A NOTE ON UNEQUAL PROBABILITY SAMPLING

By

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1. INTRODUCTION

In unequal probability sampling without replacement (Horvitz and Thompson, [2], it is well known that for $n=2$

$$\pi_i=p_i\left(1+A-\frac{p_i}{1-p_i}\right) \qquad \ldots(1.1)$$

$$\pi_{ij}=p_ip_j\left(\frac{1}{1-p_i}+\frac{1}{1-p_j}\right) \qquad \ldots(1.2)$$

Where π_i and π_{ij} denote the inclusion probabilities of ith unit and a pair of $(i, j)^{th}$ units in the sample respectively and

$$A=\sum_{i=1}^{N} p_i/(1-p_i)$$

Yates and Grundy [3] have pointed out that the unequal probability sampling scheme without replacement for $n=2$ is not equivalent to the selection of a pair of units with replacement and rejecting those samples in which the same unit is repeated.

Aggarwal and Goel [1] have proposed the following sampling scheme with unequal probabilities with replacement which gives the same inclusion probabilities π_i and π_{ij} as given by Eqs. (1.1) and (1.2) respectively.

'Select two units with replacement, one with probabilities p_i $(i=1, 2,\ldots, N)$ and the other with revised probabilities

$$p_i^{*}=\frac{p_i\,(1-p_i)^{-1}}{\sum_{i=1}^{N} p_i\,(1-p_i)^{-1}}$$

Reject the sample if the same unit is selected at both the draws. Go on drawing samples of size two till we get a sample of two different units.'

It is obvious that for $n=2$, the sampling scheme proposed by Aggarwal and Goel (1977) is complicated than usual sampling scheme with unequal probability without replacement. In this note, for $n=2$ a simple sampling scheme with unequal probability with replacement is proposed which satisfies the inclusion probabilities π_i and π_{ij} as given by Eqs. (1.1) and (1.2) respectively.

2. Proposed Sampling Scheme and Calculation of Inclusion Probabilities

Let p_i denote the probability of drawing ith unit at any particular draw for $i=1, 2, \ldots N$. Let $\sum_{i=1}^{N} p_i=1$. Let the units selected at the first draw be u_{i_1}. We go on making subsequent draws till we get a unit different from u_{i_1}.

It can be easily seen that for the proposed sampling scheme, the inclusion probability for ith unit in the sample is given by

$$\pi_i=p_i \sum_{j(\neq i)} p_j \; (1+p_i+p_i^2+\ldots)+\sum_{j(\neq i)} p_j p_i (1+p_j+p_j^2+\ldots$$

$$=\frac{p_i}{1-p_i}\sum_{j(\neq i)} P_j+p_i \sum_{j(\neq i)} \frac{p_j}{1-p_j}$$

$$=p_i\left(1+A-\frac{p_i}{1-p_i}\right)$$

and the inclusion probability for ith and jth units in the sample is given by

$$\pi_{ij}=p_i p_j (1+p_i+p_i^2+\ldots)+p_j p_i \; (1+p_j+p_j^2+\ldots)$$

$$=p_i p_j\left(\frac{1}{1-p_i}+\frac{1}{1-p_j}\right)$$

Thus the proposed sampling scheme will also give the same estimator $\hat{Y}_{HT}$ (Horvitz, Thompson estimator of population total) and its variance as obtained under Horvitz and Thompson's sampling scheme.

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REFERENCES

[1] Aggarwal, S.K. and Goel, B.B.P.S. (1977). : A note on unequal probability sampling. *Jour. Ind. Soc. Agri. Stat.* **29**, No. 2, pp. 98-100.

[2] Horvitz, D.G. and Thompson, D.J. (1952). : A generalisation of sampling without replacement from a finite universe. *Jour. Amer. Stat. Assoc.*, Vol. **47**, pp. 663-685.

[3] Yates, F. and Grundy, P.M. (1953). : Selection wor from within strata with probabilities proportional to size. *Jour. Royal Stat. Soc.*, B **15**, pp. 253-261.