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New Quartile based Variant of the Ranked Set Sampling Scheme

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SUMMARY

A new variant of ranked set sampling namely Stratified Balanced Groups Quartile Ranked Set Sampling (SBGQRSS) is proposed for estimating the population mean with samples of size *m=3k (k=1,2,3,...)* drawn independently from each stratum. The SBGQRSS scheme yields an unbiased estimator for symmetric distributions and biased but consistent estimators for asymmetric distributions. The performance of the mean estimator based on SBGQRSS is compared with simple random sampling (SRS) theoretically and by simulation studies as well. The simulation study revealed an increased efficiency of SBGQRSS over ranked set sampling (RSS), and balanced groups ranked set sampling (BGRSS) to estimate the population mean. A real data set on the parameters of high-density plantation (HDP) apple is used to illustrate the methodology of the SBGQRSS scheme.

Keywords: Ranked set sampling, Balanced groups ranked set sampling, Population mean, High-density plantation, Simulation.

1. INTRODUCTION

The efficiency of sampling design concerning effectiveness, exactness, and speed can be ensured by utilizing the information of an auxiliary variable which is correlated to the variable of interest. When the information is taken on every unit in the population, then in such situations some of the variables which are known for every unit of the population are employed to improve the sampling plan or to enhance estimation of the variables of interest. Such variables are termed as auxiliary variables or concomitant variables. The problem of improving the estimators of unknown parameters of interest using the information on auxiliary variables, highly correlated with the study variable, has received considerable attention of statisticians in survey sampling and practice. The information on the auxiliary variable may be available in one form or the other or can be made available by diverting a part of the survey resources at a moderate cost. In whatever form, the information on auxiliary variables is available, one may always utilize it to devise sampling strategies that are better than those in which no auxiliary information is used.

Let $X_1,\ X_2,\ \cdots,\ X_m$ be a simple random sample of size *m* selected from a probability density function (pdf) $f(x)$ and a cumulative distribution function (CDF) $F(x)$, with a finite population mean μ and variance σ^2. Let $X_{11h},\ X_{12h},\ \cdots,\ X_{1mh};\ X_{21h},\ X_{22h},\ \cdots,\ X_{2mh};\ \ldots;\ X_{m1h},\ X_{m2h},\ \cdots,\ X_{mmh}$ be *m* independent simple random samples each of size *m* in the h^{th} cycle $(h=1,2,\ldots,n)$. The usual SRS estimator of the population mean is (Kamarulzaman Ibrahim, 2011):

$\bar{X}_{SRS} = \frac{1}{mn}\sum_{h=1}^{n}\sum_{i=1}^{m} X_{ih}$, with variance given by $Var\left(\bar{X}_{SRS}\right) = \frac{\sigma^2}{mn}$.

Let $X_{i(1:m)h},\ X_{i(2:m)h},\ \ldots,\ X_{i(m:m)h}$ be the order statistics of the i^{th} sample $X_{i1h},\ X_{i2h},\ \cdots,\ X_{imh}$, $(i=1,2,\ldots,m)$ in the h^{th} cycle $(h=1,2,\ldots,n)$. The CDF, PDF, mean, and variance of the i^{th} order statistics $X_{(i:m)}$, respectively, are defined by:

$$F_{(i:m)}(x) = \binom{m}{i}\int_0^{F(x)} v^{i-1}(1-v)^{m-i}\,dv,$$

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$$f_{(i:m)}(x) = \binom{m}{i}[F(x)]^{i-1}[1-F(x)]^{m-i} f(x),$$

$$\mu_{(i:m)} = \int_{-\infty}^{\infty} x f_{(i:m)}(x)dx \text{ and}$$

$$\sigma^2_{(i:m)} = \int_{-\infty}^{\infty} \left(x-\mu_{(i:m)}\right)^2 f_{(i:m)}(x)dx$$

The ranked set sampling method can be described as follows:

Step 1: Randomly chose m^2 units from the target population.

Step 2: Allocate the m^2 selected units as randomly as possible into *m* sets, each of size *m*.

Step 3: Without yet knowing any values for the study variable, rank the units within each set with respect to variable of interest based on professional judgment or a concomitant variable that is correlated with the variable of interest.

Step 4: Choose a sample for actual measurement by including the i^{th} smallest ranked unit of the i^{th} set $(i = 1, 2, ..., m)$ to obtain a sample of size *m*.

Step 5: Repeat Steps 1 to 4 for ***k*** cycles to obtain a sample of size *mk* for an actual measurement.

Then, $X_{1(1:m)h}$, $X_{2(2:m)h}$, …, $X_{m(m:m)h}$ denote the measured RSS. The usual RSS estimator of the population mean is given by

$$\bar{X}_{RSS} = \frac{1}{mk}\sum_{h=1}^{k}\sum_{i=1}^{m} X_{i(i:m)h}, \quad \text{with variance}$$

$$Var\left(\bar{X}_{RSS}\right) = \frac{\sigma^2}{mk} - \frac{1}{km^2}\sum_{i=1}^{m}\left(\mu_{(i:m)}-\mu\right)^2.$$

(Takahasi and Wakimoto 1968) showed that in degenerate distributions the efficiency of RSS relative to SRS is

$$1 \le eff\left(\bar{X}_{RSS}, \bar{X}_{SRS}\right) = \frac{Var\left(\bar{X}_{SRS}\right)}{Var\left(\bar{X}_{RSS}\right)} \le \frac{m+1}{2},$$

Also, Takahasi and Wakimoto (1968) showed that

$$f(x) = \frac{1}{m}\sum_{i=1}^{m} f_{(i:m)}(x), \quad \mu = \frac{1}{m}\sum_{i=1}^{m}\mu_{(i:m)}, \quad \text{and}$$

$$\sigma^2 = \frac{1}{m}\sum_{i=1}^{m}\sigma^2_{(i:m)} + \frac{1}{m}\sum_{i=1}^{m}\left(\mu_{(i:m)}-\mu\right)^2.$$

Similarly, the balanced groups ranked set sampling (Jemain *et al.*, 2008) can be described as:

Step 1: Randomly choose $m = 3k$ sets, where $k = 1,2,3,\ldots$ each of size *m* from the target population, and rank the units within each set with respect to the variable of interest.

