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A Class of Accelerated Life Testing Models Based on the Gamma Distribution

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SUMMARY

In this paper we study some models for accelerated failure times based on the gamma distribution. These models are useful as alternatives to analyses using lognormal or inverse Gaussian distributions that are common in literature [see Bhattacharyya and Fries (1982b), In, Survival Analysis, J. Crowly and R.A. Johnson (eds.), IMS lecture notes-monograph : series-2, Institute of Mathematical Statistics, Hayward, California, 101-118]. We have investigated maximum likelihood method using four possible choices of the parameters of the gamma distribution that provides a reciprocal linear relation for the mean failure time as a function of the 'stress level'. The models are illustrated using the standard example from Nelson (1981), [Applied Life Data Analysis, Wiley, New York] on failure times of motorettes under high temperatures.

Keywords : Accelerated failure time, Gamma distribution, Reciprocal linear regression.

1. INTRODUCTION

"Accelerated life test" applies to the type of study where failure times can be accelerated by applying higher "stress" to the component, where higher "stress" may bring quicker failure of the component. For example, some components may fail quicker at higher temperatures, however, it may take a long time before failure occurs at lower temperatures. The factor that may accelerate failure is generally called "stress" factor, such as the temperature in the example just mentioned above. The need for accelerated life test in practice arises where the objective is to study the relation between failure and stress conditions and low stress conditions are costlier to study as they may require a long time before failure occurs. And therefore it becomes hard to ascertain the reliability of the component quickly. In contrast, accelerated life testing provides economic collection of data in order to be able to estimate the reliability that can then be projected at the lower stress levels.

Accelerated life testing methods are useful for obtaining information on the life of products or materials over a range of conditions involving different stress factors which are encountered in practice. Typical type of stress factors that are encountered in engineering applications include temperature, voltage, pressure, vibration, cyclic rate, load etc. or some combination of them. In other fields of application, similar problems may arise where the relationship between life and concomitant variables is of the nature described above. The approach considered here can also be applied in an agricultural research setup where screening of genotype for resistance to certain diseases is of interest. In such experiments the time that it takes for the plants of a genotype to survive may be interpreted as the failure time and this could be a function of the level of disease that may represent the stress level. For example, in screening lentil genotypes for resistance to fusarium wilt, a number of genotypes are grown in field plots infested with fusarium wilt (see

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Bayya *et al.* 1997) and the plants are observed over time for resistance to the infestation.

Accelerated life tests serve various other purposes such as identifying design failures, estimating the reliability improvement by eliminating certain failure patterns, determining burn in time and conditions, quality control, determining whether to release a design to manufacturing or product to a customer, demonstrating product reliability for customer specifications, determining the consistency of the engineering relationship and adequacy of statistical models, developing relationship between reliability and operating conditions and so on. Actually management must specify accurate estimates for their design purposes and statistical test planning helps towards this goal. Without a good experimental design, analysis and interpretations of data may not be adequate and thus may result in improper decision.

The analysis of accelerated life testing models based on the log-normal models has been thoroughly developed in a series of papers by Nelson (1971, 1972a, 1972b, 1972c), finally culminating into an excellent text book (see Nelson 1990). Singpurwalla (1972) also discusses inference procedures based on these models. Bhattacharya and Fries (1982a) propagated inverse Gaussian distribution as an alternative to Birnbaum-Saunders model for fatigue life distribution and later explored its use as an accelerated life test model (see Bhattacharya and Fries (1982b)) and model checking procedures (Bhattacharyya and Fries (1986)).

