A NOTE ON MINIMUM VARIANCE LINEAR UNBIASED ESTIMATORS IN MULTISTAGE SAMPLING DESIGN ON SUCCESSIVE OCCASIONS

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SUMMARY

There are situations, where the population consists of more than two stage units for which partial replacement of units from the sample drawn on the previous occasion can be done in number of ways. In this paper, the theory of multistage successive sampling is considered to estimate the population mean on each of the h occasions. The minimum variance linear unbiased estimator (MVLUE) of the population mean and its variance are obtained when partial retention is made at some one stage only. This is done first, under the assumption, that certain population parameters, analogous to population correlation and regression coefficients, are known and later when they are estimated from the sample.

1. INTRODUCTION

Unistage successive sampling on h occasions was developed by Jessen [6], Yates [22], Patterson [7], Tikkiwal [12, 13, 14, 15, 16, 17,20, 21], Eckler, and Prabhu Ajgaonkar [8][9]. Two stage sampling on successive occasions is considered by Tikkiwal [18][19], Singh [11], Abraham *et al* [1], Singh and Kathuria [10] and Kahturia and Singh [4] to estimate the population mean on the current occasion under certain restrictive assumptions. Agarwal and Tikkiwal [2] have obtained the results for two stage successive sampling under less restrictive assumptions.

The theory of multistage successive sampling is considered in this paper to estimate the population mean on each of the h occasions. The minimum variance linear unbiased estimator (MVLUE) of the population mean and its variance are obtained

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when partial retention is made at some one stage only. This is done first, in Sec. 5, under the assumption that certain population parameters, analogous to population correlation and regression coefficients, are known and later, when they are estimated from the sample. The results for two stage design due to Agarwal and Tikkiwal [2] can be immediately obtained from these results. The results for three stage sampling are given in brief in Sec. 7 as a special case of Sec. 5.

2. Sampling Scheme and Replacement Pattern

Let us have a L-stage finite population consisting of $N^{(1)}$ first stage units (fsu's). Let each of the $N^{(k)}$ kth stage unit consist of $N^{(k+1)}$ $(k+1)$th stage units $(k=1, 2..., L-1)$. The purpose of study is to estimate the population mean with maximum precision by resorting to a partial replacement of units say at kth stage where k can assume any value from 1 to L. The procedure for selection and replacement of units is as follows :

On the first occasion, first a sample of $n^{(1)}$ fsu's is selected out of $N^{(1)}$ with simple random sampling without replacement (SRSWOR) scheme. Then $n^{(p)}$ pth stage units are selected again with SRSWOR scheme out of its $N^{(p)}$ pth stage units in the population from each of the selected $(p-1)$th stage units for $p=2,...k-1, k+1,..., L$. At the kth stage, $n_1^{(k)}$ kth stage units are selected out of its $N^{(k)}$ kth stage units in the population from each of the selected $(k-1)$th stage units.

For the successive second and higher occasions, the units upto $(k-1)$th stage for $k\geqslant 2$, are completely retained from one occasion to the other. For selecting kth stage units on the ith occasion $(i=1, 2...,h)$, the sample of size n kth stage units consists of two parts:

(i) The $n_i'^{(k)}$ kth stage units which are also observed for the same variate at least on the previous occasion.

(i) The $n_i''^{(k)}$ kth stage units which are selected afresh with SRSWOR from those kth stage units of the population which are not selected in the sample upto $(i-1)$th occasion. Thus,

$$n_i^{(k)}=n_i'^{(k)}+n_i''^{(k)}$$

Further, whenever a kth stage unit is retained from one occasion to the other, it is observed with its higher stage units selected earlier.