Step 2: Allocate the 3k selected sets randomly into three groups, each group consisting of k sets and each set having a size equal to *m*.

Step 3: For each 3k sets in step (2), select for measurement the lowest rank unit from each set in the first group, the median from each set in the second group, and the largest rank unit from each set in the third group respectively. By this way, we have selected a sample of size $m = 3k$ units in one cycle. Steps 1-3 can be repeated *n* times to increase the sample size to $m = 3kn$.

If the sample size is odd, the BGRSS estimator of the population mean μ is

$$\hat{\mu}_{BGRSSO} = \frac{1}{m}\left(\sum_{i=1}^{k} X_{i(1:m)} + \sum_{i=k+1}^{2k} X_{i\left(\frac{m+1}{2}:m\right)} + \sum_{i=2k+1}^{3k} X_{i(m:m)}\right)$$

with variance

$$Var\left(\hat{\mu}_{BGRSSO}\right) = \frac{1}{9k^2}\left(\begin{array}{l}\sum_{i=1}^{k} Var\left(X_{i(1:m)}\right) + \sum_{i=k+1}^{2k} Var\left(X_{i\left(\frac{m+1}{2}:m\right)}\right) \\ + \sum_{i=2k+1}^{3k} Var\left(X_{i(m:m)}\right)\end{array}\right)$$

If the sample size is even, the BGRSS estimator of the population mean is

$$\hat{\mu}_{BGRSSE} = \frac{1}{m}\left(\sum_{i=1}^{k} X_{i(1:m)} + \sum_{i=k+1}^{2k}\left(\frac{1}{2}\left(X_{i\left(\frac{m}{2}:m\right)} + X_{i\left(\frac{m+2}{2}:m\right)}\right)\right) + \sum_{i=2k+1}^{3k} X_{i(m:m)}\right)$$

with variance

$$\mathrm{Var}(\hat{\mu}_{BGRSSE}) = \frac{1}{9k^2}\left\{\sum_{i=1}^{k} Var\left(X_{i(1:m)}\right) + \sum_{i=k+1}^{2k} Var\left(\frac{1}{2}\left[X_{i(\frac{m}{2}:m)} + X_{i(\frac{m+2}{2}:m)}\right]\right) + \sum_{i=2k+1}^{3k} Var\left(X_{i(m:m)}\right)\right\}$$

In order to increase the efficiency for estimating various population parameters using available information on the auxiliary variate, variants of RSS scheme like Extreme ranked set sampling (ERSS) (Samawi *et al.*, 1996), Median ranked set sampling (MRSS) (Muttlak, 1997), Multistage median ranked set sampling (MMRSS) for estimating the population mean and median (Jemain *et al.* 2006, 2007), Percentile double ranked set sampling (PDRSS) (Al-Omari and Jaber 2008), Paired double ranked set sampling (Haq *et al.* 2016), Mixed ranked set sampling (MxRSS) (Haq *et al.* 2014), Neoteric ranked sampling (Zamanzade and Al-Omari 2016), Stratified percentile ranked set sampling method for estimating the population mean (Al-Omari *et al.* 2011), Stratified quartile ranked set sampling and double percentile RSS methods for estimating the population mean (Syam *et al.* 2012, 2013) have been developed. In addition to estimation procedures, goodness of-fit-tests for Laplace distribution have been investigated in ranked set sampling (Al-Omari and Zamanzade 2017), entropy estimators of a continuous random variable in ranked set sampling (Al-Omari and Haq 2019) have been proposed, EDF-based tests of exponentiality in pair ranked set sampling (Zamanzade 2018) have been carried out, reliability estimation in multistage ranked set sampling (Mahdizadeh and Zamanzade 2017) and many other works have been carried out using the established schemes. Motivated by the already developed schemes a new variant of RSS is proposed and is named as "Stratified Balanced Group Quartile Ranked Set Sampling" and abbreviated as SBGQRSS. It is a new modified form of ranked set sampling method suggested for estimating the population mean of both symmetric as well as asymmetric distributions. In this scheme, the groups are balanced in the sense that the same number of observations are taken from amongst each of the groups using the quartiles and each of the groups comprises of the same number of sets and the same number of units.

In the subsequent sections of this paper, a brief methodology for the proposed scheme is described for selecting a representative sample of units using a hypothetical example. The estimators of the population mean for both odd and even cases along with their variances are derived. In the last section, the simulation study to illustrate the efficiency of the proposed scheme for various distributions is given and the application of the proposed scheme is described using a real dataset on High density apple.

