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Almost Filtration of Bias Precipitates : A New Approach

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Summary

In the present investigation, a funnel associated with a filter paper to filter the bias precipitates appearing in the estimators of the ratio $R = \bar{Y}/\bar{X}$ and the product, $P = \bar{Y}\bar{X}$ have been proposed. The associated filter is the imposition of a very reasonable linear constraint to filter the bias precipitates. A numerical example is also given.

Key Words : Simple random sampling, Bias, Linear variety of estimators, Asymptotic mean square error, Finite population, Estimation of ratio and product.

Introduction and Notations

For a simple random sample (SRS) of size n drawn with replacement, from a population of size N, let $\bar{y}$ and $\bar{x}$ be the sample mean estimators of $\bar{Y}$ and $\bar{X}$, the population means of characters y and x respectively. The usual estimators $\hat{R} = \bar{y}/\bar{x}$ and $\hat{P} = \bar{y}\,\bar{x}$ for estimating the ratio $R = \bar{Y}/\bar{X}$ and product $P = \bar{Y}\bar{X}$ respectively, are well known in literature. Prasad [5] has increased the efficiency of estimators of R by certain scalar multiplications but his estimators remain biased. Singh and Sahoo [11] have proposed a general class of almost unbiased estimators for the ratio and product of population means of two characters and their class includes the estimators proposed by Beale [1], Robson [8], Tin [14] and Sahoo [9]. Singh [10] has also proposed an almost unbiased estimator of product P and compared it with the usual biased and adjusted unbiased estimators. Recently, Liu [4] has proposed an improved estimator of product P.

We shall propose here a funnel associated with a filter to remove the bias precipitates appearing in the estimators of ratio and product by making use of prior information on the population mean of the auxiliary variable. It can be seen that the reactants used for bias filtration depend only on the well known optimum choice of $K = \rho \frac{C_y}{C_x}$, as reported by Srivastava ([12], [13]) Chakarbarty [2], Vos [15], Walsh [16] and Reddy ([6], [7]).

For simplicity assume that population size N is quite large as compared to the sample size n, so that the finite population correction may be ignored throughout. Write

$$\delta_0 = \frac{\bar{y}}{\bar{Y}} - 1 \quad \text{and} \quad \delta = \frac{\bar{x}}{\bar{X}} - 1$$

so that $E(\delta_0) = E(\delta) = 0$

and $E(\delta_0^2) = n^{-1} C_y^2, \; E(\delta^2) = n^{-1} C_x^2, \; E(\delta_0 \delta) = n^{-1} \rho \, C_y C_x$

and $C_y^2 = \frac{S_y^2}{\bar{Y}^2}, \; C_x^2 = \frac{S_x^2}{\bar{X}^2}$ and $\rho = \frac{S_{xy}}{(S_x S_y)}$.

2. *Linear Variety of Estimators of R*

Suppose $\hat{R}_1 = \hat{R}, \; \hat{R}_2 = \hat{R}\frac{\bar{x}}{\bar{X}}$ and $\hat{R}_3 = \hat{R}\frac{\bar{X}}{\bar{x}}$ such that $\hat{R}_1, \hat{R}_2, \hat{R}_3 \in G$, where G denotes the set of all possible estimators for estimating the population ratio, R. By definition, the set G will be a linear variety if

$$\hat{R}_s = \sum_{i=1}^{3} \alpha_i \hat{R}_i \in G \qquad (2.0.1)$$

for $\sum_{i=1}^{3} \alpha_i = 1$ and $\alpha_i \in R_0$

Here α_i (i = 1, 2, 3) denote the amount of the reactants used for bias precipitates filtration and R_o stands for the set of real numbers.

