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Modelling Volatility Influenced by Exogenous Factors using an Improved GARCH-X Model

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SUMMARY

Generalized Autoregressive Conditional Heteroscedastic (GARCH) model has gained popularity since its inception due to its ability to forecast volatility. Usually, GARCH model captures the volatility based on its past volatility and past squared residuals, but does not consider the effect of exogenous variable(s) on the volatility process owing to its univariate nature. In the domain of econometric modelling, where exogenous variables play a crucial role, GARCH model with intervention of exogenous variable(s) is more feasible than the traditional GARCH model. Hence, this study aims to empirically introduce and implement an improved GARCH-X model which can account for the effects of influencing factors (X) both into the mean and variance equation simultaneously of the standard GARCH model. In this manuscript, we have briefly discussed GARCH and GARCH-X models along with their implementation procedure. The proposed model is compared with the traditional GARCH model using domestic price index of edible oils in India along with the influencing factors like foreign exchange rate and international price index of edible oils as exogenous variables (X). Supremacy of using exogenous factors in volatility modelling is concluded from this comparison.

Keywords: Exogenous variables, GARCH model, GARCH-X model, Price index, Volatility.

1. INTRODUCTION

Modelling and forecasting of price indices of agricultural commodities efficiently are of utmost importance to policymakers as well as for various stakeholders. While designing new schemes for boosting the Indian economy, Government needs a reliable forecast of various aspects viz. agriculture, health, economy etc. Time series analysis can contribute immensely to meet this need by providing timely and reliable forecasts. Most commonly used method to analyze linear time series data is Autoregressive Integrated Moving Average Model (ARIMA) model. The model was proposed by Box and Jenkins (1976) and has earned enormous popularity in a very short time due to its power and flexibility (Hoff, 1983). But, with the presence of volatility in the data sets, this model is not very effective to capture the variations in time series. In this recent era, volatility plays the central role in price fluctuations (Black and Scholes, 1973). Volatility is usually quoted as an annualized percentage standard deviation (Alexander, 1998). Measurement of volatility is very much essential to analyze and forecast the financial datasets efficiently. Accurate measurement of food price volatility is important not only because volatility causes uncertainty to producers, consumers, and policymakers, but also costs are inevitably incurred(McMillan, 2003).Uncontrolled factors are the main cause of price and quantity fluctuation of agriculture commodities. This price fluctuation leads to volatility in agricultural markets. To overcome the problem of volatility, Engle in 1982 introduced a non-linear time series model Autoregressive Conditional Heteroscedastic (ARCH) model. An improvement or Generalized version of ARCH model is Generalized Autoregressive Conditional Heteroscedastic model (GARCH), developed by Bollerslev (1986). ARCH introduced the feature of observed autocorrelation in the return volatility of the financial assets and the GARCH model added the general characteristics of conditional heteroscedasticity (Hansen and Lunde, 2001) to it. GARCH models are accepted widely due to their extended applicability to various financial data

