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Sampling From Two Dimensional Populations

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Summary

Estimation theory for sampling from two dimensional populations with various sampling procedures along space and time dimensions are considered. Both aligned and unaligned sampling for various situations are investigated. Among the sampling schemes considered along space dimension are simple random sampling and two stage sampling while systematic sampling is considered along the time dimension.

Key words : Two dimensional populations, two-stage sampling, systematic sampling.

Introduction

Populations spread in two dimension such as spread along space and time are commonly met in practice. In agricultural surveys wherever the ultimate sampling unit is observed repeatedly, the population may be considered as spread in space and time. Some common examples are milk yield, fruit and vegetable production, fish production etc. The problem of two dimensional populations have been studied earlier in different contexts. Some of the relevant references are Quenouille [7], Das [3], Sukhatme *et al.* [6] and Manwani and Singh [5]. Studies on two dimensional populations are available in the context of spatial distributions also. Bellhouse [2] considered sampling along two dimensions with random sampling along both dimensions and with random sampling along one and systematic sampling along the other dimension.

In the present study, sampling from two dimensional populations is viewed as varying probability sampling and various sampling methods with aligned or unaligned sampling along either of the dimensions are considered in a somewhat broader perspective.

2. *General Approach for Variance and its Estimator*

Consider a population of NM units spread in two dimensions with N rows and M columns. For selecting a sample of nm units, units may be selected at random from the entire population or alternatively n and m units may be selected along the rows and columns respectively. In this alternative, selection from either or both the dimensions may be aligned or unaligned.

Let y be the study character. The unit belonging to i-th row and j-th column will be referred as ij-th unit. Define

π_{ij} = Inclusion probability of ij-th unit,

$\pi_{ij,i'j'}$ = pairwise inclusion probability of (ij, i'j')-th unit,

π_i^* = inclusion probability of i-th row,

π_j^{**} = inclusion probability of j-th column,

$\pi_{i,i'}^*$ = inclusion probability of i and i'-th rows, and

$\pi_{j,j'}^{**}$ = inclusion probability of j and j'-th columns.

It is simple to find π_{ij}. For computation of pairwise inclusion probabilities in case of aligned sampling along either one or both the dimensions, consider the following situations.

Alignment in one dimension - Consider alignment along columns only. Then, the pairwise inclusion probabilities are

$$\underset{(j\neq j')}{\pi_{ij,ij'}} = \pi_i^{*2}\,\pi_{j,j'}^{**}\ ;\ \underset{(i\neq i')}{\pi_{ij,i'j}} = \pi_{i,i'}^{*}\,\pi_j^{**}\ ;$$

$$\underset{(i\neq i',\ j\neq j')}{\pi_{ij,i'j'}} = \pi_i^{*}\,\pi_{i'}^{*}\,\pi_{j,j'}^{**}$$

Similarly, in case of row-wise alignment

$$\underset{(j\neq j')}{\pi_{ij,ij'}} = \pi_i^{*}\,\pi_{j,j'}^{**}\ ;\ \underset{(i\neq i')}{\pi_{ij,i'j}} = \pi_{i,i'}^{*}\,\pi_j^{**2}\ ;$$

$$\underset{(i\neq i',\ j\neq j')}{\pi_{ij,i'j'}} = \pi_{i,i'}^{*}\,\pi_j^{**}\,\pi_{j'}^{**}$$

Alignment in both the dimensions - In this situation, the pairwise inclusion probabilities are

$$\underset{(j\neq j')}{\pi_{ij,ij'}} = \pi_i^{*}\,\pi_{j,j'}^{**}\ ;\ \underset{(i\neq i')}{\pi_{ij,i'j}} = \pi_{i,i'}^{*}\,\pi_j^{**}\ ;$$

$$\pi_{ij,i'j'} = \pi^{*}_{i,i'}\ \pi^{**}_{j,j'} \quad (i \neq i',\ j \neq j')$$