3. Notations

Let $X_{ij_1j_2\ldots j_L}$ denote the variate value of the j_L th element of the $(L-1)$th stage unit which in turn ultimately of the j_1th fsu on the ith occasion in population and $x_{ir_1r_2\ldots r_L}$ denote the variate value of the unit on the ith occasion at the $(r_1, r_2, \ldots, r_L)$th vector draw in the sample where r_K $(k=1, 2, \ldots, L)$ denotes the r_Kth draw at the kth stage. The suffixes k and i, used below in the subsequent sections, vary from 1 to L and 1 to h respectively unless otherwise stated.

$$\bar{X}_{ij_1j_2\ldots j_{k-1}}=\left[\underset{t=k}{\overset{L}{\pi}} N^{(t)}\right]^{-1}\sum_{j_k=1}^{N^{(k)}}\ldots\sum_{j_L=1}^{N^{(L)}} X_{ij_1j_2\ldots j_L}\ ;k\geqslant 2.$$

$$\bar{X}_i=\frac{1}{N^{(1)}}\sum_{j_1=1}^{N^{(1)}}\bar{X}_{ij_1}\ ;$$

the population mean under estimation per Lth stage unit on the ith occasion.

For $\quad i=1,$

$$\bar{x}_{1n_1}(k)=\left[\ n_1^{(k)}\underset{t\neq 1}{\overset{L}{\pi}} n^{(t)}\right]^{-1}\sum_{r_1=1}^{n^{(1)}}\cdots\sum_{r_k=1}^{n_1^{(k)}}\cdots\sum_{r_L=1}^{n^{(L)}} x_{ir_1r_2\ldots r_L}\ ;$$

and for $i>1$. $\bar{x}_{in_i'}(k)$ and $\bar{x}_{in_i''}(k)$ are similarly defined.

For $\quad k=1,$

$$_1S_i^2=\frac{1}{N^{(1)}-1}\sum_{j_1=1}^{N^{(1)}}(\bar{X}_{ij_1}-\bar{X}_i)^2\ ;$$

and for $k>1$,

$$_kS_i^2=\left[\ (N^{(k)}-1)\underset{t=1}{\overset{k-1}{\pi}} N^{(t)}\ \right]^{-1}\sum_{j_1=1}^{N^{(1)}}\cdots\sum_{j_k=1}^{N^{(k)}}(\bar{X}_{ij_1\ldots j_k}-\bar{X}_{ij_1\ldots j_{k-1}})^2$$

for $k=2, \ldots, L-1$;

$$_LS_i^2=\left[\ (N^{(L)}-1)\underset{t=1}{\overset{L-1}{\pi}} N^{(t)}\ \right]^{-1}\sum_{j_1=1}^{N^{(1)}}\cdots\sum_{j_L=1}^{N^{(L)}}(X_{ij_1\ldots j_L}-\bar{X}_{ij_1\ldots j_{L-1}})^2\ ;$$

similarly, ${}_1S_{ii'}$, and ${}_kS_{ii'}$, for $k>1$ are defined.

$$ {}_kV_i^2={}_kS_i^2+\sum_{t=k+1}^{L}\left[\prod_{p=k}^{t-1} a^{(p)}\right]^{-1}\left(\frac{1}{n^{(t)}}-\frac{1}{N^{(t)}}\right){}_tS_i^2; \quad k=1,2,\ldots L-1, $$

$$ {}_LV_i^2={}_LS_i^2 $$

where $$ a^{(p)}=\begin{cases} 1 & \text{for } p=k \\ n^{(p)} & \text{for } p>k \end{cases} $$

similarly ${}_kV_{ii}$, for $k<L$ and ${}_LV_{ii}$, are defined.

$$ {}_kB_{ii'}=\frac{{}_kV_{ii'}}{{}_kV_i^2}; \quad {}_kR_{ii'}=\frac{{}_kV_{ii'}}{{}_kV_i.{}_kV_{i'}}; $$

$$ g_k=\prod_{t=1}^{k} n^{(t-1)} $$

where $n^{(0)}=1$;

$$ D_{ki}^2=\sum_{t=1}^{k-1}\frac{1}{g_t}\left(\frac{1}{n^{(t)}}-\frac{1}{N^{(t)}}\right){}_tS_i^2-\frac{1}{g_k.N^{(k)}}{}_kS_i^2; \quad k=2,\ldots,L $$

$$ D_{1i}^2=-\frac{{}_1S_i^2}{N^{(1)}}; $$

similarly, $D_{kii'}$ and $D_{ii'}$ are defined.