2. THE SUGGESTED SBGQRSS

The stratified balanced group quartile ranked set sampling may be described as follows:

1. Divide the population of sampling units into population sub-groups, called strata denoted by L using a concomitant or auxiliary variable as a basis for stratification. For full benefit from stratification, the size of the h^{th} subpopulation, denoted by N_h for $h = 1,2,..., L$, must be known.
2. Randomly select $m_h = 3k$, where $k = 1,2,...$ and $h = 1,2,..., L$ sets each of size m_h from each stratum, and rank the units within each set with respect to the variable of interest. Allocate the $3k$ selected sets randomly into three groups, each of size k sets, which are called as the balanced groups. Here, k is any positive integer. However, for practical purposes, k should be small in order to have a small sample size, so that the ranking is easy and errors in the ranking are reduced.
3. For each group in step (2), select for measurement from the first group the $q_1(m_h + 1)^{th}$ smallest ranked unit, from the second group the $q_2(m_h + 1)^{th}$ ranked unit and from the third group select the $q_3(m_h + 1)^{th}$ largest ranked unit in case m_h is odd. If m_h is even, select for measurement from the first group the $q_1(m_h + 1)^{th}$ smallest ranked unit, from the second group the $\frac{1}{2}\{q_2(m_h) + q_2(m_h + 1)\}^{th}$ ranked unit and from the third group select the $q_3(m_h + 1)^{th}$ largest ranked unit. This step involves the use of quartiles to obtain a sample from the defined balanced groups. Note that we always take the nearest integer of $q_1(m_h + 1)^{th}$, $q_2(m_h + 1)^{th}$, $q_2(m_h)^{th}$ and $q_3(m_h + 1)^{th}$ where $q_1 = 0.25$, $q_2 = 0.50$ and $q_3 = 0.75$. Since in the SBGQRSS scheme we choose $m_h = 3k$, so the minimum set size defined is 3 and using $(m_h + 1)^{th}$ term in the product of q_1, q_2 and q_3 allows to choose exactly the first, second and the third unit from the defined groups and in this way, the SBGQRSS is reduced to the original RSS procedure. The selection of the $\frac{1}{2}\{q_2(m_h) + q_2(m_h + 1)\}^{th}$ ranked unit from the second group in case of even m_h is aimed to select the median unit. The basic advantage of selecting the median unit is that it is not skewed so much by a small proportion of extremely large or small values. Because of this, the median is of central importance in robust statistics, as it is the most resistant statistic, having a breakdown point

of 50%: so long as no more than half the data are contaminated, the median will not give an arbitrarily large or small result. Selecting the median ranked unit also results in the reduction in MSE towards the estimation of the population mean.

In this way, we have a measured sample of size $m_h = 3k$ units in one cycle from each stratum. The Steps (2-3) can be repeated v times to increase the sample size to $3kv$ out of $9k^2v$ units from each stratum. The final sample size from each stratum using SBGQRSS is $m=\sum_{h=1}^{L} m_h$ for one cycle.

The SBGQRSS differs from the usual RSS (Mclyntre 1952), QRSS (Muttlak 2003), and the BGRSS (Jemain *et al.* 2008) methods. In the usual RSS, we identify and measure the i^{th} smallest ranked unit of the i^{th} sample $(i=1,2,..., m)$. In the QRSS method given by Muttlak, if the sample size n is even, from the first $n/2$ samples the $q_1(n+1)^{th}$ smallest ranked unit, and from the second $n/2$ samples the $q_3(n+1)^{th}$ smallest ranked unit are selected for measurement. If the sample size n is odd, from the first $(n-1)/2$ samples the $q_1(n+1)^{th}$ smallest ranked unit, and from the last $(n-1)/2$ samples the $q_3(n+1)^{th}$ smallest ranked unit and from the remaining sample the median ranked unit is selected for measurement. In the BGRSS method, the measured units consist of $m/3$ minima, $m/3$ medians, and $m/3$ maxima. But in SBGQRSS method, the population is stratified based on a concomitant variable, and a random sample from each of the strata is divided into 3 balanced groups. Furthermore, the $q_1(m_h+1)^{\text{th}}$ unit is selected from the 1^{st} group, the $q_2(m_h+1)^{\text{th}}$ unit from the 2^{nd} group and $q_3(m_h+1)^{\text{th}}$ unit from the 3^{rd} group respectively for an odd sample size, while in case of an even sample size the selection of the ordered units from the first and the third groups remains the same and only in the second group the $\frac{1}{2}\{q_2(m_h)+q_2(m_h+1)\}^{th}$ ranked unit is selected. Thus, if the quartiles are used to select the sample units from the balanced groups of each stratum then the whole procedure is called as stratified balanced group quartile ranked set sampling. To illustrate the SBGQRSS method, consider the following example.

2.1 Example

Suppose that we have two strata, *i.e.* $L=2$ and $h=1,2$. Let $X_{ih(q_1(m_h+1))}$, $X_{ih(q_2(m_h))}$, $X_{ih(q_2(m_h+1))}$ and $X_{ih(q_3(m_h+1))}$ be the $q_1(m_h+1)^{th}$, $q_2(m_h)^{th}$, $q_2(m_h+1)^{th}$ and $q_3(m_h+1)^{th}$ order statistics, respectively, of the i^{th} sample in the h^{th} stratum. Assume that from the first stratum we select a sample of size $m_1=6$ and from the second stratum we want a sample of size $m_2=9$. Then the process as shown below:

Stratum 1: If $k=2$, then $m_1=6$. Now, select 6 samples each of size 6, so that we have 6 sets each of size 6 as follows:

$$\{X_{11(1:6)}, X_{11(2:6)}, X_{11(3:6)}, X_{11(4:6)}, X_{11(5:6)}, X_{11(6:6)}\}$$
$$\{X_{21(1:6)}, X_{21(2:6)}, X_{21(3:6)}, X_{21(4:6)}, X_{21(5:6)}, X_{21(6:6)}\}$$
$$\{X_{31(1:6)}, X_{31(2:6)}, X_{31(3:6)}, X_{31(4:6)}, X_{31(5:6)}, X_{31(6:6)}\}$$
$$\{X_{41(1:6)}, X_{41(2:6)}, X_{41(3:6)}, X_{41(4:6)}, X_{41(5:6)}, X_{41(6:6)}\}$$
$$\{X_{51(1:6)}, X_{51(2:6)}, X_{51(3:6)}, X_{51(4:6)}, X_{51(5:6)}, X_{51(6:6)}\}$$
$$\{X_{61(1:6)}, X_{61(2:6)}, X_{61(3:6)}, X_{61(4:6)}, X_{61(5:6)}, X_{61(6:6)}\}$$

Now, first, rank the units within each set with respect to a variable of interest using an auxiliary variable, and then allocate them into 3 groups where each group contains two sets of size six units. The groups appear as shown below:

1st GROUP:

$$A_1 = \{X_{11(1:6)}, X_{11(2:6)}, X_{11(3:6)}, X_{11(4:6)}, X_{11(5:6)}, X_{11(6:6)}\}$$
$$A_2 = \{X_{21(1:6)}, X_{21(2:6)}, X_{21(3:6)}, X_{21(4:6)}, X_{21(5:6)}, X_{21(6:6)}\}$$

2nd GROUP:

$$A_3 = \{X_{31(1:6)}, X_{31(2:6)}, X_{31(3:6)}, X_{31(4:6)}, X_{31(5:6)}, X_{31(6:6)}\}$$
$$A_4 = \{X_{41(1:6)}, X_{41(2:6)}, X_{41(3:6)}, X_{41(4:6)}, X_{41(5:6)}, X_{41(6:6)}\}$$

3rd GROUP:

$$A_5 = \{X_{51(1:6)}, X_{51(2:6)}, X_{51(3:6)}, X_{51(4:6)}, X_{51(5:6)}, X_{51(6:6)}\}$$
$$A_6 = \{X_{61(1:6)}, X_{61(2:6)}, X_{61(3:6)}, X_{61(4:6)}, X_{61(5:6)}, X_{61(6:6)}\}$$

For h=1, select $X_{i1(q_1(m_1+1):6)}$ units from the first group for $i=1,2$, average of $X_{i1(q_2(m_j):6)}$ and $X_{i1(q_2(m_j+1):6)}$ units from the second group for $i=3,4$ and $X_{i1(q_3(m_j+1):6)}$ units from the third group for $i=5,6$. Take the nearest integer of $q_1(m_1+1)$, $q_2(m_1)$, $q_2(m_1+1)$, and $q_3(m_1+1)$ where $q_1=0.25$, $q_2=0.50$, and $q_3=0.75$.

Thus, from the first stratum,

$$\left\{X_{11(2:6)},\ X_{21(2:6)}, \frac{1}{2}(X_{31(3:6)},\ X_{31(4:6)}), \frac{1}{2}(X_{41(3:6)},\ X_{41(4:6)}),\ X_{51(5:6)},\ X_{61(5:6)}\right\}$$

is an SBGQRS sample of size 6.

Stratum 2: If $k=3$, then $m_2=9$. Now, select 9 samples each of size 9, so that we have 9 sets each of size 9 as follows:

$$\{X_{12(1:9)}, X_{12(2:9)}, X_{12(3:9)}, X_{12(4:9)}, X_{12(5:9)}, X_{12(6:9)}, X_{12(7:9)}, X_{12(8:9)}, X_{12(9:9)}\}$$

$$\{X_{22(1:9)}, X_{22(2:9)}, X_{22(3:9)}, X_{22(4:9)}, X_{22(5:9)}, X_{22(6:9)}, X_{22(7:9)}, X_{22(8:9)}, X_{22(9:9)}\}$$

$$\{X_{32(1:9)}, X_{32(2:9)}, X_{32(3:9)}, X_{32(4:9)}, X_{32(5:9)}, X_{32(6:9)}, X_{32(7:9)}, X_{32(8:9)}, X_{32(9:9)}\}$$

$$\{X_{42(1:9)}, X_{42(2:9)}, X_{42(3:9)}, X_{42(4:9)}, X_{42(5:9)}, X_{42(6:9)}, X_{42(7:9)}, X_{42(8:9)}, X_{42(9:9)}\}$$

$$\{X_{52(1:9)}, X_{52(2:9)}, X_{52(3:9)}, X_{52(4:9)}, X_{52(5:9)}, X_{52(6:9)}, X_{52(7:9)}, X_{52(8:9)}, X_{52(9:9)}\}$$

$$\{X_{62(1:9)}, X_{62(2:9)}, X_{62(3:9)}, X_{62(4:9)}, X_{62(5:9)}, X_{62(6:9)}, X_{62(7:9)}, X_{62(8:9)}, X_{62(9:9)}\}$$

$$\{X_{72(1:9)}, X_{72(2:9)}, X_{72(3:9)}, X_{72(4:9)}, X_{72(5:9)}, X_{72(6:9)}, X_{72(7:9)}, X_{72(8:9)}, X_{72(9:9)}\}$$

$$\{X_{82(1:9)}, X_{82(2:9)}, X_{82(3:9)}, X_{82(4:9)}, X_{82(5:9)}, X_{82(6:9)}, X_{82(7:9)}, X_{82(8:9)}, X_{82(9:9)}\}$$

$$\{X_{92(1:9)}, X_{92(2:9)}, X_{92(3:9)}, X_{92(4:9)}, X_{92(5:9)}, X_{92(6:9)}, X_{92(7:9)}, X_{92(8:9)}, X_{92(9:9)}\}$$

