A NOTE ON CONSTRUCTION OF BALANCED TERNARY DESIGNS

B.N. TYAGI[1] AND S.K.H. RIZWI[2]

INTRODUCTION

Nigam [2] has given a method of constructing Balanced Ternary Designs (BTD) by using the incidence matrices of two BIB designs with the same number of treatments. In these designs the number of blocks of the ternary designs is generally quite large. The purpose of the present note is to suggest some modifications so as to effect reduction in the number of blocks of the ternary (or n-ary) designs and to use partially balanced incomplete block (PBIB) designs also for the purpose.

RESULTS

(a) Balanced Ternary Designs through BIB designs :

Following Nigam (1974), let there be a BIB design with parameters v, b, r, k, λ whose $b \times v$ incidence matrix is N and another BIB design with v treatments and $N^* = n\ I_v$ as incidence matrix, where I_v is the indentity matrix of order v and n is a positive integer. If we add the elements of pairs (i,j) of rows where the rows i and j are chosen from the incidence matrix N and N^*, we get a balanced ternary or 4-ary design (depending on whether $n=1$ or $n>1$) with parameters

$$V=v,\ B=vb,\ R=(n+k)b,\ K=n+k, =2rn+\lambda v \qquad ...(1)$$

The design given by (1) has only vb blocks of size $n+k$. The number of blocks given in (1) would never exceed those obtained through Nigam's method.

The number of blocks can be further reduced if we add the elements of j^{th} row of N^* to only those rows of N which contain

[1]Additional Agricultura l Census Commissionar, Uttar Pradesh.
[2]State Soil Conservation Research, Rehmankhera, Lucknow.

unity in the j^{th} column. These vr rows from the blocks of a balanced ternary design with parameters

$$V=v,\ B=vr,\ R=(n+k)r,\ K=n+k=(2n+k)\lambda \qquad \ldots(2)$$

For proving the constancy of the inner product of any two column vectors of the incidence matrix, it can be easily seen that the pairs of elements given non-zero product would be $(n+1, 1), (1, n+1)$ each with the frequency of λ and $(1, 1)$ with the frequency of $(k-2)\ \lambda$. Thus the sum of products would be $(2n+k)\ \lambda$.

It may be observed here that for all values of n, the design would be ternary with $0, 1, n+1$ as elements of the incidence matrix. Taking $n=1$, the block size would be $k+1$ only.

Alternately, if we add the elements of the j^{th} row of N^* (having $n>1$) those rows of N which contain zero in the j^{th} column, then the $v\ (b-r)$ blocks so formed constitute a balanced ternary design with parameters

$$V=v,\ B=v\ (b-r),\ R=(n+k)\ (b-r),\ K=n+k\ ;$$
$$\pi=(r-\lambda)\ (2n+k-1) \qquad \ldots(3)$$

Example 1

Let us consider the case of a 4-treatment design associated to the BIB design with parameters $v=4,\ b=6,\ r=3,\ k=2,\ \lambda=1$ for which the incidence matrix is

$$N=\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Now if the elements of the first row of the identity matrix I_4 are added to those of the first, second, and third rows of N, the elements of second row of I_4 are added to those of the first, fourth and fifth rows of N and so on, we get the following incidence matrix $\hat{N}$ in 12 blocks only.

$$\hat{N}= \begin{bmatrix} 2 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 \\ 1 & 2 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 1 \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Nigam [2] has also developed ternary design for 4 treatments in 12 blocks, but the block size was 4 whereas in the above design the block size is 3. Economy in experimental resources is its chief advantage.

(*b*) *Balanced Ternary designs through PBIB designs:*

Saha [3] obtained a class of balanced ternary designs through certain Group Devisible (GD) designs by collapsing two rows of the incidence matrix of the PBIB (GD) design. We give here a different method for obtaining Balanced Ternary designs (BTD) by using a class of PBIB designs.