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(Engle, 2001). Lama *et al.* (2015) explained GARCH model and exponential GARCH (EGARCH) model along with their estimation procedures for modelling and forecasting of price index series. Some of the applications of GARCH models in agriculture can be found in Paul and Ghosh (2009), Ghosh *et al.* (2010), Paul and Ghosh (2014) and Paul (2015). Few researchers have also questioned the usefulness of the GARCH models for certain data settings. Figlewski (1997), argued that GARCH models were unable to explain much of the variability when evaluated out-of-sample, despite the fact that the GARCH models had good in-sample fit, and concluded that GARCH models were of little value. Andersen *et al.* (2003) argued that standard GARCH models are not as well suited for periods characterized by rapid fluctuations in volatility. These motivated the improvement of GARCH models. Since, the GARCH model is not sufficient to forecast the time series which are influenced by the external factors. Time series model with exogenous variables has the capacity to identify the underlying patterns in time-series data and to quantify the impact of environmental influences. Time series analysis with explanatory variables encompasses methods to model and predict correlated data taking into account additional information (Maçaira *et al.,* 2018). In order to improve the forecast accuracy of GARCH model, the external demand influencing factors (X) needs to be incorporated into the forecasting model. GARCH-X model allows the effect of external variables on the conditional variance to be taken into consideration. This new feature becomes more important to improve GARCH model estimations; especially in the event of unaccounted information from other factors which can affect GARCH estimates. When these external information are not taken into consideration, GARCH model may provide biased estimates of persistence in variance. Several researchers have documented the usefulness of GARCH model with introduction of various covariates (exogenous variables). Brenner *et al.* (1996) and Patton (2001) used interest rate levels, forward-spot spreads and interest rate spreads were used as covariates respectively by Hodrick (1989) and Hagiwara and Herce (1999). Brenner *et al.* (1996) referred this specification of GARCH as the GARCH-X model. A GARCH-X model was introduced by Apergis (1998) to model the relationship between stock prices and certain macroeconomic fundamentals affect stock market volatility. In the above models, the exogenous variables are modelled in variance model of the GARCH process. Anggraeni *et al.* (2014) studied GARCH-X with the introduction of external regressors as proxy to macroeconomic variables of interest and these external regressors are tested both on mean model and variance model of GARCH-X separately. But the exogenous variables can affect both the mean and variance model collectively. Then we have to allow the effect of covariate in both models simultaneously. In this study, this gap has been taken under consideration and introduce another type of GARCH-X model which acknowledge the effect of repressors on both mean and variance model.

In this manuscript we study the domestic price index of edible oil in India which exhibits volatility, with two relevant exogenous variables International price index of the edible oil and India/U.S foreign exchange rate. With the help of these data sets we compare traditional GARCH model with types of GARCH-X models.

2. GARCH MODEL

GARCH process allows lagged conditional variances to enter into the models. GARCH model is a weighted average of past squared residuals and also includes declining weights for the residuals that never reach zero. The GARCH model exhibits adaptive or learning behavior as it progresses. (Bentley, 2012). Let us assume that $y_t, y_{t-1}, y_{t-2}, \ldots, y_2, y_1$ are the observation of time series model. GARCH (p, q) model is conditional distribution of residual given the information available but here the conditional variance is linear function of two lags.

$$\epsilon_t = \sqrt{\sigma^2{}_t}\cdot \varepsilon_t\,; \qquad \epsilon_t/\varphi_{t-1} \sim N(0, \sigma^2{}_t),$$

$$\sigma^2{}_t = a_0 + \sum_{i=1}^{p} a_i \,\epsilon^2{}_{t-i} + \sum_{j=1}^{q} b_j \sigma^2{}_{t-j} \tag{1}$$

where $a_0 > 0,\ a_i \geq 0,\ i = 1,2,\ldots,q$ for all i and $b_j \geq 0,\ \ j = 1,2,\ldots,p$ and $\sum_{i=1}^{q} a_i + \sum_{i=1}^{q} b_j < 1$. ε is distributed i.i.d. with zero mean and unit variance. GARCH model is devoid of autoregressive component. We add $\eta_t = \epsilon^2{}_t - \sigma^2{}_t$ to both sides of the equation to express the model in the form of ARMA model.

$$\epsilon^2{}_t = a_0 + \sum_{i=1}^{\max(p,q)} (a_i + b_j)\, \epsilon^2{}_{t-i} + \sum_{j=1}^{q} b_j \eta_{t-j} \tag{2}$$

Mean model (ARIMA) of the GARCH model can be represented by the following equation

$$\varphi(B)(1-B)^d Y_t = \theta(B)\varepsilon_t \quad (3)$$

where, $\varphi(B) = 1-\varphi_1 B - \varphi_2 B^2 - \cdots - \varphi_p B^p$ = the autoregressive operator of order p;

$\varphi_1, \varphi_2, \ldots, \varphi_p$ = the corresponding autoregressive parameters;

$\theta(B) = 1-\theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q$ = the moving average operator of order q;

$\theta_1, \theta_2, \ldots, \theta_q$ = the associated non-seasonal moving average parameters;

$(1-B)^d$ = the differencing operator of order d to produce stationarity of the dth differenced data;

In the above model equation B is used as a backshift operator on Y_t and is defined as $B^i(Y_t) = B^i(Y_{t-1})$.