Now, rank the unit within each set with respect to a variable of interest, and then allocate them into 3 groups where each contains three sets each of size nine units. After ranking the groups appear as shown below:

1st GROUP:

$$A_1 = \{X_{12(1:9)}, X_{12(2:9)}, X_{12(3:9)}, X_{12(4:9)}, X_{12(5:9)}, X_{12(6:9)}, X_{12(7:9)}, X_{12(8:9)}, X_{12(9:9)}\}$$

$$A_2 = \{X_{22(1:9)}, X_{22(2:9)}, X_{22(3:9)}, X_{22(4:9)}, X_{22(5:9)}, X_{22(6:9)}, X_{22(7:9)}, X_{22(8:9)}, X_{22(9:9)}\}$$

$$A_3 = \{X_{32(1:9)}, X_{32(2:9)}, X_{32(3:9)}, X_{32(4:9)}, X_{32(5:9)}, X_{32(6:9)}, X_{32(7:9)}, X_{32(8:9)}, X_{32(9:9)}\}$$

2nd GROUP:

$$A_4 = \{X_{42(1:9)}, X_{42(2:9)}, X_{42(3:9)}, X_{42(4:9)}, X_{42(5:9)}, X_{42(6:9)}, X_{42(7:9)}, X_{42(8:9)}, X_{42(9:9)}\}$$

$$A_5 = \{X_{52(1:9)}, X_{52(2:9)}, X_{52(3:9)}, X_{52(4:9)}, X_{52(5:9)}, X_{52(6:9)}, X_{52(7:9)}, X_{52(8:9)}, X_{52(9:9)}\}$$

$$A_6 = \{X_{62(1:9)}, X_{62(2:9)}, X_{62(3:9)}, X_{62(4:9)}, X_{62(5:9)}, X_{62(6:9)}, X_{62(7:9)}, X_{62(8:9)}, X_{62(9:9)}\}$$

3rd GROUP:

$$A_7 = \{X_{72(1:9)}, X_{72(2:9)}, X_{72(3:9)}, X_{72(4:9)}, X_{72(5:9)}, X_{72(6:9)}, X_{72(7:9)}, X_{72(8:9)}, X_{72(9:9)}\}$$

$$A_8 = \{X_{82(1:9)}, X_{82(2:9)}, X_{82(3:9)}, X_{82(4:9)}, X_{82(5:9)}, X_{82(6:9)}, X_{82(7:9)}, X_{82(8:9)}, X_{82(9:9)}\}$$

$$A_9 = \{X_{92(1:9)}, X_{92(2:9)}, X_{92(3:9)}, X_{92(4:9)}, X_{92(5:9)}, X_{92(6:9)}, X_{92(7:9)}, X_{92(8:9)}, X_{92(9:9)}\}$$

For $h=2$, select $X_{i2(q_1(m_2+1):9)}$ units from the first group *i.e.* for $i=1,2,3$, $X_{i2(q_2(m_2+1):9)}$ units from the second group *i.e.* for $i=4,5,6$ and $X_{i2(q_3(m_2+1):9)}$ units from the third group for $i=7,8,9$. Take the nearest integer of $q_1(m_2+1)$, $q_2(m_2+1)$ and $q_3(m_2+1)$ where $q_1=0.25$, $q_2=0.50$ and $q_3=0.75$.

Thus, from the second stratum, we have;

$$\{X_{12(3:9)}, X_{22(3:9)}, X_{32(3:9)}, X_{42(5:9)}, X_{52(5:9)}, X_{62(5:9)}, X_{72(8:9)}, X_{82(8:9)}, X_{92(8:9)}\}$$

Therefore, the SBGQRSS units are:

$$\left\{\begin{array}{c} X_{11(2:6)}, X_{21(2:6)}, \frac{1}{2}(X_{31(3:6)}, X_{31(4:6)}), \\ X_{61(5:6)}, X_{12(3:9)}, X_{22(3:9)}, X_{32(3:9)}, X_{42(5:9)}, \\ \frac{1}{2}(X_{41(3:6)}, X_{41(4:6)}), X_{51(5:6)}, \\ X_{52(5:9)}, X_{62(5:9)}, X_{72(8:9)}, X_{82(8:9)}, X_{92(8:9)} \end{array}\right\}$$

Thus, we get a final SBGQRSS sample of size m equal to $\sum_{h=1}^{2} m_h = m_1 + m_2 = 6 + 9 = 15$. The mean of these units is used as an estimator of the population mean.

3. ESTIMATION OF POPULATION MEAN

The stratified balanced group quartile ranked set sampling estimator of the population mean is defined as:

CASE 1: When m_h is odd *i.e.* for $k = 1, 3, 5, 7, \ldots$

$$\bar{X}_{SBGQRSSO} = \sum_{h=1}^{L} \frac{W_h}{m_h} \left[\sum_{i=1}^{k} X_{ih(q_1(m_h+1):m_h)} + \sum_{i=k+1}^{2k} X_{ih(q_2(m_h+1):m_h)} + \sum_{i=2k+1}^{3k} X_{ih(q_3(m_h+1):m_h)} \right]$$

where, $W_h = \frac{N_h}{N}$, N_h is the h^{th} stratum size and N is the total population size. The variance of SBGQRSSO is given by:

$$Var(\bar{X}_{SBGQRSSO}) = Var\left[\sum_{h=1}^{L} \frac{W_h}{m_h} \left[\sum_{i=1}^{k} X_{ih(q_1(m_h+1):m_h)} + \sum_{i=k+1}^{2k} X_{ih(q_2(m_h+1):m_h)} + \sum_{i=2k+1}^{3k} X_{ih(q_3(m_h+1):m_h)} \right]\right]$$

$$= \sum_{h=1}^{L} \frac{W_h^{\,2}}{m_h^2} \left[\sum_{i=1}^{k} Var(X_{ih(q_1(m_h+1):m_h)}) + \sum_{i=k+1}^{2k} Var(X_{ih(q_2(m_h+1):m_h)}) + \sum_{i=2k+1}^{3k} Var(X_{ih(q_3(m_h+1):m_h)}) \right]$$

Take the nearest integer of $q_1(m_h + 1)$, $q_2(m_h + 1)$ and $q_3(m_h + 1)$ where $q_1 = 0.25$, $q_2 = 0.50$ and $q_3 = 0.75$.

