambda_2^2W_x^2 - 2\lambda_2W_{yx}\right\}\right] - \left[1 - \frac{A_j^2}{B_j}\right] > 0$
3.	$MSE\left(t_3^{rss}\right) - MSE_{min}\left(t_{pj}^{rss}\right) > 0$	$\left[\theta\left\{C_y^2 + \lambda_3^2C_x^2 - 2\lambda_3\rho_{yx}C_yC_x\right\} - \left\{W_y^2 + \lambda_3^2W_x^2 - 2\lambda_3W_{yx}\right\}\right] - \left[1 - \frac{A_j^2}{B_j}\right] > 0$
4.	$MSE\left(t_4^{rss}\right) - MSE_{min}\left(t_{pj}^{rss}\right) > 0$	$\left[\theta\left\{C_y^2 + \lambda_4^2C_x^2 - 2\lambda_4\rho_{yx}C_yC_x\right\} - \left\{W_y^2 + \lambda_4^2W_x^2 - 2\lambda_4W_{yx}\right\}\right] - \left[1 - \frac{A_j^2}{B_j}\right] > 0$
5.	$MSE\left(t_5^{rss}\right) - MSE_{min}\left(t_{pj}^{rss}\right) > 0$	$\left[\theta\left\{C_y^2 + \lambda_5^2C_x^2 - 2\lambda_5\rho_{yx}C_yC_x\right\} - \left\{W_{y(i)}^2 + \lambda_5^2W_{x(i)}^2 - 2\lambda_5W_{yx(i)}\right\}\right] - \left[1 - \frac{A_j^2}{B_j}\right] > 0$

The optimum value of Searls characterizing scalar that minimizes the $MSE\left(t_{pi}^{rss}\right)$ is obtained by

$$\frac{\partial}{\partial \kappa_j} MSE\left(t_{pj}^{rss}\right) = 0 \text{ giving, } \kappa_{j(opt)} = \frac{A_j}{B_j};\ j = 1,2,3,4$$

The minimum values of the MSE's of the introduced estimators for these optimum values of Searls characterizing scalars are,

$$MSE_{min}\left(t_{pj}^{rss}\right) = \bar{Y}^2\left[1 - \frac{A_j^2}{B_j}\right];\ j = 1,2,3,4 \qquad (4)$$

4.1 Efficiency Comparisons

The efficiency conditions for better performance of the introduced estimator overexisting competing estimators have been presented in the following Table:

In order to obtain the results empirically, let us consider the example given in Khan and Ismail (2019) taken from Cochran (1977). In this data, the observations on the total number of inhabitants in thousands from 49 cities during 1920 and 1930 were collected. The study variable Y is taken as inhabitants in 1930 and the auxiliary variable X as inhabitant in 1920.Now from these 49 cities, the 16 simple random samples for both Y and X are taken and then we divided them in 4 sets each with size 4 and we rank each set. From these four ranked sets, the i^{th} ranked unit is drawn from i^{th} set, which give $m = 4$ ranked set samples. This process is repeated twice to get 8 ranked set samples. The population parameters for the main as well as for the auxiliary variable under consideration are presented in Table 4.

Table 4. Population Parameters under consideration

$N = 49$	$n = 8$	$m = 4$	$r = 2$
$\hat{X} = 103.1429$	$\hat{Y} = 127.7959$	$S_y = 32.8720$	$S_x = 56.2342$
$\rho = 0.18$	$\mu_{1(x)} = 70$	$\mu_{2(x)} = 42$	$\mu_{3(x)} = 76$
$\mu_{4(x)} = 144$	$\mu_{1(y)} = 70$	$\mu_{2(y)} = 87$	$\mu_{3(y)} = 93.5$
$\mu_{4(y)} = 103.5$	$\beta_{1(x)} = 2.20$	$\beta_{2(x)} = 7.22$	

The MSE's of the suggested and the other competitive estimators of $\hat{Y}$ under RSS scheme for given data set have been given in Table-5. The Percentage Relative Efficiency (PRE) of different estimators with respect to t_1^{rss} has also been given in Table 5. The PRE of the introduced and the competing estimators with respect to t_1^{rss} is calculated as

$$PRE\left(t_z^{rss}, t_1^{rss}\right) = \frac{MSE\left(t_1^{rss}\right)}{MSE\left(t_z^{rss}\right)} X100\ ; z = j, k$$

where $j = 1,2\ldots 5$ and $k = p1, p2, p3, p4$ (5)

Table 5. MSE of various estimators and PRE with respect to t_1^{rss}

S. No.	Estimator	MSE	PRE
1.	t_1^{rss}	376.7045	100.0000
2.	t_2^{rss}	367.5767	102.4834
3.	t_3^{rss}	359.1788	104.8795
4.	t_4^{rss}	364.2033	103.4326
5.	t_5^{rss}	359.6667	104.7372
Proposed Estimators			
6.	t_{p1}^{rss}	284.2564	132.5230
7.	t_{p2}^{rss}	**277.8962**	**135.5560**
8.	t_{p3}^{rss}	284.8567	132.2437
9.	t_{p4}^{rss}	278.0568	135.4777

From Table-5, it is easy to see that the MSE's of the introduced estimators are uniformly smaller than the competing estimators. Further, while the MSEs of the competing estimators lie in the interval [359.18, 376.70] those of the suggested estimators are much smaller and lie in the interval[277.89, 284.86]. Clearly the proposed estimators dominate the existing estimators and among the proposed estimators, the estimator t_{p2}^{rss} has the minimum MSE (277.8962) and highest PRE (135.5560)of all. The following Figure-1 represents the PRE of the suggested and competing estimators with respect to the estimator t_1^{rss} for the given real data set.