For samples selected under any one of the above situations, the Horvitz-Thompson estimator [4] of the population total

$$Y = \sum_{i=1}^{N} \sum_{j=1}^{M} y_{ij} \qquad \text{is of the form}$$

$$\hat{Y}_{HT} = \sum_{i=1}^{n} \sum_{j=1}^{m} y_{ij} / \pi_{ij} \tag{1}$$

The variance expression of $\hat{Y}_{HT}$ by Yates and Grundy [8] is

$$V_{YG}(\hat{Y}_{HT}) = 1/2 \sum_{i'}\sum_{j' \neq ij=1}^{NM} (\pi_{ij}\pi_{i'j'} - \pi_{ij,i'j'}) \left(\frac{y_{ij}}{\pi_{ij}} - \frac{Y_{i'j'}}{\pi_{i'j'}}\right)^2 \tag{2}$$

Broadly, there are three types of pairs of units involved in any situation irrespective of the sampling procedure adopted along any dimension

(a) $i = i'$, $j \neq j'$ — pair of units belonging to the same row,

(b) $i \neq i'$, $j = j'$ — pair of units belonging to the same column and

(c) $i \neq i'$, $j \neq j'$ — pair of units belonging to different rows and different columns.

Accordingly, the variance expression (2) takes the following form

$$V_{YG}(\hat{Y}_{HT}) = ½ \Big[\sum_{i=1}^{N} \sum\sum_{j \neq j'=1}^{M} (\pi_{ij}\pi_{ij'} - \pi_{ij,ij'}) \left(\frac{y_{ij}}{\pi_{ij}} - \frac{y_{ij'}}{\pi_{ij'}}\right)^2 +$$

$$\sum\sum_{i \neq i'=1}^{N} \sum_{j=1}^{M} (\pi_{ij}\pi_{i'j} - \pi_{ij,i'j}) \left(\frac{y_{ij}}{\pi_{ij}} - \frac{y_{i'j}}{\pi_{i'j}}\right)^2 +$$

$$\sum\sum_{i \neq i'=1}^{N} \sum\sum_{j \neq j'=1}^{M} (\pi_{ij}\pi_{i'j'} - \pi_{ij,i'j'}) \left(\frac{y_{ij}}{\pi_{ij}} - \frac{y_{i'j'}}{\pi_{i'j'}}\right)^2 \Big] \tag{3}$$

$$= \tfrac{1}{2}(Z_1 + Z_2 + Z_3) \tag{4}$$

where Z_1, Z_2 and Z_3 are the respective terms appearing in (3). Also, an unbiased estimator of (2) through Yates and Grundy [8] takes the form

$$\hat{V}(\hat{Y}_{HT}) = \tfrac{1}{2}\Big[\sum_{i=1}^{n}\sum_{j\neq j'=1}^{m}\left(\frac{\pi_{ij}\pi_{ij'} - \pi_{ij,ij'}}{\pi_{ij,ij'}}\right)\left(\frac{y_{ij}}{\pi_{ij}} - \frac{y_{ij'}}{\pi_{ij'}}\right)^2 +$$

$$\sum_{i\neq i'=1}^{n}\sum_{j=1}^{m}\left(\frac{\pi_{ij}\pi_{i'j} - \pi_{ij,i'j}}{\pi_{ij,i'j}}\right)\left(\frac{y_{ij}}{\pi_{ij}} - \frac{y_{i'j}}{\pi_{i'j}}\right)^2 +$$

$$\sum_{i\neq i'=1}^{n}\sum_{j\neq j'=1}^{m}\left(\frac{\pi_{ij}\pi_{i'j'} - \pi_{ij,i'j'}}{\pi_{ij,i'j'}}\right)\left(\frac{y_{ij}}{\pi_{ij}} - \frac{y_{i'j'}}{\pi_{i'j'}}\right)^2\Big]$$

$$= \tfrac{1}{2}(Z_1' + Z_2' + Z_3') \tag{5}$$

where Z_1', Z_2' and Z_3' are the respective terms on the R.H.S. of (5).