CASE 2: When m_h is even *i.e.* for $k = 2, 4, 6, 8, \ldots$

$$\bar{X}_{SBGQRSSE} = \sum_{h=1}^{L} \frac{W_h}{m_h} \left[\sum_{i=1}^{k} X_{ih(q_1(m_h+1):m_h)} + \sum_{i=k+1}^{2k} \left(\frac{1}{2}\left[X_{ih(q_2(m_h):m_h)} + X_{ih(q_2(m_h+1):m_h)} \right] \right) + \sum_{i=2k+1}^{3k} X_{ih(q_3(m_h+1):m_h)} \right]$$

where, $W_h = \frac{N_h}{N}$, N_h is the stratum size and N is the total population size. The variance of SBGQRSSE is given by:

$$Var(\bar{X}_{SBGQRSSE}) = Var\left[\sum_{h=1}^{L} \frac{W_h}{m_h} \left[\sum_{i=1}^{k} X_{ih(q_1(m_h+1):m_h)} + \sum_{i=k+1}^{2k} \left(\frac{1}{2}\left[X_{ih(q_2(m_h):m_h)} + X_{ih(q_2(m_h+1):m_h)} \right] \right) + \sum_{i=2k+1}^{3k} X_{ih(q_3(m_h+1):m_h)} \right]\right]$$

$$= \sum_{h=1}^{L} \frac{W_h^{\,2}}{m_h^2} \left[\sum_{i=1}^{k} Var(X_{ih(q_1(m_h+1):m_h)}) + \frac{1}{4} \sum_{i=k+1}^{2k} \left[Var(X_{ih(q_2(m_h):m_h)}) + Var(X_{ih(q_2(m_h+1):m_h)}) + 2Cov(X_{ih(q_2(m_h):m_h)}, X_{ih(q_2(m_h+1):m_h)}) \right] + \sum_{i=2k+1}^{3k} Var(X_{ih(q_3(m_h+1):m_h)}) \right]$$

Take the nearest integer of $q_1(m_h + 1)$, $q_2(m_h)$, $q_2(m_h + 1)$ and $q_3(m_h + 1)$ where $q_1 = 0.25$, $q_2 = 0.50$ and $q_3 = 0.75$.

3.1 LEMMA

$\bar{X}_{SBGQRSSE}$ and $\bar{X}_{SBGQRSSO}$ are unbiased estimators of the mean of symmetric distributions.

Proof:

Case1: When m_h is even *i.e.* for $k = 2, 4, 6, 8, \ldots$ and $h = 1, 2, \ldots, L$

$$E(\bar{X}_{SBGQRSSE}) = E\left(\sum_{h=1}^{L} \frac{W_h}{m_h} \left[\sum_{i=1}^{k} X_{ih(q_1(m_h+1):m_h)} + \sum_{i=k+1}^{2k} \left(\frac{1}{2}\left[X_{ih(q_2(m_h):m_h)} + X_{ih(q_2(m_h+1):m_h)} \right] \right) + \sum_{i=2k+1}^{3k} X_{ih(q_3(m_h+1):m_h)} \right]\right)$$

$$= \left(\sum_{h=1}^{L} \frac{W_h}{m_h} \left[\sum_{i=1}^{k} E(X_{ih(q_1(m_h+1):m_h)}) + \sum_{i=k+1}^{2k} E\left(\frac{1}{2}\left[X_{ih(q_2(m_h):m_h)} + X_{ih(q_2(m_h+1):m_h)} \right] \right) + \sum_{i=2k+1}^{3k} E(X_{ih(q_3(m_h+1):m_h)}) \right]\right)$$

Now,

$$\mu_{ih(q_1)} + \frac{1}{2}\left[\mu_{ih(q_2(m_h))} + \mu_{ih(q_2(m_h+1))} \right] + \mu_{ih(q_3)} = 3\mu_h$$

where, $\mu_{ih(q_1)}$, $\mu_{ih(q_2)}$, and $\mu_{ih(q_3)}$ are the means of the order statistics corresponding to the first, second, and third quartiles, respectively. Since the distribution is symmetric about μ, then as per David and Nagaraja (2003) we have,

$$\mu_{ih(q_1)} + \mu_{ih(q_2)} + \mu_{ih(q_3)} = 3\mu_h$$

$$E(\bar{X}_{SBGQRSSE}) = \sum_{h=1}^{L} \frac{W_h}{m_h}\left[\frac{m_h}{3}\mu_{ih(q_1)} + \frac{m_h}{3}\frac{1}{2}(2\mu_{ih(q_2)}) + \frac{m_h}{3}\mu_{ih(q_3)}\right]$$

$$= \sum_{h=1}^{L} \frac{W_h}{m_h}\left[\frac{m_h}{3}\left(\mu_{ih(q_1)} + \mu_{ih(q_2)} + \mu_{ih(q_3)}\right)\right]$$

