A NOTE ON ESTIMATION OF μ^2 IN NORMAL DENSITY

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SUMMARY

Let $x_1, x_2, \ldots, x_n$ be a random sample of size n fr m a normal population having mean μ and variance $C^2\mu^2$. An estimator T_1^* of μ^2 has been developed which is biased and has smaller mean-squared error (MSE) than the usual unbiased estimator $T=\bar{x}^2-\frac{s^2}{n}$,

INTRODUCTION

Let $x_1, x_2, \ldots, x_n$ be a random sample of size n from a normal population having been μ and variance $C^2\mu^2$. In normal population the usual unbiased estimator for estimating μ^2 is

$$T=\bar{x}^2-\frac{s^2}{n}.$$

Searls [1] proposed an estimator of the population mean, where he utilized the known coefficient of variation. Such an estimator is although biased, has smaller mean-squared error than the usual unbiased estimator $\bar{x}$. Singh and Pandey [2] developed estimators of the population variance σ^2 of a normal population utilizing known coefficient of variation. Singh, Pandey and Hirano [3] considered a general population and suggested an estimator of the population variance σ^2 utilizing the known coefficient of kurtosis. In the present note we have tried to improve the estimator T by utilising a known value of coefficient of variation. We have considered an estimator

$$T_1=\alpha_1\bar{x}^2+\alpha_2\frac{s^2}{n},$$

where α_1 and α_2 are unknown scalars and are so determined that the mean-squared error of T_1 is minimum. The estimator thus obtained is denoted by T_1^*. Obviously T_1^* is a better estimator than the usual unbiased estimator T.

ESTIMATOR T_1^* AND ITS PROPERTIES

The proposed estimator is $T_1=\alpha_1\bar{x}^2+\alpha_2\frac{s^2}{n}$ which involves two unknown scalars α_1 and α_2. We determine the scalars to minimize the MSE of T_1. We have

$$\text{MSE}(T_1)=\alpha_1^2\, v(\bar{x}^2)+\frac{\alpha_2^2}{n^2}\, v(s^2)+\left\{\alpha_1\left(\mu^2+\frac{\sigma^2}{n}\right)+\alpha_2\frac{\sigma^2}{n}-\mu^2\right\}^2 \quad ..(1)$$

The values of α_1 and α_2 which minimize mean-squred error of T_1 are

$$a_1=\frac{n(n+C^2)}{C^4\left[\frac{\beta_2+2n-3}{n}+\frac{4\sqrt{\beta_1}}{C}+\frac{4n}{C^2}\right]+C^2(n-1)\left[\frac{C^2(\beta_2+2n-3)+4nc\sqrt{\beta_1}+4n^2}{\beta_2(n-1)+(3-n)}\right]+(n+C^2)2} \quad ...(2)$$

and

$$\alpha_2=\frac{n}{\left\{\left[C^2\frac{\beta_2}{n}+\frac{3-n}{n(n-1)}\right]+\frac{(n+c^2)^2}{c^2(n-1)}\left[\frac{\beta_{2(n-1)}+(3-n)}{C^2(\beta_2+2n-3)+4nc\sqrt{\beta_1}+4n^2}\right]+1\right\}} \quad ...(3)$$

where $C^2=\sigma^2/\mu^2$.

For normal populations $\beta_1=0$ and $\beta_2=3$, so from (2) and (3), we get

$$\alpha_1=\frac{n(n+C^2)}{C^2(n+1)(C^2+2n)+(n+C^2)^2} \quad ...(4)$$

and

$$\alpha_2=\frac{n(n-1)(C^2+2n)}{C^2(n+1)(C^2+2n)+(n+C^2)^2} \quad ...(5)$$

Therefore, if we assume the cofficient of variation to be known, the proposed estimator is

$$T_1^*=\frac{1}{C^2(n+1)(C^2+2n)+(n+C^2)^2}\left[n(n+C^2)\,\bar{x}^2+(n-1)(C^2+2n)s^2\right] \quad ...(6)$$

From expression (1), we obtain

$$\text{MSE}\ (T_1^*)=\frac{2C^2\mu^4(C^2+2n)}{C^2(n+1)(C^2+2n)+(n+C^2)^2} \quad ...(7)$$

Since T is a special case of T_1 with $\alpha_1=1$ and $\alpha_2=-1$, the proposed estimator T_1^* is better than the usual unbiased estimator T. If $\alpha_1=-\alpha_2=M$, the proposed estimator will become

$$T_2=M\left(\bar{x}^2-\frac{s^2}{n}\right)$$

The value of M which minimizes the mean-squared error of T_2 is

$$M=\frac{1}{\frac{2C^4}{n(n-1)}+\frac{4C^2}{n}+1}$$

Since T_2 is a sub-class of T_1, the estimator T_1^* is better than the estimator T_2, and since the unbiased estimator T is a special case of T_2 with $M=1$, the proposed estimator T_2 is better estimator than the usual unbiased estimator T.

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