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On Confounding in Factorial Experiments

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SUMMARY

A simple method is provided for identification of confounded interactions in 2^N (N = 2, 3, 4 . . .) factorial experiment. The existing method for the same is complicated particularly when large number of factors are taken into consideration and also when there is simultaneous confounding in the same layout. A few results regarding the simultaneous confounding in 2^N factorial experiment are also discussed.

Keywords : Confounding; Interactions; Factorial experiment; Non feasible combination of blocks; Principal block; Principal block group.

Introduction

The technique of confounding in factorial experiments has its own importance because of its usefulness. Usually we decide which effects are to be confounded and then write down the design. However there are situations when statisticians are presented with data from an experiment designed by some one else and they have to work out and identify which effects are confounded. A simple and a new procedure of identification of confounded interactions is provided as follows.

2. *The Procedure*

We consider the problem in two cases.

Case I : When there are only two blocks and only one interaction is confounded :

(*i*) If we call the block containing symbol (i) (i.e. all the treatments are at lower level) as the 'principal block', first identify the block which is not the principal block.

(*ii*) Collect the treatment labels which contain single letters from that non-principal block.

(*iii*) Combine such treatment labels multiplicatively. What we get now is the confounded interaction.

(*iv*) Change the small letters to capital letters to indicate effects as opposed to treatments.

Illustration 1. In 2^3 factorial experiment in the following layout block II is 'non-principal block' in which p and n are the treatment labels which contain single letters. Then np interaction is the confounded interaction.

Block I	np	npk	(1)	k
Block II	p	n	pk	nk

Case II : When there are more than two blocks and more than one interactions are confounded.

(i) Define a 'principal block group' as follows.

The principal block group is a combination of those blocks one of which must be the block containing the symbol (1) and some other block (or blocks) such that the total number of treatment combinations in the entire group must $\frac{2^N}{2}$.

The group of all remaining blocks is called as the 'non principal block group'.

(ii) Collect the treatment labels which contain single letters from the non principal block group.

(iii) Combine such treatment labels multiplicatively, what we get now is the confounded interaction.

(iv) Change the small letters to the capital letters to indicate effects as opposed to treatments.

(v) Each non principal block group should be examined to know the corresponding confounded interactions.

Illustration 2 : Following is the layout 2^5 factorial experiment in which there are 32 treatment combinations which are distributed in 4 blocks of size 8 each.

Block								
I	(1)	np	vns	vnr	vpr	sr	vps	npsr
II	vn	vp	s	nps	r	npr	vnsr	vpsr
III	n	p	vs	vnps	vr	vnpr	nsr	psr
IV	v	vnp	ns	ps	nr	pr	vsr	vnpsr

Consider block I together with block II as principal block group. Obviously block III together with block IV constitute non- principal block group. From non principal block group, the treatment labels which contain single letters are n, p, v. Combining them we get vnp as the confounded interaction. Next consider block I together with block III as principal block group. Hence from block II and IV we see that s, r, v are the treatment labels which contain single letters. Hence vsr is confound interaction. Finally block I together with block IV constitute the principal block group and hence from block II and III we see that npsr is confounded. It is noted that third effect is the generalised interaction of the other effects.

2. *Non feasible Combination of Blocks :*

Let in a 2^4 factorial experiment the treatment combinations be distributed in 8 blocks of size two each, as follows:

Block No.

1	2	3	4	5	6	7	8
(1)	a	b	ab	c	ac	bc	abc
d	ad	bd	abd	cd	acd	bcd	abcd

In this case there is one non-principal block group containing blocks 4, 6, 7 and 8 in which there are no treatment labels which contain single letters. Hence we can not conclude which interaction is confounded. We define such type of combinations of non-principal block groups as 'NON FEASIBLE' combinations.

Theorem : In case of simultaneous confounding in 2^N factorial experiment if each block is of size two only and the block containing symbol (1) also contains the treatment label which contain single letter then for new identification technique of the confounded interactions, the number of groups forming non feasible combinations of blocks must be greater than or equal to

$$\binom{2^{N-1}-N}{2^{N-2}-N} \qquad \text{when } N \geq 5$$

Proof : In case of 2^N factorial experiment if the block size is required to be two, then we have $\frac{2^N}{2}$ blocks in all. there are N number of treatment labels which contain single letter which can be distributed in N or less than N blocks. The non feasible combination is possible only when the principal block group contains all those blocks which contain treatment labels with single letter. Such principal block group is formed by atleast $\left(\frac{2^N}{4} - N\right)$ blocks which do not contain any treatment labels with single letter. Hence $\left(\frac{2^N}{4} - N\right)$ blocks can be selected from $\left(\frac{2^N}{2} - N\right)$ blocks in $\binom{2^N - 1 - N}{2^{N-2} - N}$ number of ways. Hence the theorem.

Particular Cases :

[A] For N = 2 there will be two blocks of size two and two treatment labels containing single letter. As such there will be no block possible without a treatment label containing single letter.

[B] For N = 3 there will be four blocks of size two and here also non feasible combination of blocks is not possible.

[C] For N = 4, as stated in section 2 there is only one non feasible combination of blocks possible.

Remarks :

(i) This new procedure of identification of confounded interactions is found to be applicable in all layouts of only 2^N factorial experiments (N = 2, 3, 4 . . .), where there is no double confounding.

(ii) In some situations we come across certain cases which are much more easily handled by mod devices. However, this new procedure which is viable with no knowledge of mod arithmetic is found to be easier than mod arithmetic in certain situations.

REFERENCES

[1] Federer, W.T., 1967. "Experimental designs theory and applications"; Oxford and IBH Publishing Co., N.Delhi.

[2] Montgomery, 1984. "Design and analysis of experiment" Second edition, John Wiley and Sons.