2.1 *Mean Square Error*

Using (2.0.2), the relation (2.0.1) in terms of δ_0 and δ may be written as

$$\hat{R}_s = R + R\left[\delta_0 - (\alpha_1 + 2\alpha_3)\,\delta + O(\delta^2)\right] \qquad (2.1.1)$$

Let us choose

$$\alpha_1 + 2\alpha_3 = K_1 \text{ (say, another constant)} \qquad (2.1.2)$$

We, then, have

$$MSE(\hat{R}_s) = n^{-1} R^2\left[C_y^2 + K_1^2 C_x^2 - 2K_1 \rho \, C_y C_x\right] \qquad (2.1.3)$$

MSE (upto terms of order n^{-1}) at (2.1.3) is minimised for

$$K_1 = \rho \frac{C_y}{C_x} \tag{2.1.4}$$

and $$\text{Min. MSE}(\hat{R}_s) = n^{-1} R^2 C_y^2 (1 - \rho^2) \tag{2.1.5}$$

Relation (2.1.5) gives the minimum mean square error of the linear variety of estimators of R suggested at (2.0.1)

2.2 *Funnel for Estimators of R.*

From (2.0.2), (2.1.2) and (2.1.4), we have

$$\sum_{i=1}^{3} \alpha_i = 1 \tag{2.2.1}$$

$$\alpha_1 + 2\,\alpha_3 = \rho \frac{C_y}{C_x} \tag{2.2.2}$$

From (2.2.1) and (2.2.2), we have three unknowns to be determined from only two equations. It is, therefore, not possible to find out unique values for the amount of reactants α_i's (i = 1, 2, 3), we shall associate a filter with the funnel by imposing a linear constraint as

$$\sum_{i=1}^{3} \alpha_i\, B(\hat{R}_i) = 0 \tag{2.2.3}$$

where $B(\hat{R}_i)$ denotes the bias in the i^{th} (i = 1, 2, 3) estimator of population ratio. Now (2.2.1), (2.2.2) and (2.2.3) may be written as

$$\begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 1 \\ B(\hat{R}_1), & B(\hat{R}_2), & B(\hat{R}_3), \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} K_1 \\ 1 \\ 0 \end{bmatrix} \tag{2.2.4}$$

The value of α_i 's (i = 1, 2, 3) obtained by solving the equation (2.2.4) filters the bias precipitates from the proposed linear variety at (2.0.1).

2.3 *Bias Filtration of Order $o(n^{-1})$*

We outline the manner in which one can use the funnel associated with a filter to extract the bias precipitates of order $o(n^{-1})$ appearing in (2.0.1). For the case under consideration,

$$B(\hat{R}_1) = n^{-1}R\left[C_x^2 - \rho\, C_x C_y\right],\; B(\hat{R}_2) = 0,\; B(\hat{R}_3) = n^{-1}R\left[3C_x^2 - 2\rho\, C_x C_y\right]$$

Using (2.2.4), we get

$$\alpha_1 = \rho\frac{C_y}{C_x}\left(3 - 2\rho\frac{C_y}{C_x}\right) \tag{2.3.1}$$

$$\alpha_2 = \left(1 - \rho\frac{C_y}{C_x}\right)^2 \tag{2.3.2}$$

and

$$\alpha_3 = -\rho\frac{C_y}{C_x}\left(1 - \rho\frac{C_y}{C_x}\right) \tag{2.3.3}$$

Use of these α_i's (i=1,2,3) filter the bias up to terms of order $O(n^{-1})$. The same process may be repeated by considering $B(\hat{R}_i)$, (i=1, 2, 3) to the order $O(n^{-2})$ if the bias in $\hat{R}_s$ is to be reduced to the order $O(n^{-3})$ and so on. Also, the results of the paper could be extended in a straight forward way when the information on more than one auxiliary variable is available.

The proposed method can also be extended in straight forward manner, to the case of estimating the population product $P = \bar{Y}\bar{X}$.

2.4 *Numerical Example*

For the purpose of a numerical illustration, consider the population of size N=20, given by Horwitz and Thompson [3]. Here y denotes the number of households in a block, while x stands for the eye estimated number of households. Also, for this population, $\rho = 0.8662$, $C_y = 0.4264$ and $C_x = 0.38895$. Proceeding on the lines indicated, it is seen that

$$\alpha_1 = 1.04532, \quad \alpha_2 = 0.00254, \quad \alpha_3 = -0.04786.$$

Using these values of α_i's (i = 1, 2, 3), one can use the estimator at $\hat{R}_s$ (2.0.1). In practice, one can use the value of $\rho\frac{C_y}{C_x}$ from some earlier survey or pilot study in constructing the estimator $\hat{R}_s$.

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