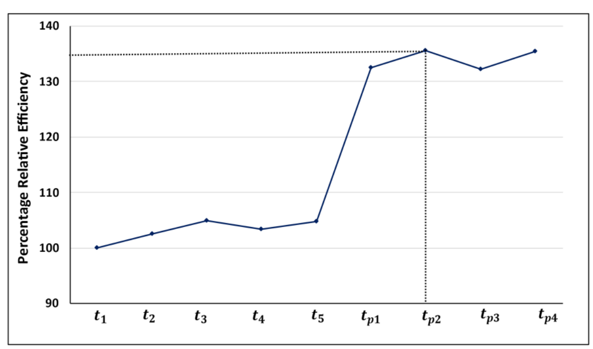

Fig. 1. PRE of different estimators over the estimator t_1^{rss} in Table 5 for real data

5. SIMULATION STUDY

In this section, the theoretical findings have been verified using a simulation study using R programming language. This will aid in comparing the results for the proposed and the existing estimators for the artificially generated population. Assuming a bivariate normal distribution, the simulated population has been generated using the parameters of the real-life population considered earlier so that the means, variances and the coefficient of correlation between the variables in the generated population are taken as $\mu_y = 127.7959$, $\mu_x = 103.1429$, $\sigma_y^2 = 1080.568$, $\sigma_x^2 = 3162.285$ and $\rho_{yx} = 0.18$.

The following steps have been used for the generation of required simulated population:

(a) Generate a bivariate normal population of size $N = 1000$.

(b) Randomly select a sample of size $(10)^2$ from this simulated population.

(c) Allocate these $(10)^2$ selected units at random in 10 sets each of size 10.

(d) Apply RSS in these 10 sets each of size 10 and get a Ranked Set Sample of size 10.

(e) Steps (b), (c) and (d) are repeated 5 times to get a sample of size 50.

(f) Step (e) is repeated 1000 times.

(g) If $t_{(j)i}^{rss}$ is the i^{th} value of the estimator t_j^{rss}, then the Empirical MSE of the estimator t_j^{rss} is computed using $MSE\left(t_j^{rss}\right) = \frac{1}{1000}\sum_{i=1}^{1000}\left(t_{(j)i}^{rss} - \bar{Y}\right)^2$.

(h) The Percentage Relative Efficiency (PRE) of the suggested and competing estimators with respect to the estimator t_1^{rss} have been computed using (5).

(i) The PREs, Relative Biases and Relative RMSEs of different competing and suggested estimators with respect to the estimator t_1^{rss} are presented in Table-6. The Empirical Percentage Absolute Relative Bias (PARB) and the Relative Root Mean Squared Error (RRMSE) have also been obtained which are given by $PARB(t) = \frac{\left|E(t) - \bar{Y}\right|}{\bar{Y}} X100$ and $RRMSE(t) = \frac{\sqrt{E\left(t - \bar{Y}\right)^2}}{\bar{Y}}$ respectively.

Table 6. PREs of different estimators with respect to t_1^{rss}

S. No.	Estimator	PRE	PARB	RRMSE
1.	t_1^{rss}	100.0000	15.4226	0.1639
2.	t_2^{rss}	104.7354	15.1835	0.1601
3.	t_3^{rss}	107.3891	14.8481	0.1581
4.	t_4^{rss}	107.7424	14.3672	0.1579
5.	t_5^{rss}	106.8996	14.4948	0.1585
Proposed Estimators				
6.	t_{p1}^{rss}	138.6124	12.2537	0.1392
7.	t_{p2}^{rss}	145.8852	12.1156	0.1357
8.	t_{p3}^{rss}	139.1164	12.2441	0.1389
9.	t_{p4}^{rss}	142.7957	12.1864	0.1371

The following figure shows the PRE of various estimators under consideration over t_1^{rss}.