3. *Some Specific Sampling Procedures*

For populations spread in space and time, it is convenient to observe the selected space units at systematically selected time units. Of several alternative choice of sampling designs over space and time, on practical considerations, we consider following two cases for the choice of space units while systematic sampling is assumed to be followed over time in both the cases:

(a) Equal probability sampling

(b) Two stage random sampling (unequal primary sampling units (psu's)).

3.1 *Equal probability sampling along the space dimension and systematic sampling along the time dimension*

Let N rows of the two dimensional population stand for N space units and M columns denote the M time units. Consider selection of n space units by simple random sampling without replacement (SRSWOR) and m time units by linear systematic sampling along the time dimension with sampling interval k (assuming $M = mk$). The use of SRSWOR along space dimension is, though, oversimplification but it may be taken as a prelude to more general design to be considered subsequently. Denoting this sampling scheme as $r_i\, sy_j$; $i, j = 0, 1$ where the notations r and sy stand for SRSWOR and systematic sampling along space and the time dimensions respectively and the suffixes O and 1 denote unaligned and aligned nature of selection. Consider the following situations :

a) $r_o\, sy_1$ — alignment along time dimension only
b) $r_1\, sy_o$ — alignment along space dimension only
c) $r_1\, sy_1$ — alignment along both the dimensions.

The first two cases are straight way two stage sampling procedures with time units and space units as psu's respectively. Define the following notations.

$$M = mk \text{ and } j = uv \ (u = 1, 2, \ldots, k\,;\ v = 1, 2, \ldots, m)$$

$Y_{ij} = y_{iuv}$ = y value for the unit belonging to i-th ($i = 1, 2, \ldots, N$) row and j-th ($j = 1, 2, \ldots, M$) column = y-value for the unit corresponding to i-th space unit, v-th time unit ($v = 1, 2, k$) in the u-th systematic sample ($u = 1, 2, \ldots, m$).

$$\overline{Y}_{i.} = M^{-1} \sum_{j}^{M} Y_{ij} \qquad \overline{Y}_{.j} = N^{-1} \sum_{i}^{N} Y_{ij}$$

$$\overline{Y}_{..} = (NM)^{-1} \sum_{i}^{N} \sum_{j}^{M} Y_{ij} \qquad S^2 = (NM-1)^{-1} \sum_{i}^{N} \sum_{j}^{M} (Y_{ij} - \overline{Y}_{..})^2$$

$$S^2_{br} = (N-1)^{-1} \sum_{i}^{N} (Y_{i.} - \overline{Y}_{..})^2$$

$$S^2_{ir} = (M-1)^{-1} \sum_{j}^{M} (Y_{ij} - \overline{Y}_{i.})^2 \ ; \ S^2_{wr} = (N)^{-1} \sum_{i}^{N} S^2_{ir.}$$

$$S^2_{bc} = (M-1)^{-1} \sum_{j}^{M} (\overline{Y}_{.j} - \overline{Y}_{..})^2$$

$$S^2_{jc} = (N-1)^{-1} \sum_{i}^{N} (Y_{ij} - \overline{Y}_{.j})^2 \ ; \ S^2_{wc} = (M)^{-1} \sum_{i}^{M} S^2_{jc}$$

$$\overline{y}_{iu.} = m^{-1} \sum_{v}^{m} y_{iuv} \ , \ \overline{y}_{.uv} = n^{-1} \sum_{i}^{n} y_{iuv} \ ; \ \overline{y}_{.u.} = m^{-1} \sum_{v}^{m} y_{.uv}$$

$$S^2_{syi} = (k-1)^{-1} \sum_{u}^{k} (\overline{y}_{iu.} - \overline{Y}_{i.})^2 \ ; \ \overline{S}^2_{sy} = N^{-1} \sum_{i}^{N} S^2_{syi}$$

