Jour. Ind. Soc. Ag. Statistics
45(2), 1993 : 235-242

Measurement of Economic Efficiency – Frontier Function Approach

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(Received : May, 1992)

Summary

The economic efficiency combines technical and allocative efficiency. The core of economic theory is concerned with the allocative or price efficiency – the marginal value products of some or all factors might be equal to their marginal factor costs. The other important aspect of economic decision making process is to produce the greatest possible output from a given set of inputs. It means that the technical decision is efficient. Cobb-Douglas production was the basic model applied to measure allocative efficiency in agriculture. But Cobb-Douglas function ignored the problem of technical efficiency by assuming that all the technologies are identical across farms. In this paper, a methodology was suggested to measure both allocative and technical efficiency. A probabilistic frontier function was estimated using linear programming to avoid the drawbacks of non-frontier approaches.
Key Words : OLS, LP, Outliers, Objective function, Technical efficiency, Allocative efficiency, Economic efficiency.

Introduction

Measurement of economic efficiency includes technical efficiency and price efficiency. Technical efficiency refers to the proper choice of production function among all those actively in use by farms. Price efficiency refers to the proper choice of input combinations. Cobb-Douglas production assumes that all farms are efficient technically and derive the maximum output from any chosen level of inputs. The production function needs assumption of constant returns to scale and perfect competitive market. It neglects the differences in the environments of farms compared. These assumptions are unrealistic because the extent of utilization of inputs to their optimum level depends on the knowledge of farmer about the chosen technology. The issue of economic efficiency in agriculture has now been broadened from the earlier emphasis on price efficiency to consider technical efficiency also. The present paper discusses a methodological approach applying frontier production function to measure economic efficiency. This model is successful in measuring both technical and price efficiencies.

2. *Materials and Methods*

Rice farms commanded under Srivilliputhur tank in Kamarajar district of Tamil Nadu had been selected for the study. Thirty rice farmers were selected at random and the data related to the year 1990-91. The frontier production function is defined as the relationship that describes the maximum possible output for the given combination of inputs [Ferguson, (4)]. A production function estimated by the ordinary least-squares (OLS) method shows an average response and does not represent the frontier. Farrell (3) used a deterministic approach in which he estimated a cost frontier by using linear programming (LP), requiring all observations to lie on or above the cost frontier. Aigner and Chu (1) transformed Farrell's cost frontier into a production frontier. They observed all observations to lie on or below the production frontier. Since the outliers under a deterministic approach effect the results, Timmer (5) converted the deterministic frontier into a probabilistic frontier function. This approach deletes outlier observations or extreme observations until the estimated coefficients are stabilised. Timmer's probabilistic approach is presented below and used in the study. The usual Cobb-Douglas production function in log forms (capital letters) can be written as

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 \ldots + \beta_n X_n + e \qquad (1)$$

where Y is the output of the farm; X_1 to X_n are the inputs used and e is the random error term that contains a systematic efficiency term as well. The equation (1) can be written as

$$Y_t = \sum_{i=0}^{n} \beta_i X_{it} + e_t \qquad t = 1, 2, \ldots m \qquad (2)$$

$$Y_t = \hat{Y}_t + e_t$$

where one column of X_i is a vector of ones to allow for an intercept. If all error terms are constrained to one side of the estimated production surface the resulting function is an envelope. To be an efficient frontier, equation (2) can be estimated such that

$$\sum_{i=0}^{n} \beta_i X_{it} = \hat{Y}_t \geq Y_t \qquad (3)$$

The efficient farms satisfy the equality condition of $e_t = 0$ or $\hat{Y}_t = Y_t$. All other farms have a smaller actual output than would be achieved if they too were efficient. To force the estimated production surface to lie as closely as possible to the actual set of output points, a minimising constraint should be placed on sum function of sum of the resulting error terms. The problem then is to minimise $\sum e_t$

$$\text{subject to the constraint } \hat{Y}_t \geq Y_t \qquad t = 1, 2, \ldots m \tag{4}$$

This forms a linear programming problem. The production frontier in equation (4) can be transformed into a probabilistic frontier with the deletion of outliers one by one, until all coefficients are stabilised.

