*Jour. Ind. Soc. Ag. Statistics*
*46(3), 1994 : 315–317*

# A New Type of Slope Rotatable Central Composite Design

B. Re. Victorbabu and V.L. Narasimham
*Nagarjuna University, Guntur-522 510*
(Received : September, 1993)

SUMMARY

A new method of construction of slope rotatable central composite designs (SRCCDs) is introduced.

*Keywords* : Response surface designs; Slope rotatability; Slope rotatable central composite designs.

## *Introduction*

Hader and Park [3] introduced slope rotatable central composite designs (SRCCDs). Victorbabu and Narasimham [4], [5], [6] studied in detail the conditions to be satisfied by a general second order slope rotatable design (SOSRD) and also constructed SOSRDs using BIB design. For definitions and notations refer to [3] and [4]. It is clear from slope rotatability condition (in page 2471 of [41] that the solution for design levels (like a,b, etc.,) depend on c and $n_0$. In contrast in rotatable design the design levels a, b, etc., are same for any $n_o$ (c = 3)and any number of central points can be added to the second order rotatable design (SORD) without changing the values non-zero design levels. In SOSRD depending on the number of central points desired, we have to choose the non- zero design levels a,b, etc., suitably. This interdependence of the parameters $n_0$, c and design levels is utilised to evolve a new type of construction of slope rotatable central composite designs.

## 2. *A New Type of Slope Rotatable Central Composite Design*

The most widely used design for fitting a second order model is the central composite design. Central composite designs are constructed by adding suitable factorial combinations to those obtained from $\frac{1}{2^p} \times 2^v$ fractional factorial design (here $2^{t(v)} = \frac{1}{2^p} \times 2^v$) denotes a suitable fractional replicate of $2^v$, in which no interaction with less than five factors is confounded. In coded form the points of $2^v$ $(2^{t(v)})$ factorial have coordinates $(\pm 1, \pm 1, \ldots, \pm 1)$ and 2v axial (star) points have coordinates of the form $(\pm a, 0, \ldots, 0)$, $(0, \pm a, \ldots, 0)$, . . . , $(0, 0, \ldots, \pm a)$ etc., and if necessary $n_0$ central points may be replicated sometimes. The method of construction of a new type of SRCCD is given in the following theorem (2.1).

*Theorem 2.1* : A central composite design will be a v-dimensional SRCCD with c (pre-fixed) in $N = 2^{t(v)} + 2v + n_0$ design points if,

$$a^2 = [(c-1)2^{t(v)-1}]^{1/2} \text{ and} \tag{2.1}$$

$$n_0 = \frac{\{2^{t(v)} + 2a^2\}^2 [v(c-5)+4]}{2^{t(v)}[v(c-5)+(c-3)^2]} - [2^{t(v)} + 2v] \tag{2.2}$$

and $n_0$ turns out to be an integer.

*Proof* : Follows by verifying the conditions to be satisfied by a SOSRD as given in [3] and [4].

*Example* : We illustrate the method by constructing a new type of SRCCD with c=5 for 3-factors in N=32 design points. We have,

$$(2)\ \text{(i)}\ \Sigma x_{iu}^2 = 8 + 2a^2 = N\lambda_2,\ \text{(ii)}\ \Sigma x_{iu}^4 = 8 + 2a^4 = 5N\lambda_4$$

$$(3)\ \Sigma x_{iu}^2 x_{ju}^2 = 8 = N\lambda_4 \tag{2.3}$$

(2) (ii) and (3) of (2.3) lead to a=2 and (2.2) gives $n_0 = 18$.

If v is such that t(v) is odd, then positive solution for exists and hence a new type of positive solution for $n_0$ exists and hence a new type of SRCCD exists for such v with c=5. A list of a new type of SRCCDs for v=3,6,9,10 and 11 ($v \le 17$) constructed by above method with c=5 are given below.

| v | a | $n_0$ | N |
|---|---|---|---|
| 3 | 2.000 | 18 | 32 |
| 6 | 2.8284 | 28 | 72 |
| 9 | 4.0000 | 54 | 200 |
| 10 | 4.0000 | 52 | 200 |
| 11 | 4.0000 | 50 | 200 |

From (2.2), we note that integral solution for $n_0$ does note exist for $v = 2, 4, 5, 7, 8, 13, 14, 15, 16, 17$ ($v \le 17$) with c=5. In these cases we take $[n_0]$ or $[n_0] + 1$, D ($[n_0]$ denotes Gauss symbol in (2.2)) central points and construct nearly SRCCD.

It can be observed that a necessary condition for the existence of a positive integral solution for $n_0$ is $c \ge 5$. As such one can construct above type of SRCCD (if they exist) with any other value of $c \ge 5$, for example c = 6, 7, 7.5, 8 etc.

We have also observed by taking higher value of c, we get designs with lesser number of design points.

## ACKNOWLEDGEMENTS

The authors are thankful to the referee and the editor for their constructive suggestions which have very much improved the earlier version of this paper. The southers are thankful to the authorities of Nagarjuna University for providing the necessary facilities to carry this work.

## REFERENCES

[1] Box, G.E.P. and Hunter J.S., 1975. Multifactor experimental designs for exploring response surfaces, *Ann Math. Satists.* **2**, 195-241.

[2] Das, M.N. and Narasimham, V.L., 1962. Construction of rotatable designs through balanced incomplete block designs, *Ann Math. Stat.*, **33**, 1421-1439.

[3] Hader, R.J. and Park, S.H., 1978. Slope Rotatable central composite designs, *Technometrics*, **20**, 413-417.

[4] Victorbabu, B.Re. and Narasimham, V.L., 1991a. Construction of second order slope rotatable designs through balanced incomplete block designs, *Commn. Statist., Theory and Methods*, **20**, 2467-2478.

[5] Victorababu, B.Re. and Narasimham, V.L., 1991b. Construction of second order slope rotatable designs through a pair of balanced incomplete block designs, *J. Ind. Soc. Agric. Statist.*, **43(3)**, 291-295.

[6] Victorababu, B.Re. and Narasimham, V.L., 1993. Construction of three level second order slope rotatable designs using balanced incomplete block designs, *Pak. J. Statist*, **9 B**, 91-95.