The models considered in Nelson (1990) assume that the logarithm of failure time follows a linear regression as a function of inverse of the stress. This assumption has been questioned by several authors. For example, Bhattacharyya and Fries (1982b) argue that it makes more sense to assume the reciprocal of failure times to follow a linear regression in the stress variable(s). His model is based on failure times distributed as inverse Gaussian. Our aim is to study such models with life times following gamma distribution. Several such models are described in the next section. The basic goal is to choose the parameters of the gamma distribution so as to ascertain an inverse relation between the mean failure time and the stress level. The next section (Section 3) outlines maximum likelihood procedure for estimation of the parameters. And Section 4 discusses the application of these models for a standard data set analyzed by several authors. We find that one of the models is a strong competitor to the log-normal and inverse Gaussian regression models used earlier.

2. THE RECIPROCAL LINEAR REGRESSION MODEL

In stochastic modeling of failure time, the fatigue life time distribution leads a prominent role in the engineering literature. Bhattacharyya and Fries (1982a) assumed that the fatigue is governed by a Wiener process, and the time to failure is interpreted as a first passage time distribution, hence may be assumed to follow an inverse Gaussian (IG) distribution. Let Y denote the failure times then denoting the mean failure time at stress level x by $\mu_x = E(Y \mid X = x)$ we may postulate the reciprocal linear model as

$$\mu_x^{-1} = \delta_0 + \delta_1 x;\ \delta_0 > 0,\ \delta_1 \geq 0,\ x > 0, \qquad (2.1)$$

i.e. the conditional distribution of $Y \mid X = x$ is considered to be $IG(\mu_x, \lambda)$, where λ is the dispersion parameter.

Since, the fatigue life distribution may not follow a Wiener process, IG distribution may not always be appropriate. Hence, we would like to consider some other general family of distributions in accelerated life testing. Due to proximity of the gamma distribution to the log-normal and inverse Gaussian family, we are inclined to use the gamma distribution as the model for failure times, *i.e*. the failure times at stress level x may be assumed to follow Gamma(α_x, β_x) distribution where the density of a Gamma(α, β) distribution is given by

$$f(y) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} y^{\alpha-1} \exp(-y/\beta),\ y > 0;\ \alpha \geq 1,\ \beta > 0 \qquad (2.2)$$

The models that we propose below assume the dependence of the parameters α and β on x.

In a practical situation, we only consider $\delta_0 + \delta_1 x > 0$ on a finite interval of x which corresponds to the range of stress x. But we assume that the origin is taken at the lower point of this interval, i.e. $\delta_0 \geq 0$. Different choice of α_x and β_x will generate different models that may be motivated from the following considerations. A constant β_x implies that the distribution shape may change with respect to x but not the scale, where as, a constant α_x implies that the distribution changes with x according to the changes in scale. Moreover, since, $\mu_x = \alpha_x\beta_x$ and, since the mean failure time at level x is considered to be a decreasing function of x, a general model for μ_x may be given by $1/\mu_x = g(\alpha_x\beta_x)$, where g

is an increasing function, this may be achieved by assuming α_x to be decreasing in x for fixed β_x and/or β_x to be decreasing in x for fixed α_x. Obviously, we may assume the dependence of both the parameters on x, however, here we will assume this for only one of the parameters. In addition to providing a simple model this assumption has the potential to specify a reciprocal linear model as given in the following choices

Model I : $\alpha_x = 1/(\alpha_0 + \alpha_1 x)$ and $\beta_x = \beta$

Model II : $\beta_x = 1/(\beta_0 + \beta_1 x)$ and $\alpha_x = \alpha$

Model III : $\alpha_x = (\alpha_0^* + \alpha_1^* /x)$ and $\beta_x = \beta$

Model IV : $\beta_x = (\beta_0^* + \beta_1^* /x)$ and $\alpha_x = \alpha$

The next section discusses the maximum likelihood estimation of the parameters.

3. MAXIMUM LIKELIHOOD ESTIMATION OF THE PARAMETERS

In order to obtain estimates of the parameters, consider the observations (x_i, y_i); $i = 1, 2, ..., n$ from n runs of an accelerated life test experiment where y_i denote the failure time corresponding to the stress setting x_i. Here the random variables $y_1, y_2, ..., y_n$ are considered to be independent and that $y_i \sim$ Gamma($\alpha_{x_i}, \beta_{x_i}$).