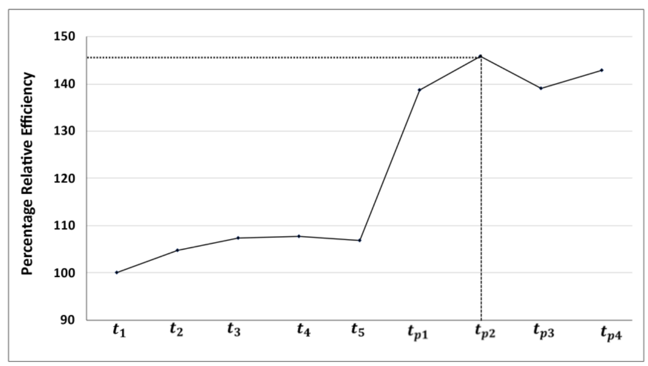

Fig. 2. PRE of different estimators over estimator t_1^{rss} for simulated data

6. CONCLUDING REMARKS

Over the years, the improved estimation of population mean has dominated the research scenario in sampling. Numerous estimators under different sampling schemes are suggested that are better than the existing competing estimators. Continuing the idea, the present paper suggests a new class of Searls type estimator for using auxiliary variable under RSS scheme. The optimum value of the Searls constant is obtained that leads to minimum MSE of the introduced class of estimators. The entries in Table 5 clearly indicate that the suggested family of estimators outperforms the existing competing estimators and the estimator t_{p2}^{rss} is the most efficient among the class of competing as well as the introduced family of estimators. The similar type of results have also been obtained for the simulated population and among the class of competing and suggested estimators, t_{p2}^{rss} is best as it has the highest PRE which may be verified from Table 6 and Figure 2. In view of the above, the suggested estimator t_{p2}^{rss} may be recommended for more efficient estimation of population mean under RSS in practice.

ACKNOWLEDGEMENTS

The authors express their deep sense of gratitude to the editor and the learned referee for their constructive comments and suggestions which immensely improved the earlier draft.

REFERENCES

Abbasi, A.M. and Shad, M.Y. (2017). Estimation of Population Mean and Median using Double Robust Truncation based Ranked Set Sampling, *Pakistan Journal of Statistics and Operation Research*, **13(2)**, 379-394.

Arnold, B.C., Balakrishnan N., Nagaraja, H.N. (1993). *A first course in order statistics*. Wiley, New York.

Balci, S., Akkaya, A.D., Ulgen, B.E. (2013). Modified Maximum Likelihood estimators using ranked set sampling,*Journal of Computational and Applied Mathematics*, **238**,171-179.

Biradar, B.S. and Santosha, C.D. (2015). Estimation of the Population Mean Based on Extremes Ranked Set Sampling, *American Journal of Mathematics and Statistics*, **5(1)**, 32-36.

Bhushan, S. and Kumar, A. (2021). Novel log type class of estimators under ranked set sampling, *Sankhya-B*, Doi.org/10.1007/s13571-021-00265-y

Cochran, W. G. (1977). *Sampling techniques*. Wiley, New York.

Dell, T.R. and Clutter, J.L. (1972). Ranked set sampling theory with order statistics background. *Biometrics*, **28**, 545-553.

Jemain A.A., Al-omari, A., Ibrahim, K. (2008). Some variations of Ranked set sampling, *Electronic Journal of Applied Statistical Analysis*, **1**, 1-15.

Kadilar, C., Unyazici, Y. and Cingi, H. (2009). Ratio estimator for the population mean using ranked set sampling, *Statistical Papers*, **50(2)**, 301-309.

Khan, L., and Shabbir, J. (2016). Improved ratio-type estimators of population mean in ranked set sampling using two concomitant variables, *Pakistan Journal of Statistics and Operation Research*, **12(3)**, 507-518.

Khan, L., Shabbir, J. and Khan, S.A. (2019). Efficient estimators of population mean in ranked set sampling scheme using two concomitant variables, *Journal of Statistics and Management Systems*, **22(8)**, 1467-1480.

Khan, Z., and Ismail, M. (2019). Ratio-type estimator of population mean based on ranked Set sampling. *Pakistan Journal of Statistics and Operations Research*, **15(2)**, 445-449.

Mclntyre, G.A. (1952). A method for unbiased selective sampling using ranked sets, *Australian Journal of Agricultural Research*, 3, 385-390.

Mandowara, V, L. and Mehta, N. (2012). A better estimator of population mean with power transformation based on ranked set sampling, *Statistics in Transition-New Series*, **13(3)**, 551-558.

Muttlak H.A. (2003). Investigating the use of quartile ranked set samples for estimating the population mean, *Journal of Applied Mathematics and Computation*, **146**, 437-443.

Pelle, E and Perri, P.F. (2018). Improving mean estimation in ranked set sampling using the Rao regression-type estimator, *Brazilian Journal of Probability and Statistics*, **32(3)**, 467-496.

Samawi, H.M. and Muttlak, H.A. (1996). Estimation of ratio using rank set sampling, *Biometrical Journal*, **38(6)**, 753-764.

Searls, D.T. (1964). The utilization of a known coefficient of variation in the estimation procedure, *Journal of American Statistical Association*, **59**, 1225-1226.

Stokes. S.L. (1977). Ranked set sampling with concomitant variables, *Communications in Statistics-Theory and Methods*, **6**, 1207-1211.

Takahasi, K. and Wakimoto, K. (1968). On unbiased estimates of the population mean based on the sample stratified by means of ordering, *Annals of the institute of statistical mathematics*, **20(1)**, 1-31.