4. ASSUMPTION

$$ {}_kR_{ij}=\prod_{t=i}^{j-1} {}_kR_{t,t+1} \text{ for all } i, j\ (>i). $$

5. MVLUE OF $\bar{X}_h$ AND ITS VARIANCE

Let a linear estimator e_h be given by

$$ e^h=\sum_{i=1}^{h}\cdots\sum_{r_1=1}^{n^{(1)}}\cdots\sum_{r_k=1}^{n_i^{(k)}}\cdots\sum_{r_L=1}^{n^{(L)}} w_{ir_1r_2\cdots r_L}\cdot x_{ir_1r_2\cdots r_L} \qquad \ldots(5.1) $$

where $w_{ir_1r_2\cdots r_L}$ be the weight to be associated with the variate $x_{ir_1r_2\cdots r_L}$ and depends upon the vector draw $(r_1r_2, \ldots r_L)$ but does not depend on the outcomes at the vector draw. It may be noted here that such a linear estimator belongs to $T_{11\ \ 1}$ class (Koop, [5]). The following two lemmas are presented without proof to examine the unbiasedness and minimum variance properties of e_h. The proof of Lemma5.1 is simple and the proof of Lemma 5.2 is on the lines of the proof of Tikkiwal's Lemma 2.1 [18].

Leema 5.1

The estimator e_h is an unbiased estimator of $\bar{X}_h$, the population mean on hth occasion iff

$$\sum_{r_1=1}^{n^{(1)}} \cdots \sum_{r_k=1}^{n_i^{(k)}} \cdots \sum_{r_L=1}^{n^{(L)}} w_{ir_1r_2\cdots r_L} = \begin{cases} 0 & \text{for } i \neq h \\ 1 & \text{for } i = h \end{cases} \qquad \ldots(5.2)$$

Lemma 5.2

e_h, the linear unbiased estimator of $\bar{X}_h$, is *MVLUE* iff

$$\text{Cov}\,(x_{ir_1r_2\ldots r_L}, e_h) = \lambda_{ih} \qquad \ldots(5.3)$$

where λ_{ih} is some constant for all vector draws and i. The variance of such a *MVLUE* is given by

$$\text{Var}\,(e_h) = \text{Cov}\,(x_{hr_1r_2\cdots r_L}, e_h) \qquad \ldots(5.4)$$

Now we state and prove the following theorem regarding the *MVLUE* of the population mean on the hth occasion and its variance.

Theorem 5.1

For $h \geqslant 2$, the *MVLUE*, ${}_k\hat{X}_h$, of $\bar{X}_h$, under the Assumption 4, is given by

$${}_k\hat{X}_h = (1 - {}_k\phi_h)[\bar{x}_{hn'_h}(k) + {}_kB_{h-1,h}\,({}_k\hat{X}_{h-1} - \bar{x}_{h-1n'_h}(k))] + {}_k\phi_h\,\bar{x}_{hn''_h}(k) \qquad \ldots(5.5)$$

where
$$\frac{{}_k\phi_h}{1 - {}_k\phi_h} = \frac{n_h''^{(k)}}{n_h'^{(k)}}(1 - {}_kR^2_{h-1,h}) + \frac{n_h''^{(k)}}{n_{h-1}''^{(k)}}\,{}_kR^2_{h-1,h}\cdot{}_k\phi_{h-1}; \qquad \ldots(5.6)$$

$${}_k\phi_1 = \frac{n_h''^{(k)}}{n_1^{(k)}}\,;\quad n_1''^{(k)} = n_1^{(k)} - n_2'^{(k)}.$$

The variance of the *MVLUE*, ${}_k\hat{X}_h$, when the various ϕ's, R's and B's occuring in the estimator are known in advance, is given by

$$\text{Var}\ ({}_k\hat{X}_h)=D_{kh}^2+\frac{{}_k\phi_h}{g_k n_h''^{(k)}}\,{}_kV_h^2 \qquad \ldots(5.7)$$

Proof:

The estimator ${}_k\hat{X}_h$ given by Eq. (5.5) in Theorem 5.1 can be put in the form of the estimator e_h given by Eq. (5.1). Thus Theorem 5.1 can be easily proved with the help of Lemma 5.1 and Lemma 5.2 in addition to the following lemmas.