$$S_{sy}^2 = (k-1)^{-1} \sum_{u}^{k} (\bar{y}_{.u.} - \bar{Y}_{..})^2$$

$$S_{iu}^2 = (m-1)^{-1} \sum_{v}^{m} (y_{iuv} - \bar{y}_{iu.})^2$$

The Horvitz-Thompson estimator for population total is

$$\hat{Y}_{HT} = NM\,(nm)^{-1} \sum_{i}^{n} \sum_{j}^{m} y_{ij}$$

The variance expressions corresponding to the selection procedures $r_O sy_1$ and $r_1 sy_O$ are

$$V_{r_0 sy_1} = N^2 M^2 \left[\frac{(k-1)}{k} S_{sy}^2 + \frac{1}{m}\left(\frac{1}{n} - \frac{1}{N}\right) \overline{S}_{wc}^2 \right] \quad (6)$$

and

$$V_{r_1 sy_0} = N^2 M^2 \left[\frac{(k-1)}{k} \frac{\overline{S}_{sy}^2}{n} + \left(\frac{1}{n} - \frac{1}{N}\right) S_{br}^2 \right] \quad (7)$$

Under the selection procedure r_1sy_1, it is not possible to get n unbiased estimator of variance, therefore, the problem has been approached by considering t sub-samples along the time dimension by r_1sy_1 procedure in the following manner.

A sample of nmt units is selected in the form of t samples each of size nm. For this purpose, selection of mt time units along the time dimension is done by selecting t systematic samples each of size m corresponding to t random starts selected between 1 to k by SRSWOR while selection of n units along the space dimension is done by SRSWOR. The nmt units corresponding to the points of intersection constitute the sample. The inclusion probability for y-th unit is $\pi_{ij} = nmt\,(NM)^{-1}$. the pairwise inclusion probabilities are

$$\pi_{ij,ij'} = nmt/NM, \quad (j \neq j') \quad \text{when j and j}'\text{-th units belong to the same systematic sample in i-th row,}$$

$$= nm^2\, t\,(t-1)/NM\,(M-m) \quad \text{otherwise}$$

$$\pi_{ij,i'j} = nmt\,(n-1)/(NM\,(n-1)) \quad (i \neq i')$$

$$\pi_{ij,i'j'} \underset{(i \neq i',\, j \neq j')}{} = nmt\,(n-1)/(NM\,(N-1))$$ when j and j′-th units belong to the same systematic sample in i-th and i′-th rows,

$$= nm^2\, t\,(n-1)/(NM\,(M-m)\,(N-1))$$ otherwise

and $V_{r_1 syt_1}$ may be derived using (4) to be of the form

$$V_{r_1 syt_1} = N^2M^2\left\{\left(\frac{1}{n}-\frac{1}{N}\right) S_{br}^2 + \frac{(k-t)\,N}{kt(N-1)}\left[\left(\frac{1}{n}-\frac{1}{N}\right)\overline{S}_{sy}^2 + \frac{n-1}{n}\, S_{sy}^2\right]\right\} \qquad (8)$$

For $t = 1$, $V_{r_1 syt_1}$ corresponds to $V_{r_1 sy_1}$. This case was considered by Ahuja and Srivastava [1]. When n = N, the sampling procedure reduces to systematic sampling over time in uni-dimensional populations with each unit being a cluster of N units. Similarly, when m = M = 1, there is sampling over space only, the selection procedure reduces to SRSWOR. The corresponding variance formulae follow from (8). Alternatively, $V_{r_1 sy_1}$ may be expressed as

$$V_{r_1 sy_1} = V_{r_0 sy_1} + EFa$$

where
$$EFa = N^2M^2\left\{\left[\left(\frac{1}{n}-\frac{1}{N}\right)\left(S_{br}^2 - \frac{\overline{S}_{wc}^2}{m}\right)\right] + \frac{k-1}{k}\left(\frac{1}{n}-\frac{1}{N}\right)\left[\frac{N}{N-1}\left(\overline{S}_{sy}^2 - S_{sy}^2\right) - \frac{\overline{S}_{wc}^2}{m}\right]\right\} \qquad (9)$$

The alignment effect (9) is the sum of two components, the first component arising due to random aligned sampling and the second one due to systematic sampling. It may easily be seen that when units within each of the columns are arranged randomly and independently, the alignment effect reduces to zero.