The general form of the log-likelihood function is given by

$$\log L = -\sum_{i=1}^{n}\left[\frac{y_i}{\beta_{x_i}} + (\alpha_{x_i} - 1)\log y_i - \log\Gamma\alpha_{x_i} - \alpha_{x_i}\log\beta_{x_i}\right] \tag{3.1}$$

This simplifies to different forms for the models mentioned earlier and these will be discussed below. An important case is when the shape parameters α_{x_i} are not constant with respect to x_i. In this case we use the following Euler's infinite product representation of the gamma function

$$\frac{1}{\Gamma z} = z e^{\gamma z}\prod_{n=1}^{\infty}\left[\left(1+\frac{z}{n}\right)e^{-\frac{z}{n}}\right], -\infty \le x \le \infty \tag{3.2}$$

where γ is known as Euler's constant defined as

$$\gamma = \lim_{m\to\infty}\left[1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots+\frac{1}{m}-\log m\right]$$
$$= 0.5772156649.$$

Taking the logarithm and term by term differentiation gives

$$\frac{d}{dz}\log\Gamma(z) = \Psi(z) = -\gamma - \frac{1}{z} + z\sum_{n=1}^{\infty}\frac{1}{n(n+z)} \tag{3.3}$$

that is useful for computations.

3.1 Model I

In this case we let $\beta_{x_i} \equiv \beta$ and then the parameters α_0, α_1 and β are related to the parameters δ_0 and δ_1 of the reciprocal linear model by

$$\alpha_x^{-1} = \beta(\delta_0 + \delta_1 x), \tag{3.4}$$

or

$$\alpha_0 = \beta\delta_0,\ \alpha_1 = \beta\delta_1. \tag{3.5}$$

We basically need to estimate three parameters α_0, α_1 and β and the corresponding log-likelihood becomes

$$\log L = -\sum_{i=1}^{n}\left[\frac{y_i}{\beta} + (\alpha_{x_i} - 1)\log y_i - \log\Gamma(\alpha_{x_i}) - \alpha_{x_i}\log\beta\right], \tag{3.6}$$

where $\alpha_{x_i} = 1/(\alpha_0 + \alpha_1 x_i)$. So the maximum likelihood estimators of α_0, α_1 and β can be obtained by solving the following estimating equations

$$\sum_{i=1}^{n}\hat{\alpha}_{x_i}^2\log y_i + \sum_{i=1}^{n}\left\{\gamma + \hat{\alpha}_{x_i}^{-1} - \hat{\alpha}_{x_i}\left(\sum_{p=1}^{\infty}\frac{1}{p(p+\hat{\alpha}_{x_i})}\right)\right\}\hat{\alpha}_{x_i}^2 = \sum_{i=1}^{n}\hat{\alpha}_{x_i}^2\log\beta; \tag{3.7}$$

$$\sum_{i=1}^{n}x_i\hat{\alpha}_{x_i}^2\log y_i + \sum_{i=1}^{n}\left\{\gamma + \hat{\alpha}_{x_i}^{-1} - \hat{\alpha}_{x_i}\left(\sum_{p=1}^{\infty}\frac{1}{p(p+\hat{\alpha}_{x_i})}\right)\right\}x_i\hat{\alpha}_{x_i}^2 = \sum_{i=1}^{n}x_i\hat{\alpha}_{x_i}^2\log\hat{\beta}; \tag{3.8}$$

$$\hat{\beta} = \frac{\sum_{i=1}^{n}y_i}{\sum_{i=1}^{n}\alpha_{x_i}} \tag{3.9}$$

Substituting the value of β obtained above in equations (3.7) and (3.8), we have two equations in two unknowns that can be solved by numerical methods. We have found Newton-Raphson method useful that converges quickly.

