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Some New Classes of PBIB Designs of Rectangular Type

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SUMMARY

Goswami, et al. [4] constructed rectangular designs through Q_2 arrays. The method developed in this communication provides substantial reduction in the number of replications per treatment in the series obtained.

Keywords : Partially balanced incomplete block design; Rectangular design.

Introduction

Vartak [8] introduced a three associate PBIB design, also called rectangular design (RD). The RD is defined as follows :

Let there be v = mn symbols arranged in a rectangle of m rows and n columns. With respect to each symbols, the first associates are the other (n – 1) symbols of the same row, the second associates are the other (m – 1) symbols of the same column and the remaining (m – 1) (n – 1) symbols are third associates. For this association scheme, $n_1 = n - 1, n_2 = m - 1$, and $n_3 = (n-1)(m-1)$.

A Rectangular Design (RD) is a Partially Balanced Incomplete Block Design (PBIB) design $(mn, b, r, k, \lambda_1, \lambda_2, \lambda_3)$ the symbols having usual significance.

The existing literature reveals that different classes of rectangular designs have been obtained in different communications viz. Raghavarao and Aggarwal [7], Aggarwal and Singh [1], Kageyama and Tanaka [5], Kegeyama and Mohan [6], Bhagwandas, et al. [2], Gill [3] and Goswami, et al [4].

In this communication, a method of construction of a class of Rectangular Designs has been developed which reduces the number of replications per treatment in the series of RD $[v = ms, b = s(s-1), \ r = d(s-1), \ k = (s-1), \lambda_1 = d(d-1), \lambda_2 = 0, \lambda_3 = d^2]$ where $md = s - 1$, obtained by Goswami, et, al. [4].

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2. *Method of Construction*

Theorem 2.1 : If md = (s − 1) where m and d are both positive integers greater than (< 1) and s is a prime or prime power then a RD [$v = ps, b = ms, r = (s-1), k = pd, \lambda_1 = d-1, \lambda_2 = 0, \lambda_3 = d$] can always be constructed, where $1 < p \leq m$.

Proof (by construction) : Let x be a primitive root of GF(s) where md = (s − 1).

From a block of size d with the elements $(x^0, x^m, x^{2m}, \ldots, x^{(d-1)m})$ and let it be denoted by the row vector α_1.

Next, take any element y_1 not in α_1 and form a second block $\alpha_2 = y_1(\alpha_1)$ where $y_1(\alpha_1)$ means multiplication of each element of α_1 by y_1.

Again, take another element y_2 not in α_1 and α_2 and obtain the third block $\alpha_3 = y_2(\alpha_1)$. Likewise m blocks $\alpha_1, \alpha_2, \ldots, \alpha_m$ can be obtained and at this stage all the non-zero elements are exhausted.

Arrange these block vectors to get the following matrix of order m × dp where $p \leq m$

$$\underset{(mxdp)}{A} = \begin{bmatrix} \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_p \\ \alpha_2, \alpha_3, \alpha_4, \ldots, \alpha_{p+1} \\ \cdot \\ \cdot \\ \cdot \\ \alpha_m, \alpha_1, \alpha_2, \ldots, \alpha_{p-1} \end{bmatrix} \quad \text{(No transpose)}$$

Actually the 1st column of A can be treated as m initial blocks each of size d.

By developing these blocks a BIB designs is obtained.

This is true for each vector column of A. Each of the rows of A of size dp is treated as an initial block and developed mod s each set of s blocks following the rows (block) of its previous one. This gives a matrix of order ms × pd where the blocks are taken as the row vectors.

Actually PBIB designs ate juxtaposed in the matrix and no elements is repeated in any row.

To each element in position from $d \times i + 1$ to $d \times (i + 1)$ in each row of the above matrix add s x i to get the design ($i = 0, 1, 2, \ldots, p - 1$).