An unbiased estimator of $V_{r_1 syt_1}$ through (5) is

$$\hat{V}_{r_1 syt_1} = N^2M^2\left\{\left(\frac{1}{n}-\frac{1}{N}\right) s_{br}^2 + \frac{(k-t)\,n}{kt(n-1)}\left[\left(\frac{1}{n}-\frac{1}{N}\right)\overline{s}_{sy}^2 + \frac{N-1}{N}\, s_{sy}^2\right]\right\} \qquad (10)$$

where s_{br}^2, $\overline{s}_{sy}^2$ and s_{sy}^2 represent corresponding sample mean squares with i, j, u and v being now varying as $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, mt$; $u = 1, 2, \ldots, t$; and $v = 1, 2, \ldots, m$.

3.2 *Two stage random sampling along space and systematic sampling along time dimension*

Consider a population of N psu's with P_i ssu's in the i-th (i = 1, 2, . . . , N) psu such that

$$\sum_{i}^{N} P_i = P_0\,,\ \bar{P} = P_0 N^{-1} \text{ and } w_i = P_i\,(\bar{P})^{-1}$$

Let y - values for ssu's be spread over M time units and Y_{ihj} (i = 1, 2, . . . , N ; h = 1, 2, . . . , P_i , j = 1, 2, . . . , M) be the value for i h-th ssu on the j-th time unit.

Define $Y = \sum_{i}^{N} \sum_{h}^{P_i} \sum_{j}^{M} Y_{ihj}$, $\qquad \bar{Y}_{i.j} = P_i^{-1} \sum_{h}^{P_i} y_{ihj}$,

$$\bar{Y}_{ih.} = M^{-1} \sum_{j}^{M} y_{ihj}\,, \qquad \bar{Y}_{i..} = P_i^{-1} \sum_{h}^{P_i} \bar{Y}_{ih.}\,,$$

$$\bar{Y}_{..j} = N^{-1} \sum_{i}^{N} w_i\, \bar{Y}_{i.j}\,, \qquad \bar{Y} = N^{-1} \sum_{i}^{N} w_i\, Y_{i..} = M^{-1} \sum_{j}^{M} \bar{Y}_{..j}$$

Consider the following situations, where t refers to two stage sampling and sy refers to systematic sampling.

a) $t_1\, sy_0$ — alignment along space dimension and

b) $t_1\, sy_1$ — alignment along both dimensions.

Selection of $\sum_{i}^{n} p_i$ (p_i being the number of ssu's selected from i-th psu) units by two-stage random sampling along the space dimensions and m units along the time dimension is done.

The selection procedure (a) is straightway three stage sampling with systematic sampling along the third sampling stage. Similar to section 3.1, let j-th time unit be denoted by uv where v stands for v-th time unit in the u-th systematic sample. Further, defining :

$$y_{ihj} = y_{ihuv} \qquad ; \qquad \bar{y}_{ihu} = m^{-1} \sum_{v}^{m} y_{ihuv}$$

$$\bar{y}_{ih..} = M^{-1} \sum_{u}^{k} \sum_{v}^{m} y_{ihuv} \quad ; \quad \bar{y}_{i.u.} = m^{-1} P_i^{-1} \sum_{h}^{P_i} \sum_{v}^{m} y_{ihuv}$$

$$\bar{y}_{..u.} = N^{-1} \sum_{i}^{N} w_i \bar{y}_{i.u.} \quad \bar{Y}_{i...} = k^{-1} \sum_{u}^{k} \bar{y}_{i.u} = P_i^{-1} \sum_{h}^{P_i} \bar{y}_{ih..}$$