By setting all $e_t \geq 0$, equation (3), can be written as an equality.

$$\sum \beta_i X_{it} - e_t = Y_t \tag{5}$$

The objective is to minimise $\sum e_t$

subject to $\sum \beta_i X_{it} \geq Y_t$ and $\beta_i \geq 0$

In order to solve this problem using linear programming, $\sum e_t$ should be expressed as a linear function of β_{it} and X_{it}. Equation (5) can be summed over t and solved for $\sum e_t$.

$$\sum_{t=1}^{m} e_t = \sum_{t=1}^{m} \sum_{i=0}^{n} \beta_i X_{it} - \sum_{t=1}^{m} Y_t \tag{6}$$

where

n = number of Variables (i = 0, 1, . . . , n)

m = number of observations (t = 1, 2, . . . , m)

For any particular data set, the last term in the equation (6) *i.e.* $[-\sum Y_t]$ is a constant. Any set of β_i, that minimises $\sum e_t$ for one value of $-\sum Y_t$ will minimise for any other value including zero. Hence the last term $\left[-\sum Y_t\right]$ can be dropped from equation (6) without any consequence. Minimisation of $\sum e_t$ is approximately equal to the minimisation of the sum of estimated value of output.

$$\text{Min} \sum e_t \cong \text{Min} \sum \beta_i X_{it} \tag{7}$$

For computational purpose, it is desirable to divide equation (7) by number of observations. Thus the arithmetic mean of observations of the ith input $\overline{X}_i$ is used instead of total.

$$\frac{1}{m} \sum e_t = \sum \beta_i \overline{X}_i$$

where

$$\overline{x}_i = \frac{1}{m} \sum X_i$$

Therefore, the objective function in equation (4) is altered. In expansion terms, the objective of the linear programming is to minimise

$$\hat{\beta}_0 + \hat{\beta}_1 \overline{X}_1 + \hat{\beta}_2 \overline{X}_2 + \ldots + \hat{\beta}_n \overline{X}_n$$

Subject to

$$\begin{array}{l} \hat{\beta}_0 + \hat{\beta}_1 X_{11} + \hat{\beta}_2 X_{21} + \ldots + \hat{\beta}_n X_{n1} \geq Y_1 \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \hat{\beta}_0 + \hat{\beta}_1 X_{1m} + \hat{\beta}_2 X_{2m} + \ldots + \hat{\beta}_n X_{nm} \geq Y_m \end{array} \tag{8}$$

and $\beta_i \geq 0$. This can be solved by any linear programming package.

The vector $Y_t/\hat{Y}_t$ is the index of technical efficiency with a separate measure for each farm. These measures are averaged over number of observations to reach a single value of the each farm's technical efficiency.

$$\text{Technical efficiency (TE)} = \frac{1}{m} \frac{\sum Y_t}{\sum \hat{Y}_t} \tag{9}$$

The farm specific price efficiency or allocative efficiency is estimated to be

$$AE = \hat{Y}_t / \tilde{Y}_t \tag{10}$$

Where $\hat{Y}_t$ is the maximum output of the farm t and $\tilde{Y}_t$ is the output at the optimum level of all variable inputs.

Farm specific economic efficiency is estimated by the equation

$$EE_i = (TE_i)(AE_i) \tag{11}$$

2.1 *The empirical model*

The estimated Cobb–Douglas production function is specified as

$$\ln(Y) = \beta_0 + \beta_1 \ln(F) + \beta_2 \ln(I) + \beta_3 \ln(L) + \beta_4 \ln(P) + e \qquad (12)$$

where is Y is the rice output Quintal per hectare

F is fertilizer of kgs per hectare

I is number of irrigation per hectare

L is labour value in rupees per hectare and

P is plant protection expenses per hectare

The production function in equation (12) was first estimated by OLS method. It was transformed into a deterministic frontier production function as follows. The objective function is to minimise

$$\beta_0(1) + \beta_1 \ln(\bar{F}) + \beta_2 \ln(\bar{I}) + \beta_3 \ln(\bar{L}) + \beta_4 \ln(\bar{P})$$