Parameters are estimated by Quasi-maximum likelihood estimation (QMLE) technique. Parameter estimates from QMLE are consistent and asymptotically normal under conditions investigated by Ling and McAleer (2003).

3. GARCH-X MODEL

In GARCH-X model the effect of extraneous factors on the conditional variance are taken into consideration. This refinement of GARCH models become more dominant in various practical scenario, especially in the event of an unaccounted information from other factors can effect GARCH estimations. If these additional information are not taken into consideration, GARCH model may provide biased estimates of persistence in variance.

We summarize the process of analysis of GARCH-X model into following four steps:

I. ***Model identification:*** This step involves selection of order of the GARCH (p, q) model. Tsay (1987) argued that through a Box-Jenkins methodological procedure, GARCH (1, 1) model exhibits the best fit and higher order GARCH formulations added complication and also no significant improvements in goodness of fit. We select GARCH (1,1) as predecessor of the GARCH-X model. In our study we include two exogenous variables *viz.* International price index of the edible oil and India/ US foreign exchange rate.

II. Inclusion of exogenous variables: This is the most important step in the process of GARCH-X analysis. In this step, we determine the numbers and types of relevant exogenous variables to be included in the model. There is no specific rule to decide appropriate numbers and types of the exogenous variables. But we can have the preliminary idea from the correlation analysis between the study variable and the exogenous variables. In our study we include two exogenous variables *viz.* International price index of the edible oil and India/ U.S foreign exchange rate. Another major problem is how to include exogenous variables in model.

Exogenous variables can be included in mean model as follows:

Mean model:

$$\varphi(B)(1-B)^d Y_t = \theta(B)\varepsilon_t + \sum_{s=0}^{l}\sum_{k=1}^{r}\gamma_{ks}X_{k,t-s} \quad (4)$$

Variance model:

$$\sigma^2{}_t = a_0 + \sum_{i=1}^{p} a_i \in^2{}_{t-i} + \sum_{j=1}^{q} b_j \sigma^2{}_{t-j}$$

Exogenous variables can be model in variance such as:

Mean model: $\varphi(B)(1-B)^d Y_t = \theta(B)\varepsilon_t$ (5)

Variance model: $\sigma^2{}_t = a_0 + \sum_{i=1}^{p} a_i \in^2{}_{t-i} + \sum_{j=1}^{q} b_j \sigma^2{}_{t-j} + \sum_{s=1}^{l}\sum_{k=1}^{r}\gamma_{ks}X_{k,t-s}$

We propose a new type of GARCH-X model by including the exogenous variables in both the mean and variance model of GARCH model.

Mean model:

$$\varphi(B)(1-B)^d Y_t = \theta(B)\varepsilon_t + \sum_{s=0}^{l}\sum_{k=1}^{r}\gamma_{ks}X_{k,t-s} \quad (6)$$

Variance model:

$$\sigma^2{}_t = a_0 + \sum_{i=1}^{p} a_i \in^2{}_{t-i} + \sum_{j=1}^{q} b_j \sigma^2{}_{t-j} + \sum_{s=1}^{l}\sum_{k=1}^{r}\gamma_{ks}X_{k,t-s}$$

Similar to the GARCH, conditions for stationary of the GARCH-X model are $a_0 > 0$, $a_i \geq 0$, $i = 1,2,\ldots,q$ for all i; $b_j \geq 0$, $j = 1,2,\ldots,p$; $\gamma_{ks} \geq 0$, $k = 1,2,\ldots,r$; $s = 1,2,\ldots,l$ and $\sum_{i=1}^{q} a_i + \sum_{i=1}^{q} b_j < 1$. Here k is the number of exogenous variables and s is the lag length of the exogenous variables. In our study, we take two exogenous variables with no lag for mean model and one lag for variance model. Another important assumption of these model is $a_0 + \gamma_k X_k > 0$, for all t. This assumption extends the practical applicability of this model by allowing to adopts a nonnegative covariate or exogenous variables with parameter conditions of $a_0 > 0, \gamma_k \geq 0,\ k = 1,2,\ldots,n$.