$$\bar{Y}_{....} = M^{-1} \left(\sum_{i}^{N} P_i \right)^{-1} \sum_{i}^{N} \sum_{h}^{P_i} \sum_{u}^{k} \sum_{v}^{m} y_{ihuv}$$

$$S_{br}'^2 = (N-1)^{-1} \sum_{i}^{N} (\bar{Y}_{i...} - \bar{Y}_{...})^2$$

$$S_{i(P_i)}'^2 = \frac{1}{P_i - 1} \sum_{h=1}^{P_i} (Y_{ih..} - Y_{i...})^2$$

$$\bar{S}_{sy}'^2 = (N)^{-1} \sum_{i}^{N} S_{syi}'^2 \quad ; \quad S_{syi}'^2 = (k-1)^{-1} \sum_{u}^{k} (\bar{y}_{i.u} - \bar{Y}_{i...})^2$$

$$S_{syih}'^2 = (k-1)^{-1} \sum_{u}^{k} (\bar{y}_{ihu.} - \bar{y}_{ih..})^2 \quad S_{sy}'^2 = (k-1)^{-1} \sum_{u}^{k} (\bar{y}_{..u.} - \bar{Y}_{....})^2$$

An unbiased estimator of the population total is

$$\hat{Y}_{t_1 sy_0} = N n^{-1} \sum^{n} P_i P_i^{-1} \sum_{h}^{P_i} M m^{-1} \sum_{v}^{m} y_{ihv}$$

Variance of this estimator is

$$V_{t_1 sy_0} = M^2 P_0^2 \left(\frac{1}{n} - \frac{1}{N} \right) S_{br}'^2 + \frac{1}{nN} \sum_{i}^{N} w_i^2 \left(\frac{1}{p_i} - \frac{1}{P_i} \right) S_{i(P_i)}'^2$$

$$+ \frac{1}{nN} \frac{k-1}{k} \sum_{i}^{N} \frac{w_i^2}{P_i p_i} \sum_{h}^{P_i} S_{syih}'^2 \qquad (11)$$

In order to get an unbiased estimator of variance under the selection procedure t_1sy_1, the problem has been dealt by selecting t systematic samples each of size m corresponding to t random starts selected between 1 to k by SRSWOR. The sampling along the space dimension is two stage random sampling. The unbiased estimator of population total with $\pi_{ihj} = nmtp_i\,(NMP_i)^{-1}$ is

$$\hat{Y}_{t_1syt_1} = Nn^{-1}\sum_{i=1}^{n} P_i\,p_i^{-1}\sum_{h=1}^{P_i} Mt^{-1}m^{-1}\sum_{j=1}^{m} y_{ihj}$$

The pairwise inclusion probabilities are

$$\pi_{\substack{ihj,\,ihj'\\(j\neq j')}} = nmtp_i\,/NMP_i \qquad \text{for systematic samples}$$

$$= nm^2t(t-1)p_i\,/\,NM(M-m)\,P_i \qquad \text{otherwise}$$

$$\pi_{\substack{ihj,\,ih'j\\(h\neq h')}} = nmtp_i\,(p_i-1)\,/NMP_i\,(P_i-1) \quad (h,h'=1,2,\ldots,P_i)$$

$$\pi_{\substack{ihj,\,i'h'j\\(i\neq i')}} = nmt(n-1)p_ip_i'\,/NM\,(N-1)\,P_i\,P_i' \quad \begin{bmatrix} h=1,2,\ldots,P_i \\ h'=1,2,\ldots,P_i' \end{bmatrix}$$

$$\pi_{\substack{ihj,\,ih'j'\\(h\neq h',\,j\neq j')}} = nmtp_i\,(p_i-1)\,/NMP_i\,(P_i-1) \qquad \text{for systematic samples}$$

