NOTE ON A PROPERTY OF THE COEFFICIENTS IN THE FORMULA FOR THE GENOTYPIC COVARIANCE BETWEEN PARENT AND OFFSPRING IN POLYPLOIDY

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UNDER random mating the genotypic covariance between parent and offspring is given by $\frac{1}{2}\sigma^2(A)$ for diploid organism where $\sigma^2(A)$ is the additive genetic variance. Likewise, for tetraploids the covariance is given by $\frac{1}{2}\sigma^2(A) + \frac{1}{6}\sigma^2(D)$ and for hexaploids by $\frac{1}{2}\sigma^2(A) + \frac{1}{5}\sigma^2(D) + \frac{1}{20}\sigma^2(T)$ as given by Kempthorne (1957) where $\sigma^2(D)$ and $\sigma^2(T)$ are digene and trigene variances respectively. An interesting property of the coefficients ($\frac{1}{2}$) for diploids; ($\frac{1}{2}$, $\frac{1}{6}$) for tetraploids; ($\frac{1}{2}$, $\frac{1}{5}$, $\frac{1}{20}$) for hexaploids, etc., is that their sums $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, etc., respectively, form the Faray series $r/(r+1)$. This property can be used as an alternate check upon the values of the coefficients thus obtained.

Kempthorne (1957) has given the generalised formula for the genotypic covariance between two individuals X and Y related in some way by the relation:—

$$\text{Cov.}(X, Y) = p_1(2r)\sigma^2(A) + p_2\frac{(2r)(2r-1)}{1.2}\sigma^2(D) + p_3\frac{(2r)(2r-1)(2r-2)}{1.2.3}\sigma^2(T) + \dots + p_k\frac{(2r)!}{(k)!(2r-k)!}\sigma^2 L(k) + \dots \quad (1)$$

where $\sigma^2 L(k)$ is the component of genotypic variance arising from the deviations common to all individuals which possess at least the same k genes, p_k the probability that a random set of k genes of X are identical by descent to a random set of k genes of Y and the individuals belong to $2r$-ploid class.

For parent-offspring covariances under the assumption that segregation is by chromosomes and not by chromatids any p-coefficient is given by

$$p_k = \frac{(2r-k)\,!^2\,(k)\,!\,(r)\,!}{(2r)\,!^2\,(r-k)\,!}. \tag{2}$$

The coefficient of $\sigma^2 L(k)$ for parent-offspring covariance is then given by

$$\phi(k) = \frac{(r)\,!\,(2r-k)\,!}{(2r)\,!\,(r-k)\,!}. \tag{3}$$

It can be easily shown that

$$\sum_{k=1}^{r} \phi(k) = \frac{r}{(r+1)} \tag{4}$$

which give the well-known Faray Series $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, etc., for different values of r.

The following table gives the coefficients $\phi(k)$ and their sum $\sum_{k=1}^{r} \phi(k)$ for some polyploids.

$2r$ \ k	1	2	3	4	5	$\sum_{k=1}^{r} \phi(k)$
2	1/2	..	..	..	..	1/2
4	1/2	1/6	..	..	..	2/3
6	1/2	1/5	1/20	..	..	3/4
8	1/2	3/14	1/14	1/10	..	4/5
10	1/2	2/9	1/12	1/42	1/252	5/6

Such simple relationship for other types of covariances as between full-sibs has not been found.

Summary

The coefficients of genotypic variances in the expression for the genotypic covariance between parent and offspring in a randomly breeding population are ($\frac{1}{2}$) for diploids, ($\frac{1}{2}$, $\frac{1}{6}$) for tetraploids and ($\frac{1}{2}$, $\frac{1}{5}$, $\frac{1}{20}$) for hexaploids. Their sums are $\frac{1}{2}$ for diploids, $\frac{2}{3}$ for tetraploids and $\frac{3}{4}$ for hexaploids. It can be shown that for the general $2r$-ploids the sum of the coefficients is $r/(r+1)$ the well-known Faray Series. This property provides an alternate check on the correctness of the individual coefficients.

Reference

Kempthorne, O. P. .. An Introduction to Genetic Statistics, New York (1957).