3.2 Model II

For the second model we assume that $\alpha_{x_i} \equiv \alpha$. The relation between the parameters α, β_0 and β_1 and δ_0, δ_1 are given by

$$\beta_x^{-1} = \alpha\,(\delta_0 + \delta_1 x),$$

i.e.

$$\beta_0 = \alpha\delta_0,\ \beta_1 = \alpha\delta_1. \tag{3.10}$$

The maximum likelihood equations in this case are given by

$$-\sum y_i + \hat{\alpha}\sum \hat{\beta}_{x_i}^{-1} = 0; \tag{3.11}$$

$$-\sum y_i x_i + \hat{\alpha}\sum x_i \hat{\beta}_{x_i}^{-1} = 0; \tag{3.12}$$

$$\sum \log y_i + \sum \log \hat{\beta}_{x_i} = n \log \Psi(\hat{\alpha}) \tag{3.13}$$

Eliminating $\hat{\alpha}$ from these equations gives

$$\sum y_i \sum x_i \hat{\beta}_{x_i}^{-1} = \sum y_i x_i \sum \hat{\beta}_{x_i}^{-1}, \tag{3.14}$$

and

$$\Psi\left(\frac{\sum y_i}{\sum \hat{\beta}_{x_i}^{-1}}\right) = \frac{1}{n}\left(\sum \log y_i + \sum \log \hat{\beta}_{x_i}\right). \tag{3.15}$$

These two equations can be iteratively solved for $\hat{\beta}_0$ and $\hat{\beta}_1$. However, direct numerical solutions were found to be equally convenient.

3.3 Model III

In this model the scale parameter is again constant as in model I, but α_{x_i} is given by

$$\alpha_{x_i} = \alpha_0^* + \alpha_1^* r_i,$$

where $r_i = 1/x_i$. Relationship of the parameters of the gamma model to the reciprocal linear model can not be explicitly written. All we can say is that, α_0^*, α_1^* and β are such that

$$(\alpha_0^* + \alpha_1^*/x_i)\beta = 1/(\delta_0 + \delta_1 x_i),\ i = 1, 2, ..., n.$$

For maximum likelihood estimation of α_0, α_1 and β, similar computations as for Model 1 provide the solution of $\hat{\beta}$ in terms of $\hat{\alpha}_{x_i}$ as

$$\hat{\beta} = \frac{\sum_{i=1}^n y_i}{\sum_{i=1}^n \hat{\alpha}_{x_i}} \tag{3.16}$$

and the maximum likelihood equations for $\hat{\alpha}_0^*$ and $\hat{\alpha}_1^*$ are given by

$$\sum \log y_i - n \log \hat{\beta} = \sum \Psi(\hat{\alpha}_0^* + \hat{\alpha}_1^* r_i); \tag{3.17}$$

$$\sum r_i \log y_i - \log \hat{\beta} \sum r_i = \sum r_i \Psi(\hat{\alpha}_0^* + \hat{\alpha}_1^* r_i). \tag{3.18}$$

Substituting the value of $\hat{\beta}$ from (3.16), the above equations reduce to two unknowns $\hat{\alpha}_0^*$ and $\hat{\alpha}_1^*$ that can be iteratively solved. We have used Newton-Raphson method along with the formulae for Ψ function in computations that seems quite efficient.