The parameters γ_k indicate the effects of the exogenous variables on the conditional variance of the residuals. If γ_k are positive, this implies that study variable becomes more volatile and effect of exogenous variables on the conditional variance are noteworthy. The effect of each exogenous variable is directly proportional to the magnitude of corresponding γ_k's. Precisely, the presence of exogenous variables in the conditional variance in the model may be utilized to obtain efficient and reliable forecasts of study variable.

III. ***Parameters estimation:*** Similar to the GARCH model, parameters of the GARCH-X model are estimated by Quasi-maximum likelihood estimation (QMLE) technique.

The GARCH-X model is given above where the parameters are collected in $\vartheta = (\omega, \theta)$ where $\omega = a_0$ and $\theta = (a, b, \gamma)$. The full parameter vector is decomposed into $\theta = (a, b, \gamma)$ and the intercept ω. The true, data-generating parameter is denoted $\vartheta_0 = (\omega_0, \theta_0)$, where $\theta_0 = (a_0, b_0, \gamma_0)$ and the associated volatility process $\sigma^2{}_t = \sigma^2{}_t(\vartheta_0)$. We will assume that $E[\log(a_0 \in^2{}_t + b_0)] < 0$, this assumption ensure stationarity.

These are the data-generating parameter values $\vartheta_0 = (\omega_0, \theta_0) \in (\Omega, \Theta)$ which we wish to estimate. QMLE propose to do so through the Gaussian log-likelihood with $\varepsilon\ {}_t \sim iid\ N(0,1)$.

$$L_n(\vartheta) = \sum_{t=1}^{n} l_n(\vartheta), \qquad l_n(\vartheta = -log\ \sigma^2{}_t(\vartheta) - \frac{y_t{}^2}{\sigma^2{}_t(\vartheta)} \tag{7}$$

where $\sigma^2{}_t(\vartheta)$ is the volatility process induced by a given parameter value ϑ. It is assumed to be initialized at some fixed parameter independent value. This will not restrict to be normally distributed and hence $L_n(\vartheta)$ is a quasi-log likelihood function. The QMLE of the parameters is defined as:

$$\hat{\vartheta} = \left(\hat{\omega}, \hat{\theta}\right) = \arg \max_{(\omega,\theta)\in(\Omega,\Theta)} L_n(\omega, \theta) \tag{8}$$

The intercept ω plays a special role since $\hat{\omega}$ will have radically different behavior depending on whether x_t is stationary or not. In fact, in the case where xt is nonstationary, $\hat{\omega}$ could not be identified. However when non-stationarity is generated by exogenous variables, the GARCH-X model can be consistently estimated by QMLE.

IV. ***Diagnosis the fitness of the model:*** This is the final step in process of GARCH-X model analysis. We use various selection criteria (Akaike, Bayes, Shibata and Hannan-Quinn information criteria) to prove the supremacy of the GARCH-X model over the GARCH model and also to find the most suitable model. We check the stability of the estimate of the parameters of these models with the help of Hansen-Nyblom test. We also analyze the forecasting performance of the models by measuring the forecasting accuracy in term of Root mean squared error (RMSE).