Lemma 5.3

Let $\bar{x}_{in}^{(1)}\ldots n^{(L)}$ and $\bar{x}_{i'n}^{(1)}\ldots n^{(L)}$ be the sample means for the variate X on ith and i'th occasions (i, $i'=1, 2, \ldots, h$) based on the observations froms L-stage population, where for $k=1$, $n^{(1)}$ fsu's are selected out of $N^{(1)}$ and for $k>1$, $n^{(k)}$ kth stage units are selected out of $N^{(k)}$ kth stage units of each of the $n^{(k-1)}$ $(k-1)$th stage units which in turn are selected out of $N^{(k-1)}$. If the method of selection at all the stages is one that of *SRSWOR*, then

$$\text{Cov}\ (x_{ir_1r_2\cdots r_L},\ \bar{x}_{i'}\,n^{(1)}\ldots n^{(L)})=\begin{cases}\displaystyle\sum_{t=1}^{L}\frac{1}{g_t}\left(\frac{1}{n^{(t)}}-\frac{1}{N^{(t)}}\right){}_tS_{ii'} \text{ if } r_t\in n^{(t)};\ t=1,2,\ \ ,L\\[2ex] \displaystyle\sum_{t=1}^{k-1}\frac{1}{g_t}\left(\frac{1}{n^{(t)}}-\frac{1}{N^{(t)}}\right){}_tS_{ii'}-\left[N^{(k)}\prod_{t=1}^{k-1}n^{(t)}\right]^{-1}{}_kS_{ii'}\\ \text{if } r_t\in n^{(t)} \text{ for } t<k \text{ and } r_k\notin n^{(k)} \text{ for } k\geqslant 2;\\[1ex] t=1, 2,\ldots, k-1-\dfrac{1}{N^{(1)}}\ {}_1S_{ii'} \text{ if } r_1\notin n^{(1)}\end{cases}$$

Lemma 5.4

If ${}_k\hat{X}_i$ *MVLUE* of $\bar{X}_{i'}$ then

$$\text{Cov}\ (x_{ir_1r_2\ldots r_L},\ {}_k\hat{X}_{i'})=\begin{cases}D_{kii'}+\dfrac{{}_k\phi_i}{g_k n_i''^{(k)}}\displaystyle\prod_{t=i+1}^{i'}(1-{}_k\phi_t)\,{}_kV_{ii'} \text{ for } i<i'\\[2ex] D_{ki}^2+\dfrac{{}_k\phi_i}{g_k n_i''^{(k)}}\,{}_kV_i^2 \text{ for } i=i'\\[2ex] D_{kii'}+\dfrac{{}_k\phi_{i'}}{g_k n_{i'}''^{(k)}}\,{}_kV_{ii'} \text{ for } i>i'\end{cases}$$

If the kth stage unit corresponding to $x_{ir_1r_2\cdots r_L}$ is not present in the sample upto i'th occasion, then

$$\text{Cov}\,(x_{ir_1r_2\cdots r_L},\, {}_k\hat{X}_{i'})=D_{kii'}.$$

The proof of Lemma 5.3 is easy and hence omitted. Lemma 5.4 can easily be proved first for $i'=2$ and then in general by induction assuming that it is true for $i'-1$.

Remark 5.1

We have presented Theorem 5.1 under the Assumption 4. In the situations where this assumption does not hold good, it can be easily shown that Theorem 5.1 still hold good if the common $n_i'^{(k)}$ kth stage units on the ith occasion be a sub-sample of $n_{i-1}''^{(k)}$, the new kth stage units on the $(i-1)$th occasion for $i=3, 4, \ldots, h$.