3.4 Model IV

For this model, α_{x_i} is assumed constant denoted by α and we can write $\beta_{x_i} = \beta_0^* + \beta_1^* r_i$. As in model III, we have the following relation to the reciprocal linear model

$$\alpha(\beta_0^* + \beta_1^* r_i) = 1/(\delta_0 + \delta_1 x_i).$$

No further simplification seems possible. For the maximum likelihood solution we have as for model II

$$\sum y_i \hat{\beta}_{x_i}^{-2} - \hat{\alpha}\sum \hat{\beta}_{x_i}^{-1} = 0; \tag{3.19}$$

$$\sum r_i y_i \hat{\beta}_{x_i}^{-2} - \hat{\alpha}\sum r_i \hat{\beta}_{x_i}^{-1} = 0; \tag{3.20}$$

$$\sum \log y_i - \sum \log \hat{\beta}_{x_i} = n \log \Psi(\hat{\alpha}). \tag{3.21}$$

Eliminating $\hat{\alpha}$ from the first two equations gives an equation that along with the other equation gives two unknowns $\hat{\beta}_0^*$ and $\hat{\beta}_1^*$ to solve.

In the example below, we have used iterative method of solution in this case also.

4. AN EXAMPLE

Nelson (1981) reported data on the failure of 40 motorettes with a new class-H insulation material in a motorette test performed at four elevated temperature settings at 190°C; 220°C; 240°C and 260°C: For each test temperature, the 10 motorettes were periodically examined for insulation failure and the given failure time is midway between the inspection time when the failure was found and the time of the previous inspection.

Table 1. Hours to failure for class-H insulation material

190°C	220°C	24°0C	260°C
7228	1764	1175	600
7228	2436	1175	744
7228	2436	1521	744
8448	2436	1569	744
9167	2436	1617	912
9167	2436	1665	1128
9167	3108	1665	1320
9167	3108	1713	1464
10511	3108	1761	1608
10511	3108	1953	1896

Source : Nelson (1971)

4.1 Estimates of the Parameters

For illustrative purposes, we fit the gamma regression model to these data. Nelson (1981) (see also Singpurwala (1972)) used the same data in the Arrhenius model by employing a combination of graphical and analytic techniques based upon the assumptions that the log-failure times are normally distributed with constant variance and the mean depends on temperature. Actually the main purpose of the experiment was to estimate insulation life at 180°C exceeded a minimum requirement. Bhattacharyya and Fries (1982b) fit an inverse Gaussian reciprocal linear model to be adequate for this data. They choose the x values given by $x = 10^{-8}\ (t^3 - 180^3)$, t denoting the temperature in centigrade. Babu and Chaubey (1996) used this data to fit an inverse Gaussian model given by $y_i \sim IG(\delta_i, \gamma)$ where $\mu_{x_i}^{-1} = \delta_i = \delta_0 + \delta_1 x_i$, while demonstrating the resampling procedure in this setting. We used the NLP procedure of SAS (1997) to carry out the maximum likelihood method that provides estimates of the parameters, Hessian matrix, covariance matrix, correlation matrix and confidence limits (the program is available from the authors upon request).

In the table below we summarise the estimates of parameters along with their standard errors.

The estimates for model I have smaller standard errors in general as compared to model II and a similar observation is recorded between model III and model IV. However, this is not enough for model choice; some goodness of fit criterion has to be employed (see Section 4.3).

Table 2. Maximum likelihood estimates of the parameters

Model	Parameters	Estimates	Standard error
I	α_0	0.00433	0.00123
	α_1	0.77118	0.17239
	β	107.42389	24.11420
II	β_0	0.00074	0.00026
	β_1	0.13956	0.03187
	α	19.18338	4.25277
III	α_0^*	4.16547	1.17190
	α_1^*	0.69342	0.15900
	β	124.75500	28.04153
IV	β_0^*	23.07059	8.69414
	β_1^*	5.43162	1.28748
	α	17.19921	3.80919

4.2 Confidence Intervals for Different Parameters

An approximation to 100 (1 – α)% confidence interval for a parameter θ (say) is given by

$$\hat{\theta} \pm Z_{\frac{\alpha}{2}} \left[S.E.\left(\hat{\theta}\right) \right] \tag{4.1}$$

where $Z_{\frac{\alpha}{2}}$ is the 100 (1 – α)-th percentile of the standard normal distribution. The table below gives approximate 90% confidence intervals for the parameters in the four fitted models. It is seen that for all the confidence intervals corresponding to the parameters α_1, α_1^*, β_1, β_1^* that signify the dependence of the mean on x; lower limits are positive. This implies that the assumed models validate the inverse relation between the temperature and mean failure time.