Subject to

$$\beta_0(1) + \beta_1 \ln(F_1) + \beta_2 \ln(I_1) + \beta_3 \ln(L_1) + \beta_4 \ln(P_1) \geq Y_1 \qquad (13)$$

$$\beta_0(1) + \beta_1 \ln(F_2) + \beta_2 \ln(I_2) + \beta_3 \ln(L_2) + \beta_4 \ln(P_2) \geq Y_2$$

$$\vdots \qquad\qquad \vdots$$

$$\beta_0(1) + \beta_1 \ln(F_{30}) + \beta_2 \ln(I_{30}) + \beta_3 \ln(L_{30}) + \beta_4 \ln(P_{30}) \geq Y_{30}$$

where $\bar{F}$, $\bar{I}$, $\bar{L}$, $\bar{P}$ are mean values of the respective inputs. The probabilistic function coefficients used in estimating efficiencies were obtained from equation (13) after deleting outlier observations until the estimated coefficients are stabilised. The results are presented and discussed.

3. *Results and Discussion*

Estimates of OLS, deterministic and probabilitic Cobb–Douglas production frontier for rice crop are given in Table 1. The sample mean resource use are presented in Table 2.

The OLS estimates implied that fertilizer, Irrigation and plant protection were significant factors of production in these rice farms. It might be due to under use of resources by the "average" farmers. These "average" farmers were operating in the first stage of

Table 1. Estimated Parameters of OLS, Deterministic and Probabilistic Frontier Production Functions

Parameters	OLS	Deterministic Frontier Function	Probabilistic Frontier Function
Constant	-1.8806^{NS}	0.1143	0.1079
Fertilizer	0.5421^{s}	0.4196	0.4268
Irrigation	0.4670^{s}	0.3925	0.3864
Labour	-0.0421^{NS}	0.0037	0.0032
Plant Protection	0.2329	0.1243	0.1285
R^2	0.95		
$\bar{R}^2$	0.94		
Sample Size	30	30	27

s - Significant at one per cent level.

NS - Non-significant at one per cent.

Table 2. Sample Mean Resource Use

Particulars	Quantity/Value
Fertilizer (kg/ha)	166.35
Irrigation (Numbers/ha)	29.37
Casual labour (Rs/ha)	3178.67
Plant Protection Expenses (Rs/ha)	879.37
Total Cost (Rs/ha)	7670.83
Gross Income (Rs/ha)	10787.11
Net Income (Rs/ha)	3116.28

production curve and so these input elasticities were significant. In the order of significance, the important factors of production were water, fertilizer and plant protection.

The OLS coefficient for fertilizer was highest and for labour was negative and lowest. The OLS function portrayed the response of the "average" farmer while the frontier function reflected the response of the "best practice" farmers. The constant term in the frontier functions were higher than that estimated by the OLS method. In addition, the slopes had been changed revealing appreciable amounts of resource use by "best practice" farmers. Comparing production curves, the frontier envelope shifted upward with a shift in the intercept of the production function.

The difference between deterministic and probabilistic co-efficient showed the effect of outliers on the deterministic co-efficients. The probabilistic frontier function was selected for estimating efficiency measures since it was free of effects due to outliers. From the probabilistic function coefficients, farm specific technical efficiency (TE) is estimated as 0.74 using the equation (9). The technical inefficiency is given as (1 - 0.74) = 0.26. Farm specific allocative efficiency is estimated to be 0.95 using equation (10.) The allocative inefficiency is given as (1 - 0.95) = 0.05. Farm specific economic efficiency is calculated by the equation (11).

Economic Efficiency	=	(0.74) (0.95)
	=	0.703
Economic Inefficiency	=	1 - 0.703
	=	0.297

The results showed that the output loss due to technical inefficiency (26 per cent) was higher than the loss due to allocative inefficiency (5 per cent). The study implied that the rice output of "average farmer" could be increased by 26 percent by adopting the technology followed by the "best practice" farmers. By optimum resource allocation, there existed a scope to raise output by five percent.

The economic inefficiency revealed that the production could be raised by 29.7 per cent if the technology gaps between "average farmer" and "best practice farmer" were narrowed and by optimum resource allocation in all farms. This highlighted the need for improving the farm extension services to exploit the potential in the available agricultural technologies and it would improve the technical efficiency of "average-farmers". Factor market efficiency ensuring supply of inputs to meet the quantity demanded at reasonable price could help in raising the allocative efficiency of the rice farms in the tank-bed environment.

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