The above method provides a Rectangular Design with following parametric sets

$v = ps$, $b = ms$, $r = (s - 1)$, $k = pd$, $\lambda_1 = d - 1$, $\lambda_2 = 0$, $\lambda_3 = d$, $n_1 = (s - 1)$, $n_2 = (p - 1)$ and $n_3 = (s - 1)(p - 1)$.

Illustration 2.1 : When $s = 7, m = 3$ and $d = 2$ and primitive root of 7 is 3.

$$\alpha_1 = (3^0, 3^3) \text{ i.e. } (1, 6)$$
$$\alpha_2 = (3^1, 3^4) \text{ i.e. } (2, 5)$$
$$\alpha_2 = (3^2, 3^5) \text{ i.e. } (3, 4)$$

$$A = \begin{bmatrix} \alpha_1 \, \alpha_2 \, \alpha_3 \\ \alpha_2 \, \alpha_3 \, \alpha_1 \\ \alpha_3 \, \alpha_1 \, \alpha_2 \end{bmatrix} \text{ when } p = 3$$

$$A = \begin{bmatrix} 16 & 25 & 34 \\ 25 & 34 & 16 \\ 34 & 16 & 25 \end{bmatrix}$$

If we take $p = 2$, then the 3, initial blocks to be developed mod 7 are the following 3 rows

Ist row 1, 6, 2, 6
2nd row 2, 5, 3, 4
3rd row 3, 4, 1, 6

By developing the initial block mod 7 we get the following:

From the first block	From the second block	From the third block
1, 6, 2, 5	2, 5, 3, 4	3, 4, 1, 6
2, 0, 3, 6	3, 6, 4, 5	4, 5, 2, 0
3, 1, 4, 0	4, 0, 5, 6	5, 6, 3, 1
4, 2, 5, 1	5, 1, 6, 0	6, 0, 4, 2
5, 3, 6, 2	6, 2, 0, 1	0, 1, 5, 3
6, 4, 0, 3	0, 3, 1, 2	1, 2, 6, 4
0, 5, 1, 4	1, 4, 2, 3	2, 3, 0, 5

The number $s \times i$ is to be added to elements in column $d \times i + 1$ to $d \times (i + 1)$ where $i = 0, 1, 2, \ldots, p - 1$. So, $s(i) = o$ is to added to each elements in column 1 to 2 that is, these columns remain the same. Again $s(i) = 7$

for i = 1 is to be added to each element in column 3 and 4. After these operations the design becomes.

1, 6, 9, 12	2, 5, 10, 11	3, 4, 8, 13
2, 0, 10, 13	3, 6, 11, 12	4, 5, 9, 7
3, 1, 11, 7	4, 0, 12, 13	5, 6, 10, 8
4, 2, 12, 8	5, 1, 13, 7	6, 0, 11, 9
5, 3, 13, 9	6, 2, 7, 8	0, 1, 12, 10
6, 4, 7, 10	0, 3, 8, 9	1, 2, 13, 11
0, 5, 8, 11	1, 4, 9, 10	2, 3, 7, 12

This design has parameters

$$v = 14, b = 21, r = 6, k = 4, \lambda_1 = 1\ \lambda_2 = 0, \lambda_3 = 2, n_1 = 6, n_2 = 1,$$

$n_3 = 6\ p = 2.$

The following rectangular designs can be constructed by theorem 2.1 with $r \leq 10$ and $k \leq 10$

S1. No.	v	b	r	k	λ_1	λ_2	$\lambda3$
1	2×5	10	4	4	1	0	2
2	2×7	14	6	6	2	0	3
3	3×7	21	6	6	1	0	2
4	2×7	21	6	4	1	0	2
5	4×9	36	8	8	1	0	2
6	2×9	18	8	8	3	0	4
7	3×9	36	8	6	1	0	2
8	2×9	36	8	4	1	0	2
9	2×11	22	10	10	4	0	5
10	5×11	55	10	10	1	0	2
11	4×11	55	10	8	1	0	2
12	3×11	55	10	6	1	0	2
13	2×11	55	10	4	1	0	2

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