Jour. Ind. Soc. Ag. Statistics
45 (3), 1993 : 285-297

Markov-based Models for Weather Spells-A Case Study

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(Received: February, 1989)

Summary

A Markov chain probability model has been fitted to the daily rainfall data recorded at Nagpur meteorological department. The geometric distribution based on the Markovian assumption is found to give a good fit to the wet spell distribution in general. The validity of the basic Markovian assumption has been tested for Nagpur conditions.

Key Words : Markov Chain, Geometric Distribution, Daily Rainfall, Weather Spells.

Introduction

Nagpur represents areas of dryland cultivation with cotton and orange as major cash crops. Mean annual rainfall of Nagpur is about 1200 mms, out of which the rainy months, i.e., June to September, alone contribute about 1068 mms or 85%. But yearly variations in the rainfall amounts and its distribution over the crop growing season are rather severe. Information on the rainfall patterns such as the expected dry and wet periods, distribution of weather spells etc during the crop season is therefore useful in planning field operations in advance.

Following Gabriel and Neumann [5] [6], many authors have analysed the distribution of rainfall patterns and occurrence of dry and wet weather spells, using a Markov-based geometric distribution. Basu, [2], Bhargava et al. [3]; Sundararaj and Ramachandra, [10]; Kulkarni et al., [7]; Manohar and Siddappa, [8]; Reddy et al., [9]. Such studies would be helpful in gaining insight into the characteristics of rainfall patterns, based on long series of past data. In the present investigation, an attempt is made to seek empirical evidence of the suitability of representating daily rainfall occurrence by a Markov-based geometric model for Nagpur conditions and forms the first of the series of research on climatological and crop weather modelling studies, particularly for rainfed cotton regions of Maharashtra.

The days to equilibrium, which is the time required by the system to attain its original state, i.e., dry, gives information regarding the capacity of each rainy month to withhold wet conditions. This is obtained as the number of steps or times the 'P' matrix is powered so that its diagonal elements P_{00} and P_{11} become equal to π_0 and π_1 (Kulkarni et al, [7]). (4)

The probabilities of obtaining a wet spell of length 'w' days and a dry spell of length 'd' days from the Markov-based geometric model have been obtained according to Sundararaj & Ramachandra [10]. The probability generating expression for the wet and dry spell lengths are :

$$P_{(w\,=\,x)} = (P_{11})^{x-1}\,(1 - P_{11}) \qquad (5)$$

and

$$P_{(d\,=\,y)} = (P_{00})^{y-1}\,(1 - P_{00}) \qquad (6)$$

The probability of a wet (dry) can also be estimated without making the Markovian precondition of dependence directly from the observed frequency distribution for wet (dry) spells by assuming a geometric distribution, viz.,

if X is geometrically distributed,

$$P_{(X\,=\,n)} = p^{n-1}\,(1 - p), \qquad n = 1, 2, \ldots$$

then

$$E(X) = \frac{1}{1-p}, \quad Var(X) = \frac{p}{(1-p)^2} \qquad (7)$$

where, 'p' is the probability of a wet (dry) day. An estimate of 'p' which is a maximum likelihood estimate, is obtained from the reciprocal value of the sample mean of the corresponding observed frequency distribution (Sundararaj and Ramachandra, [10]).

The 'a priori' probabilities of a rainy day (dry day) 'i' days after a rainy (dry) day and their complementaries can be predicted (Sundararaj and Ramachandra, [10]) and are given by

$$P_{i(11)} = \frac{(1 - P_{11})}{(2 - P_{11} - P_{00})}\,(P_{11} + P_{00} - 1)^i + \frac{(1 - P_{00})}{(2 - P_{11} - P_{00})} \qquad (8)$$

$$P_{i(00)} = \frac{(1 - P_{00})}{(2 - P_{11} - P_{00})}\,(P_{11} + P_{00} - 1)^i + \frac{(1 - P_{11})}{(2 - P_{11} - P_{00})} \qquad (9)$$