4. DATA DESCRIPTION

In this study, we illustrated these models with the help of domestic price index of edible oil in India which exhibit volatility, with two exogenous variables International price index of the edible oil and India/U.S foreign exchange rate. The international edible oils price index and domestic edible oils price index data were collected from the World Bank Commodity Prices Indices (Pink Sheet) and Office of the Economic Adviser respectively, Ministry of Commerce, Government of India. And the data of foreign exchange rate has been taken from International Monetary Fund (IMF), exchange rate. Each series contained 437 data points (from April, 1982 to August, 2018). Domestic price index of a commodity is directly proportional to the quantity export to the abroad (Robert, 2015). This quantity is directly affected by the International price index of that commodity and foreign exchange rate of that country. The exchange rate has an important relationship to the price level because it represents a link between domestic price index and foreign prices (The Exchange Rate and the Price Level; The Department of Economics: University of Toronto, *https://www.economics.utoronto.ca/jfloyd/modules/expl.html*). Hence we select two exogenous variables *viz.* International price index of the edible oils and India/US foreign exchange rate which also have a high correlation with the domestic price index of edible oils of India [see table 2]. In this manuscript, we have used the R software for analyzing the datasets with the help of "t series", "forecast", "f Garch", "aTSA" and "rugarch" packages.

Table 1. Summary statistics of the data series

	Domestic price index	International price index	Foreign exchange rate (Rupees)
Median	76.60	107.55	42.76
Mean	87.63	125.03	37.82
Variance	1605.67	2248.92	297.49
Std.Deviation	40.07	47.42	17.25
Coef.Variance	0.46	0.38	0.46

Table 2. Pearson correlation coefficient between variables

	Correlation domestic price index	p-value (t-test of correlation)
International price index	0.73	< 0.01
Foreign exchange rate	0.92	< 0.01

5. TESTING OF ARCH EFFECT

Before going to fit GARCH model, we have to ensure that the data sets exhibits volatility. ARCH-LM test is used to test the presence of ARCH effect in the dataset.

Table 3. Test of the ARCH effect of the dataset (at lags 4)

*Null hypothesis: There is no ARCH effect present in the data set		
	Test statistics	p-value
Domestic price index	1010.4	< 0.01

From the table 3, we can statistically infer that the dataset of domestic price index of edible oil exhibits the effect of volatility. We can also visualize the effect of volatility in the datasets in Fig. 1. We also tested seasonality of the data series, but there is no evidence of seasonality in our study variables.

6. FITTING OF GARCH AND GARCH-X MODEL

First, we would fit the traditional GARCH model. Then, we use the GARCH-X model with two external variables and compare it with the traditional GARCH model. These external variables are tested on mean model, variance model and both mean model and variance model.

Insertion of external variables in the model shows that exogenous regressors have significant influence on the conditional variance of the model. We can notice that in GARCH-X (in mean model), both exogenous variables in mean model have significant impact. In GARCH-X (in variance model), both the regressors are failed to draw any mark on the model. In GARCH-X (in both mean and variance model), result in portion of mean model in similar to GARCH-X (in mean model) and in portion of variance model, only regressors 2 (foreign exchange rate) has some impact on the model.

However, the parameter estimates value of the exogenous variable 2 (foreign exchange rate) is relatively large as compare exogenous variable 1 (International price index). Total of the parameter estimates of exogenous variable 1 and exogenous variable 2 is larger than total of alpha and beta of the model. Thus, exogenous variables are an important factor in effecting the phase of volatility.