$$= nmt^2t(t-1)p_i\,(p_i-1)\,/NM(M-m)P_i\,(P_i-1) \qquad \text{otherwise}$$

$$\pi_{\substack{ihj,\,i'h'j'\\(i\neq i',\,j\neq j')}} = n(n-1)\,p_ip_i'\,mt/N(N-1)\,P_iP_i'\,M \qquad \text{for systematic samples}$$

$$= n(n-1)\,p_ip_i'\;m^2t(t-1)/N(N-1)\,P_iP_i'\,M(M-m) \qquad \text{otherwise}$$

Variance expression for $\hat{Y}_1syt_1$, though involving complicated algebra, is omitted and the final expression through Horvitz-Thompson approach (1952) is

$$V_{t_1syt_1} = M^2P_0^2\left\{\left(\frac{1}{n}-\frac{1}{N}\right)S'^2_{br}+\frac{N(k-t)}{kt(N-1)}\left[\left(\frac{1}{n}-\frac{1}{N}\right)S'^2_{sy}+\frac{n-1}{n}\,S'^2_{sy.}\right]\right\}$$

$$+\,M^2P_0^2\left\{\frac{1}{nN}\sum_{i=1} w_i^2\left(\frac{1}{p_i}-\frac{1}{P_i}\right)S'^2_{i(P_i)}+\frac{k-t}{ktNn}\sum_i\sum_h w_i^2\left(\frac{1}{p_i}-\frac{1}{P_i}\right)\frac{1}{P_i-1}\,S^2_{syih}\right\}$$

$$-\,M^2P_0^2\left\{\frac{k-t}{ktNn}\sum_i w_i^2\left(\frac{1}{p_i}-\frac{1}{P_i}\right)\frac{P_i}{P_i-1}\,S'^2_{syi}\right\} \qquad (12)$$

The expression (12) consists of two components -(i) the first arising due to the adoption of random sampling (aligned) along the space and systematic sampling (aligned) along the time dimensions, and (ii) the second one due to sub-sampling of psu's with aligned selection along the time dimension. In case of equal sizes of psu's (say, P) and an equal number of ssu's (say,p) selected from each of the sampled psu's the sampling reduces to t_1syt_1 with equal psu's. Further, for m = M, the selection reduces to two stage random sampling with each ssu being a cluster of M elements. In case each first stage unit is sampled i.e., n = N, t_1syt_1 reduces to sampling from two dimensional population with stratified random sampling (aligned selection) along space and systematic sampling (aligned selection) along the time dimensions. The corresponding variance expression follows directly from expression (12).

Estimator of $V_{t_1syt_1}$, through Harvitz-Thompson procedure simplifies to

$$V_{t_1syt_1} = M^2P_0^2\left\{\left(\frac{1}{n}-\frac{1}{N}\right)s'^2_{br}+\frac{n(k-t)}{kt(n-1)}\left[\left(\frac{1}{n}-\frac{1}{N}\right)\bar{s}'^2_{sy}+\frac{N-1}{N}s'^2_{sy}\right]\right\}$$

$$+M^2P_0^2\left\{\frac{1}{nN}\sum_i w_i^2\left(\frac{1}{p_i}-\frac{1}{P_i}\right)s'^2_{i(p_i)}+\frac{k-t}{ktNn}\sum_i\sum_h w_i^2\left(\frac{1}{p_i}-\frac{1}{P_i}\right)\frac{1}{p_i-1}s^2_{syih}\right\}$$

$$+M^2P_0^2\left\{\frac{k-t}{ktNn}\sum_i w_i^2\left(\frac{1}{p_i}-\frac{1}{P_i}\right)\frac{p_i}{p_i-1}s'^2_{syi}\right\} \tag{13}$$

where s'^2_{br}, $\bar{s}'^2_{sy}$, s'^2_{sy}, $s'^2_{i(P_i)}$, s'^2_{syi} and s^2_{syih} denote the sample mean squares with i, h, u, v varying as (i = 1, 2, . . . , n ; h = 1, 2, . . . , p_i ; u = 1, 2, . . . , t ; v=1, 2, . . . , m).

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