Table 1. Estimates of conditional probabilities of rainfall occurrence

Preceding day		Frequency Actual day		Conditional probabilities	
		Wet	Dry	Wet	Dry
June	Wet	331	312	0.5148	0.4852
	Dry	339	1178	0.2235	0.7765
	Total	670	1490	0.3102*	0.6898*
July	Wet	691	440	0.6110	0.3890
	Dry	445	656	0.4042	0.5958
	Total	1136	1096	0.5090*	0.4910*
Aug	Wet	579	409	0.5860	0.4140
	Dry	396	848	0.3183	0.6817
	Total	975	1257	0.4368*	0.5632*
Sept	Wet	364	357	0.5048	0.4951
	Dry	340	1099	0.2363	0.7637
	Total	704	1456	0.3259*	0.6741*
Oct	Wet	105	135	0.4375	0.5625
	Dry	128	1864	0.0643	0.9357
	Total	233	1999	0.1044*	0.8956*
Nov	Wet	22	147	0.3188	0.6812
	Dry	46	2045	0.0220	0.9780
	Total	68	2092	0.0315*	0.9685*
June–Sept	Wet	1965	1518	0.5642	0.4358
	Dry	1520	3781	0.2867	0.7133
	Total	3485	5299	0.3967*	0.6032*
June-Nov	Wet	2092	1700	0.5517	0.4483
	Dry	1694	7690	0.1805	0.8195
	Total	3786	9390	0.2873*	0.7127*

*Unconditional binomial probabilities for wet and dry days respectively.

Table 3 (a) Observed and expected frequencies of dry spells

Spell length	June		July		Aug		Sept		June–Sept	
	O	E	O	E	O	E	O	E	O	E
1	111	67.6 90.0	206	175.0 220.4	150	128.0 175.6	115	80.0 111.1	582	429.8# 594.6
2	63	53.5	92	104.3	85	87.2	58	61.5	298	306.6
3	44	42.4	51	62.1	57	59.5	41	47.0	193	218.7
4	27	33.4	23	37.0	30	40.5	29	35.9	109	156.0
5	16	26.4	24	22.1	13	27.6	25	27.4	78	111.2
6	17	20.9	12	13.1	20	18.8	17	20.9	66	79.3
7	12•	16.5	10	7.8	9	12.8	16	16.0	47	56.6
8			15+	11.4	5	8.7	7	12.2	24	40.4
9					10	6.0	6	9.3	22	28.8
10					23#	12.6	5	7.1	17	20.5
11							6	5.4	15	14.7
12							16•	16.9	13	10.5
13									9	7.5
14									26	18.5
Total	290	290	433	433	402	402	341	341	1499	1499
χ^2	36.90 13.94		16.24 11.69		28.42 28.37		22.03 7.48		97.89 $1 44.26	
d.f.	6		7		9		11		13	
P	0.001 0.02–0.05		0.02–0.05 0.10–0.20		0.001–0.01 0.001–0.01		0.02–0.05 0.75–0.90		0.001 $2 0.001	

• Spells > 7 + Spells > 8 # Spells > 10 @ Spells > 12

O – Observed E – Expected

$ First line, expected frequencies based on P_{01}
Second line, expected frequencies based on P(W)

$1 and $2 Corresponding χ^2 and P–values in the large mass of data accumulated over a long period of 72 years; the influence of many other extraneous factors and so on. But we have singled out the role of 'persistence' in influencing the model prediction. Another interesting aspect of persistence observed by Yap [11] is that after a dry (wet) day, the probability of the following day being dry (wet) increases with the increasing spell length. He also observed that the rate of increase of the probabilities was more pronounced for shorter dry spells than for corresponding wet spell lengths. Such aspects also deserve investigation.

Figures (1.a) and (1.b) show the results of the model-fitting for the distribution of wet and dry spells, for the consolidated period, June to September.