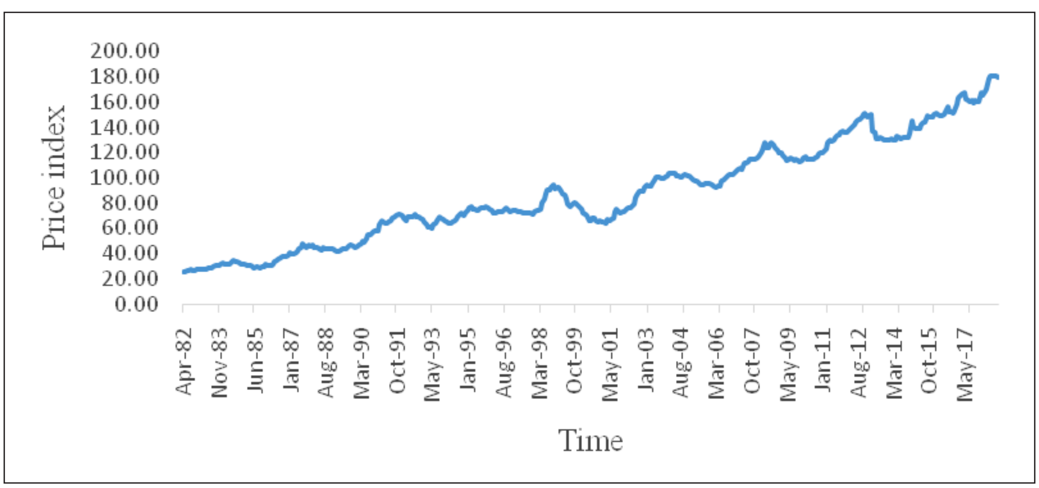

Fig. 1. Time plot of the domestic price index of edible oil dataset

Table 4. Estimate of parameters of GARCH model and different GARCH-X model

	Mu	**Mean Exo-1**	**Mean Exo-2**	**Omega**	**Alpha**	**Beta**	**Var Exo-1**	**Var Exo-2**
GARCH	72.91	….	….	1.24	0.97	0.02	….	….
	0.42	**….**	**….**	**0.52**	**0.08**	**0.05**	**….**	**….**
GARCH-X (in mean model)	-1.26	0.35	1.11	3.36	0.77	0.22	….	….
	0.92	**0.01**	**0.02**	**1.50**	**0.08**	**0.06**	**….**	**….**
GARCH-X (in variance model)	72.90	….	….	1.20	1.00	0.01	0.00 (≈0.01)	0.00 (≈0.01)
	0.43	**….**	**….**	**12.71**	**0.10**	**0.08**	**0.08**	**0.13**
GARCH-X (in both mean and variance model)	-1.32	0.34	1.11	0.00 (≈0.01)	0.84	0.15	0.00 (≈0.01)	0.15
	0.99	**0.01**	**0.03**	**0.00 (<0.01)**	**0.11**	**0.06**	**0.03**	**0.12**

***Bold numbers are the standard errors of the corresponding parameters**

Now we compare these models on the basis of various model selection criteria. Comparison of these models based on the model information criteria is presented in the table 5.

Table 5. Various information criteria of the models

Information Criteria	**GARCH**	**GARCH-X (in mean model)**	**GARCH-X (in variance model)**	**GARCH-X (in both mean and variance model)**
Akaike Information Criteria	8.89	7.45	8.90	**7.44**
Bayes Information Criteria	8.93	7.51	8.95	**7.51**
Shibata Information Criteria	8.89	7.45	8.90	**7.44**
Hannan-Quinn Information Criteria	8.90	7.48	8.92	**7.47**

Based on various model information criteria GARCH-X (in both mean and variance model)is most suitable as it has the smallest value in the all selection criteria (Table 5). GARCH-X (in mean model) is also has smallest value in Bayes information criteria but a very little large value in rest of the selection criteria. However, GARCH-X model has the smaller value in all information criteria compare to the GARCH model. Hence it seems like external regressors are important in some ways toward improving volatility estimations.

Structural instability is pervasive in economic time series relationships, and it can be quite perilous to ignore. Inferences about economic relationships can go astray, forecasts can be inaccurate, and policy recommendations can be misleading or worse (Hansen; 2001). Model stability is very important for prediction and econometric inference, because a parametric statistical model is completely described by its parameters; model stability is equivalent to parameter stability. Now we test the stability of the estimate of the parameters of these models with the help of the Hansen-Nyblom tests. Hansen-Nyblom test is extensions of Chow test (Chow; 1960) to nonlinear maximum likelihood and general econometric problems. This test was developed by Hansen (1992) and Nyblom (1989). The Hansen-Nyblom test, tests the null hypothesis that all parameters are constant against the alternative that some of the parameters are unstable.