When this sub-sampling condition does not satisfy then the estimator given by Eq. (5.5) is no more MVLUE.

6. Variance of ${}_k\hat{X}_h$ when the Parameters are Estimated from Sample.

In Sec. 5 we have obtained the MVLUE of the population mean and its variance on the hth occasion when certain population values occuring in the estimator are not estimated from the sample but are known in advance. When the estimates of these values are used then ${}_k\hat{X}_h$ neither remains linear nor unbiased. In this section we shall give the variance of ${}_k\hat{X}_h$ when certain parameters are unbiasedly estimated from the sample. For this, let the following generalisation of Tikkiwal's model (1965, Sec. 5) for multistage design hold good.

$$X_{ij_1j_2\cdots j_L}=\bar{X}_i+\delta_{ij_1}+\delta_{ij_1j_2}+\quad+\delta_{ij_1j_2\cdots j_L}\ \text{for all } i, j_1, j_2, \ldots, j_L$$

where δ_{ij_1}, $\delta_{ij_1j_2}, \ldots, \delta_{ij_1j_2\cdots j_L}$ are the random effects such that the L vectors.

$$\delta_{j_1}=(\delta_{1j_1}, \delta_{2j_1}, \ldots, \delta_{hj_1}),$$

$$\delta_{j_1j_2}=(\delta_{1j_1j_2}, \delta_{2j_1j_2}, \ldots, \delta_{hj_1j_2}),$$

$$\vdots$$

$\delta_{j_1j_2\cdots j_L}=(\delta_{1j_1j_2\cdots j_L}, \delta_{2j_1j_2\cdots j_L}, \quad, \delta_{hj_1j_2\cdots j_L})$ are mutually independent and follow non-degenerate h-variate normal laws with zero vector means and certain variance-covariance matrices. Then the non-linear estimator ${}_k\hat{X}_h$ is an unbiased estimator of $\bar{X}_h$ if the sample of $n_i^{\prime(k)}$ kth stage units is a sub-sample of $n_{i-1}^{\prime\prime(k)}$ for $i \geqslant 3$ and its variance is given by

(i) for $k=1$,

$$\frac{E({}_1\hat{\phi}_h)}{n_h^{\prime\prime(k)}} W_{(1)h} \text{ and}$$

(ii) for $k>1$

$$\sum_{t=1}^{k-1} \frac{\sigma^2_{(t)h}}{g_t n^{(t)}} + \frac{E({}_k\hat{\phi}_h)}{n_h^{\prime\prime(k)}} W_{(k)h}$$

where

$$W_{(k)h} = \begin{cases} \sigma^2_{(k)h} + \sum\limits_{t=k+1}^{L} \dfrac{\sigma^2_{(t)h}}{n^{(t)} \prod\limits_{p=k}^{t-1} a^{(p)}} & \text{for } k<L \\ \sigma^2_{(L)h} & \text{for } k=L \end{cases}$$

$$\sigma^2_{(k)h} = E(\delta^2_{hj_1j_2\cdots j_k})$$

and ${}_k\hat{\phi}_h$ is the estimator of ${}_k\phi_h$ obtained through estimates of W's as given below.

When all $n_i^{\prime(h)}$ kth stage units are not contained in $n_{i-1}^{\prime\prime(k)}$ for $i \geqslant 3$ then the non-linear estimator ${}_k\hat{X}_h$ is a consistant and asymptotically unbiased and its mean square deviation V_k is given by

$$\sum_{t=1}^{k-1} \frac{\sigma^2_{(t)h}}{g_t n^{(t)}} + \frac{{}_k\phi_h}{n_h^{\prime\prime(k)}} W_{(k)h} \leqslant V_k \leqslant \sum_{t=1}^{k-1} \frac{\sigma^2_{(t)h}}{g_t n^{(t)}} + \frac{E({}_k\hat{\phi}_h)}{n_h^{\prime\prime(k)}} W_{(k)h}$$