Table 3. Confidence intervals for the parameters

Model	Parameters	Lower limit	Upper limit
I	α_0	0.002307	0.006353
	α_1	0.487600	1.054760
	β	67.756031	147.091749
II	β_0	0.000323	0.001163
	β_1	0.087133	0.191986
	α	12.187573	26.179187
III	α_0^*	2.237695	6.093246
	α_1^*	0.431865	0.954975
	β	78.626683	170.883317
IV	β_0^*	8.768730	37.372450
	β_1^*	3.313715	7.549525
	α	10.933093	23.465328

4.3 Goodness of Fit of the Models

To determine the adequacy of the models we use the predicted values computed from the formula

$$\hat{y}_i = \hat{\mu}_{x_i} = \hat{\alpha}_{x_i}\hat{\beta}_{x_i}, \qquad (4.2)$$

where $\hat{\alpha}_{x_i}$ and $\hat{\beta}_{x_i}$ depend on the model used. For judging the goodness of fit we use the following measures

$$L_1 = \sum_{i=1}^{n} |\hat{y}_i - y_i| \qquad (4.3)$$

$$L_2 = \sum_{i=1}^{n} |\hat{y}_i - y_i|^2 \qquad (4.4)$$

For comparative purpose, we can use Bhattacharyya and Fries (1982b) result or Babu and Chaubey (1996) result with respect to IG modelling. Bhattacharyya and Fries (1982b) find the inverse Gaussian reciprocal linear model to be adequate for this data excluding the 2600C setting, where as Babu and Chaubey (1996) used this data for both batches. We reproduce the estimates from Babu and Chaubey (1996) for the reciprocal linear model

$$\hat{\delta}_0 = 0.03731,\ \hat{\delta}_1 = 7.317285 \qquad (4.5)$$

and use

$$\hat{y}_i = \frac{1}{\hat{\delta}_0 + \hat{\delta}_1 x_i} \qquad (4.6)$$

The table below summarizes the L_1, L_2 measures for different fitted models.

Table 4. L_1 and L_2 measures for different models

Model	L_1	L_2
I	19068.23003	18101706.78
II	18937.04473	18119160.63
III	19651.03725	19547356.61
IV	20946.02746	24074289.70
IG Model	18784.80094	18231932.08

Among the proposed models, model II is the best model according to the L_1 norm, however, model II comes out to be on the top according to L_2 norm. The IG-model comes out to be slightly better than model I according to the L_1 norm. Thus, models I and II may be considered to be competitors of the IG reciprocal linear model. The closeness of models I and II to the IG model is not very surprising as both these models are very similar; the form of the dependence of the mean failure time on x, is the same for both the models, the difference lies in the distribution of the failure times. Models III and IV may not be realistic in retrospect at low temperatures as they imply that at zero temperature, mean failure time is infinite.

4.4 Conclusions

It is observed that the reciprocal linear model for scale parametrization gives the better result than other types according to L_1 norm, but model I, i.e. shape parametrization is slightly better according to L_2 norm. The IG model (Babu and Chaubey 1996) provides best fit amongst all the models considered here according to the L_1 norm but this property is lost when we consider L_2 norm.

Yet another set of alternative models may be built in these situations considering the use of power transformation family of dependent variable before carrying out the regression analysis. Because power transformation family is useful for correcting skewness of the distribution of error terms, unequal error variances and non-linearity of the regression function, this may be an interesting proposition for future research. The models proposed here show very promising for applications for analyzing the relationship between non-negative random variables. However, further investigations through a simulation study may be essential that can provide clearer guidelines indicated which model could be better to fit the data in practice.

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