To find out how the probability structure is affected in the absence of the Markovian precondition of dependence of rainfall events, the estimates for the probabilities of wet and dry days, via

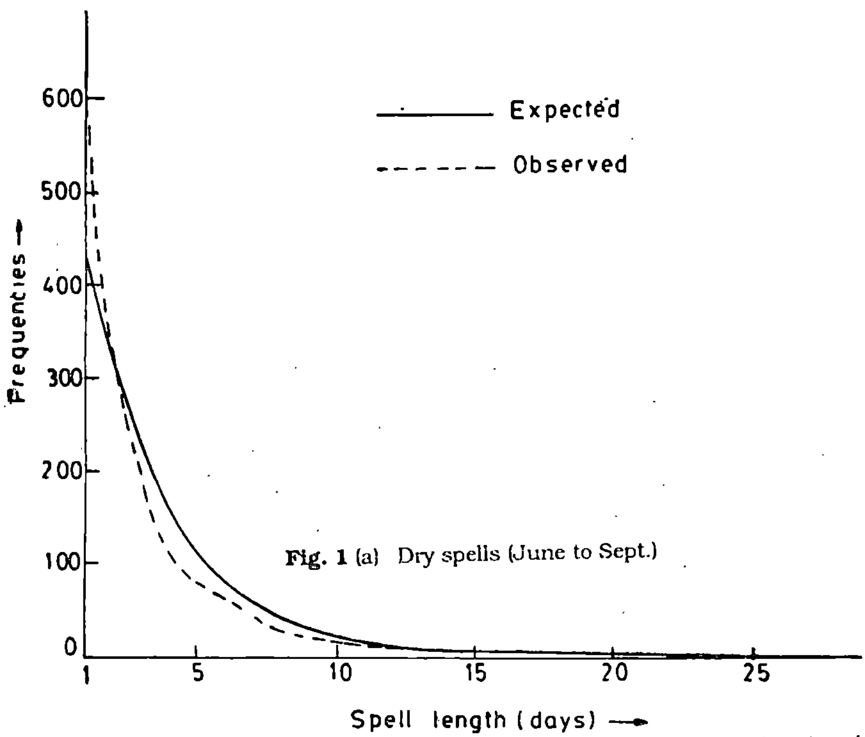

Fig. 1 (a) Dry spells (June to Sept.)

The results reveal that these estimates are closer to the Markovian conditional probabilities obtained in Table 1. and considerably different from those of the unconditional binomial probabilities reported in the same Table for all the months, thus pointing to the possible validity of the basic Markovian assumption for weather spells.

Test of the Markovian assumption itself, viz., "Given the state (wet or dry) of the previous day, the probability of rain or dry on any day is independent of events of further preceding days" was made by employing formulae (8), (9), (10) and (11) and using the chi-square test for goodness of fit. The observed and expected frequencies of wet/dry days 'i' days after a wet/dry day for i = 2 and 3 are reported in Table 5 for all the months.

The test validates the Markovian assumption for both i = 2 and 3 in July for both wet and dry days; for i = 2 for both wet and dry days in August and for i = 2 for dry days only and i = 3 for both wet and dry days in September at the 5% level of significance. The failure of the model in June requires further investigation.

In addition, the Table also shows that the probabilities of rainfall occurrence (or dry), 'i' days after a wet or dry day converge to the respective absolute probabilities of a wet (dry) day, which are 0.3102, 0.5090, 0.4368, 0.3259, 0.3967 for a wet day and 0.6898, 0.4910, 0.5632, 0.6741, and 0.6032 for a dry day respectively in June, July, August, September and June to September.

4. *Conclusions*

On the whole, as judged by the χ^2 – test of goodness of fit, the geometric distribution model based on the Markovian dependence of weather occurrence seems satisfactory for representing daily rainfall occurrence at Nagpur. The case of dry spells, however, requires further investigation. The role of persistence in influencing model prediction is suspected. The limitations mentioned earlier about the data-base itself must also be kept in view. The findings of this paper could form a basis for developing further rainfall models for use in crop planning and management.

ACKNOWLEDGMENTS

The authors wish to thank the Director, CICR, Nagpur for his keen interest and for providing encouragement for this study. Kind cooperation by the Meteorological Office at Pune and Nagpur in making available the historical daily rainfall records, is gratefully acknowledged. The authors thank the referee for his useful comments.

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