A consistant and asymptotically unbiased estimator of the variance or the mean square error of non-linear estimator ${}_k\hat{X}_h$, as the case may be, is

(i) For $k=1$,

$$\frac{{}_1\hat{\phi}_h}{n_h^{\prime\prime(1)}} \, {}_1s_h^2 \text{ and}$$

(*ii*) For $k>1$

$$\frac{{}_1s_h^2}{n^{(1)}}+\frac{{}_k\hat{\phi}_h}{n_h^{\prime\prime(k)}}\,{}_ks_h^2-\frac{{}_ks_h^2}{n_h^{(k)}\prod\limits_{t=1}^{k-1} n^{(t)}}$$

where

$${}_1s_h^2=\frac{1}{(n_h^{(1)}-1)}\sum_{r_1=1}^{n^{(1)}}(\bar{x}_{hr_1}-\bar{x}_h)^2$$

and

$${}_ks_h^2=\left[(n_h^{(k)}-1)\prod_{t=1}^{k-1} n^{(t)}\right]^{-1}\sum_{r_1=1}^{n^{(1)}}\cdots\sum_{r_k=1}^{n_h^{(k)}}(\bar{x}_{hr_1r_2\cdots r_k}-\bar{x}_{hr_1r_2\cdots r_{k-1}})^2$$

provide unbiased estimates $W_{(k)h}$ for different k.

7. Three Stage Sampling

It has been observed in Sec. 1 that the results on two stage successive sampling follow as a special case of the results presented in Secs. 5 and 6. In large number of sampling enquiries, three stage sampling is used. The various results for such enquiries can also be worked out from the general discussion. However, for illustration purpose, the results flowing from Theorem 5.1 are presented below for three stage design. There is partial replacement (*i*) at the first stage only, *i.e.* $k=1$; (*ii*) at the second stage only, *i.e.* $k=2$; and (*iii*) at the third stage only, *i.e.* $k=3$.

The MVLUEs of the population mean on the $h(2)$th occasion are obtained by putting $k=1, 2, 3$ in Eqs. (5.5) and (5.6). Their variances are given by

$$\text{Var}({}_1\hat{X}_h)=\frac{{}_1\phi_h}{n_h^{\prime\prime(1)}}\,{}_1V_h^2-\frac{{}_1S_h^2}{N^{(1)}} \qquad \ldots(7.1)$$

where

$${}_1V_h^2={}_1S_h^2+\left(\frac{1}{n^{(2)}}-\frac{1}{N^{(2)}}\right){}_2S_h^2+\frac{1}{n^{(2)}}\left(\frac{1}{n^{(3)}}-\frac{1}{N^{(3)}}\right){}_3S_h^2 \qquad \ldots(7.2)$$

$$\text{Var}({}_2\hat{X}_h)=\left(\frac{1}{n^{(1)}}-\frac{1}{N^{(1)}}\right){}_1S_h^2+\frac{1}{n^{(1)}}\left(\frac{{}_2\phi_h}{n_h^{\prime\prime(2)}}\,{}_2V_h^2-\frac{{}_2S_h^2}{N^{(2)}}\right) \qquad \ldots(7.3)$$

where

$$ {}_2V_h^2 = {}_2S_h^2 + \left(\frac{1}{n^{(3)}} - \frac{1}{N^{(3)}} \right) {}_3S_h^2 \qquad \ldots(7.4) $$

and

$$ \text{Var}({}_3\hat{X}_h) = \left(\frac{1}{n^{(1)}} - \frac{1}{N^{(1)}} \right) {}_1S_h^2 + \frac{1}{n^{(1)}} \left(\frac{1}{n^{(2)}} - \frac{1}{N^{(2)}} \right) {}_2S_h^2 $$

$$ + \frac{1}{n^{(1)}n^{(2)}} \left(\frac{{}_3\phi_h}{n_h''^{(3)}} - \frac{1}{(N^{3})} \right) {}_3S_h^2 \qquad \ldots(7.5) $$

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