Let there be a PBIB design with paramaters v, b, r, k, n_l, λ_l ($1=1, 2...m$) such that $v-k=2n$, where n is an integer>1. Let us add the j^{th} row of N^* to those rows of the incidence matrix of the PBIB design which contain zero in the j^{th} column. We then have the following

Theorem

The $v\,(b-r)$ blocks so formed constitute a B T D with the parameters

$$V=v,\ B=v\,(b-r),\ R=(n+k)\,(b-r),\ K=n+k,\ \pi=2nr \qquad ...(4)$$

Proof: V, B, R and K require no proof.

For proving the constancy of the inner product of any pair of column vectors of the the incidence matrix of the *BTD*, let us take the i^{th} and j^{th} columns. The contribution to the inner product would come from the pairs (1, 1), (1, 0) and (0, 1) of the elements of incidence matrix of PBIB design which would be transformed to (1, 1), $(1, n)$ and $(n, 1)$ with frequencies $\lambda_l (v-k)$, $(r-\lambda_l)$ and $(r-\lambda_l)$ respectively if i^{th} and j^{th} treatments are l^{th} associates $(l=1, 2 \ldots \ldots m)$. The inner product would thus be given by

$$\pi = \lambda_l(v-k)+2n(r-\lambda_l)$$
$$=2nr+\lambda_l(v-k-2n)$$

which would be independent of λ_l if $v-k=2n$. Thus the above ternary design would always be balanced.

It may be remarked here that if $v-k \neq 2n$, the ternary design would be partially balanced having the association scheme of the associated PBIB design.

(c) *Use of Youden Squares for two—way elimination of heterogeneity*:

Youden squares are incomplete latin squires in which each treatment occurs in a column. These incomplete block designs allow for two—way elimination of hetrogeneity. We take the case of the ternary design so obtained with vr blocks of size $k+n$ units each. These units can always be so arranged that in each of the first k columns each treatment occurs r times and no treatment occurs more than once in a row. The next n columns would be such that every treatment occupies all the vrn units of vr rows exactly r times.

It may, however, be remarked that the methods given in this note always result in designs with the number of blocks as an integral multiple of the treatment number. Thus the treatments within blocks of the ternary design having vr blocks, following Agrawal [1], can always be so arranged that every treatment occurs exactly r times in the first k columns and in the remaining n columns also, every treatment occurs exactly r times. Similar arguments hold good when ternary designs are formed with vb blocks or $v\ (b-r)$ blocks. Hence all these designs allow for elimination of heterogeneity in two ways.

Example 2.

As an example we rearrange the 4 treatments within blocks of the design in example 1 in the following way.

1.	1	2	1
2.	1	3	1
3.	1	4	1
4.	2	1	2
5.	2	3	2
6.	2	4	2
7.	3	1	3
8.	3	2	3
9.	3	4	3
10.	4	1	4
11.	4	2	4
12.	4	3	4

ACKNOWLEDGEMENTS:

We are greateful to the referees for their valuable comments.

REFERENCES

[1] Agrawal H. (1966) : Some generalisation of distinct representatives with application to statistical designs. Ann. Math. Stat., 37, 525-526.

[2] Nigam, A.K. (1974) : Construction of balanced *n*-ary Block designs & partially balanced arrays, Journal of the Indian Soc. of Agri. Statistics, 26 (2), 48-58.

[3] Saha, G. M. (1975) : On construction of Balanced Ternary designs. Sankhya, 37 (B), 220-227.

OBITUARY

It is with profound sorrow that we have to report the sudden demise of Dr. B.V. Sukhatme, one of the life members of the Indian Society of Agricultural Statistics, on Monday, the 30th April, 1979. He was 54. He was Professor of Statistics in Iowa State University of Science and Technology, Ames, Iowa, U.S.A. Prior to his assignment in U.S.A., he was Professor at the Indian Agricultural Statistics Research Institute, New Delhi. As a member of the Editorial Board of the Journal of the Society he rendered valuable services. His untimely demise is a great loss to the Statistical community and to the Society in particular.