Table 6. Hansen-Nyblom test of the models

	GARCH	**GARCH-X (in mean model)**	**GARCH-X (in variance model)**	**GARCH-X (in both mean and variance model)**
Joint statistics	1.47	4.80	4.81	7.06
Individual statistics				
Mu	0.93	0.38	0.93	0.40
Mean Exo-1	….	0.38	….	0.50
Mean Exo-2	….	0.09	….	0.15
Omega	0.03	1.11	0.03	1.63
Alpha	0.33	0.53	0.19	0.58
Beta	0.09	0.59	0.09	0.73
Var Exo-1	….	….	0.07	0.90
Var Exo-2	….	….	0.03	0.72
Asymptotic Critical Value (at 1% level of significance)				
Joint statistics	1.6	2.12	2.12	2.59
Individual statistics	0.75	0.75	0.75	0.75

From the table 6, we found the joint statistics of the parameters of the GARCH-X model are not stable, whereas joint statistics of the parameters of the GARCH model is stable. But if check the stability of the individual parameters in models, we can notice that only one parameter is unstable in GARCH-X (in mean model) and GARCH-X (in variance model) as similar to GARCH model. In GARCH-X (in both mean and variance model), only two parameters (omega and Var Exo-1) are unstable. If examine deeply the GARCH-X (in both mean and variance model) in table 4, we realize that these unstable parameters have no significant impact on the model.

It is paramount to analyze these models on the basis of their forecasting performance. We compare these model's forecasting accuracy by their measuring their Root mean squared error (RMSE) value.

Table 7. Comparison of RMSE of the models

	GARCH	GARCH-X (in mean model)	GARCH-X (in variance model)	GARCH-X (in both mean and variance model)
RMSE	42.64	19.27	42.65	19.25

From the table 7, it is clearly visible that GARCH-X (in both mean and variance model) has the minimum RMSE value as compare to rest of the models presented in this study.

Table 8. Diebold-Mariano Test between GARCH and GARCH-X model

*Null hypothesis: The two forecasts have the same accuracy	
Test statistics	p-value
15.18	< 0.01

We use the Diebold-Mariano test to determine whether prediction is significantly different or not based on residuals generated by the models. The null hypothesis is that the two forecasts have the same accuracy. The alternative hypothesis is taken as the first forecast is less accurate than the second forecast. The results show that GARCH-X model produces better forecast than the GARCH model. In the Fig. 2 shows the predicted values (proposed GARCH-X model) against actual values plot and gives us a picture of model performance.

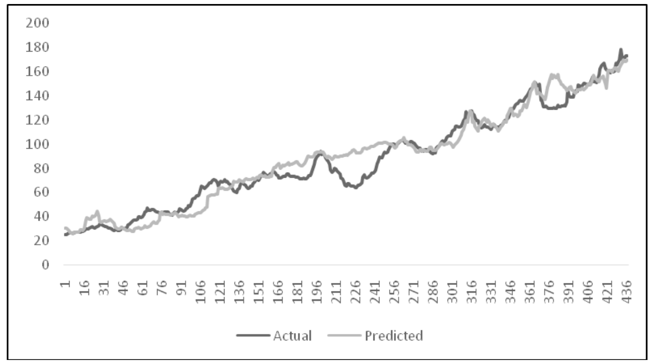

Fig. 2. Predicted against actual plot

7. CONCLUSIONS

This manuscript investigated the volatility behavior of domestic price index of edible oil and the potential impact of exogenous variables *viz.* International price index of edible oil and India/U.S. foreign exchange rate on the volatility of our study variable. Our study shows that the volatility of domestic price index of edible oil is driven more by external variables than its own-variance. Inclusion of relevant external variables into GARCH-X model are examined to be capable to enhance the overall volatility estimation. We empirically compare the GARCH model and types of GARCH-X models. From this empirical study, we can infer that the GARCH-X (in both mean and variance model) establish its supremacy over GARCH model, GARCH-X (in mean model) and GARCH-X (in variance model). The findings of this study have provided direct support for the potential use of accurate forecasts in decision-making for the wholesalers, retailers, stakeholders as well as government of India for policy making. Further research can be done on the optimum criteria to select numbers and types of exogenous variables to include in the GARCH-X model. Stability of estimates of parameters of the GARCH-X model is also a crucial lacuna which can be addressed in future.

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