$$= \sum_{h=1}^{L} \frac{W_h}{m_h}\left[\frac{m_h}{3}(3\mu_h)\right]$$

$$= \sum_{h=1}^{L} W_h \mu_h$$

$$= \mu$$

Case 2: When m_h is odd *i.e.* for $k = 1, 3, 5, 7, \ldots$ and $h = 1, 2, \ldots, L$

$$E(\bar{X}_{SBGQRSSO}) = E\left(\sum_{h=1}^{L} \frac{W_h}{m_h}\left[\sum_{i=1}^{k} X_{ih(q_1(m_h+1):m_h)} + \sum_{i=k+1}^{2k} X_{ih(q_2(m_h+1):m_h)} + \sum_{i=2k+1}^{3k} X_{ih(q_3(m_h+1):m_h)}\right]\right)$$

$$= \sum_{h=1}^{L} \frac{W_h}{m_h}\left[\sum_{i=1}^{k} E(X_{ih(q_1(m_h+1):m_h)}) + \sum_{i=k+1}^{2k} E(X_{ih(q_2(m_h+1):m_h)}) + \sum_{i=2k+1}^{3k} E(X_{ih(q_3(m_h+1):m_h)})\right]$$

$$= \sum_{h=1}^{L} \frac{W_h}{m_h}\left[\sum_{i=1}^{k} \mu_{ih(q_1)} + \sum_{i=k+1}^{2k} \mu_{ih(q_2)} + \sum_{i=2k+1}^{3k} \mu_{ih(q_3)}\right]$$

where, $\mu_{ih(q_1)}$ is the mean of the first quartile for the first $\frac{m_h}{3}$ samples in the h^{th} stratum, $\mu_{ih(q_2)}$ is the mean of the second quartile for the middle $\frac{m_h}{3}$ samples in the h^{th} stratum, and $\mu_{ih(q_3)}$ is the mean for the last $\frac{m_h}{3}$ samples in the h^{th} stratum. Since the distribution is symmetric about μ, then

$$\mu_{ih(q_1)} + \mu_{ih(q_2)} + \mu_{ih(q_3)} = 3\mu_h$$

Therefore,

$$E(\bar{X}_{SBGQRSSO}) = \sum_{h=1}^{L} \frac{W_h}{m_h}\left[\frac{m_h}{3}\mu_{ih(q_1)} + \frac{m_h}{3}\mu_{ih(q_2)} + \frac{m_h}{3}\mu_{ih(q_3)}\right]$$

$$= \sum_{h=1}^{L} \frac{W_h}{m_h}\frac{m_h}{3}\left[\mu_{ih(q_1)} + \mu_{ih(q_2)} + \mu_{ih(q_3)}\right]$$

$$= \sum_{h=1}^{L} \frac{W_h}{m_h}\frac{m_h}{3}[3\mu_h]$$

$$= \sum_{h=1}^{L} W_h \mu_h$$

$$= \mu$$

4. SIMULATION STUDY

To compare the performance of the proposed estimator of population mean μ using the SBGQRSS scheme with RSS, and BGRSS schemes, seven non-uniform probability distributions are considered. 50,000 random samples are generated from the population of each distribution and variance or mean squared error (MSE) of the averages of drawn samples are compared.

From Table 1 it is clear that the proposed scheme is most efficient in estimating the population mean with respect to SRS for various distributions with the same number of units as compared to the conventional RSS and BGRSS

5. APPLICATION OF SBGQRSS

To outline the SBGQRSS scheme, the data on Trunk Cross-sectional Area and Yield of the Gala RedLum apple variety during the year 2016 maintained at the Plate I High density plantation at SKUAST-Kashmir is used. The descriptive statistics for the variables used in the empirical study are shown in Table 2 below:

Let us suppose that we want to draw a sample of size $m = \sum_{h=1}^{2} m_h = 12$, and let the population of 135 high density apple be stratified with $L = 2$ and $h = 1, 2$, based on the concomitant variable TCA which is easily measurable for the whole population. Let $X_{ih(q_1(m_h+1))}$, $X_{ih(q_2(m_h))}$, $X_{ih(q_2(m_h+1))}$, $X_{ih(q_3(m_h+1))}$ be the $q_1(m_h + 1)^{th}$, $q_2(m_h)^{th}$, $q_2(m_h+1)^{th}$ and $q_3(m_h+1)^{th}$

Table 1. Efficiency of RSS, BGRSS, and SBGQRSS w.r.t SRS for estimating the population mean

Sample Size (*m*) / Distribution	6			9		
	RSS	BGRSS	SBGQRSS	RSS	BGRSS	SBGQRSS
Normal(0,1)	2.140	2.720	3.143	3.281	3.371	4.162
Logistic(0,1)	2.482	2.641	3.291	3.344	3.689	4.791
Laplace(0,1)	2.489	3.691	4.262	3.412	3.461	4.998
Weibull(6,1)	2.764	2.994	3.268	3.680	3.697	4.234
Beta(7,4)	2.721	3.014	3.112	2.721	3.014	3.110
t(3)	2.101	2.974	5.654	2.101	2.974	5.923
Weibull(6)	2.705	2.912	3.146	3.597	3.721	4.129

Table 2. Descriptive Statistics of the variables of Gala RedLum

Descriptive statistics	Yield	TeA
Mean	10.220	6.299
Median	10.293	6.127
Confidence Level (95.0%)	0.388	0.128
Population Variance	5.199	0.566
Kurtosis	0.536	0.344
Skewness	0.559	0.856
Range	12.972	3.701
Minimum	5.698	5.083
Maximum	18.669	8.784

order statistics, respectively, of the i^{th}sample in the h^{th} stratum. Assume that from the first stratum we select a sample of size 6 *i.e.* $m_1 = 6$ and from the second stratum we want a sample of size 6 *i.e.* $m_2 = 6$. Then the procedure of SBGQRSS is shown below:

Stratum 1:

Step 1: On dividing the population into two strata using TCA of Gala, from the first stratum a random sample of 36 plants are chosen. The plants are represented using an ordered pair (TCA, YIELD) as:

SET 1: {(**5.52**, 7.60), (**5.86**, 9.21), (**5.64**, 7.94), (**5.83**, 9.02), (**5.47**, 7.48), (**5.84**, 9.03)}

SET 2: {(**5.66**, 8.04), (**5.46**, 7.33), (**5.66**, 8.04), (**5.69**, 8.08), (**6.01**, 9.61), (**5.73**, 8.57)}

SET 3: {(**5.47**, 7.48), (**5.30**, 7.08), (**5.57**, 7.71), (**5.70**, 8.14), (**6.02**, 9.90), (**6.01**, 9.61)}

SET 4: {(**5.69**, 8.07), (**5.23**, 6.34), (**5.64**, 7.94), (**5.30**, 6.88), (**5.65**, 7.99), (**5.67**, 8.05)}

SET 5: {(**6.02**, 9.90), (**5.30**, 6.88), (**5.64**, 7.94), (**5.66**, 8.04), (**5.71**, 8.19), (**5.47**, 7.48)}

SET 6: {(**5.89**, 9.29), (**5.30**, 7.08), (**5.74**, 8.72), (**5.80**, 8.92), (**5.98**, 9.42), (**5.71**, 8.20)}

Step 2: For each set in Step1, rank the pairs within each set based on the TCA (shown in bold) from lowest to highest as shown below

SET 1: {(5.47, 7.48), (5.52, 7.60), (5.64, 7.94), (5.83, 9.02), (5.84, 9.03), (5.86, 9.21)}

SET 2: {(5.46, 7.33), (5.66, 8.04), (5.66, 8.04), (5.69, 8.08), (5.73, 8.57), (6.01, 9.61)}

SET 3: {(5.30, 7.08), (5.47, 7.48), (5.57, 7.71), (5.70, 8.14), (6.01, 9.61), (6.02, 9.90)}

SET 4: {(5.23, 6.34), (5.30, 6.88), (5.64, 7.94), (5.65, 7.99), (5.67, 8.05), (5.69, 8.07)}

SET 5: {(5.30, 6.88), (5.47, 7.48), (5.64, 7.94), (5.66, 8.04), (5.71, 8.19), (6.02, 9.90)}

SET 6: {(5.30, 7.08), (5.71, 8.20), (5.74, 8.72), (5.80, 8.92), (5.89, 9.29), (5.98, 9.42)}

Step 3: Divide the above defined sets into 3 balanced groups each of size 2 sets *i.e.*

GROUP 1

SET 1: {(5.47, 7.48), (5.52, 7.60), (5.64, 7.94), (5.83, 9.02), (5.84, 9.03), (5.86, 9.21)}

SET 2: {(5.46, 7.33), (5.66, 8.04), (5.66, 8.04), (5.69, 8.08), (5.73, 8.57), (6.01, 9.61)}

GROUP 2

SET 3: {(5.30, 7.08), (5.47, 7.48), (5.57, 7.71), (5.70, 8.14), (6.01, 9.61), (6.02, 9.90)}

SET4: {(5.23, 6.34), (5.30, 6.88), (5.64, 7.94), (5.65, 7.99), (5.67, 8.05), (5.69, 8.07)}

GROUP 3

SET 5: {(5.30, 6.88), (5.47, 7.48), (5.64, 7.94), (5.66, 8.04), (5.71, 8.19), (6.02, 9.90)}

SET 6: {(5.30, 7.08), (5.71, 8.20), (5.74, 8.72), (5.80, 8.92), (5.89, 9.29), (5.98, 9.42)}

Step 4: From the first group select the $q_1(m_h+1)^{th}$ *unit,* from the second group average of the $q_2(m_h)$ *and* $q_2(m_h+1)^{th}$ unit and from the third group $q_3(m_h+1)^{th}$ unit. Taking the nearest integer of $q_1(m_h+1)$, $q_2(m_h)$, $q_2(m_h+1)$ and $q_3(m_h+1)$ where $q_1=0.25$, $q_2=0.50$ and $q_3=0.75$. Thus, from the first group {7.60, 8.04}, from the second group $\{\frac{1}{2}(7.71+8.14), \frac{1}{2}(7.94+7.99)\}$ and from the third group {8.19, 9.29} are respectively selected for estimating the population mean of Yield of Gala from the first stratum. Thus, from stratum 1 we have the sample:

$\{7.60,\ 8.04,\ \frac{1}{2}(7.71+8.14),\ \frac{1}{2}(7.94+7.99),\ 8.19, 9.29\}$

Step 5: Now, consider stratum 2 with $n=6$, and again repeat the steps from 1-3, leading to the below balanced groups:

GROUP 1

SET 1: (6.15, 10.47), (6.43, 11.16), (6.52, 11.32), (7.14, 12.37), (7.47, 13.13), (8.09, 15.72)

SET 2: (6.36, 11.03), (6.44, 11.19), (7.28, 12.72), (7.32, 12.79), (7.63, 13.57), (8.32, 16.22)

GROUP 2

SET 3: (6.44, 11.19), (6.92, 12.0), (7.06, 12.2), (7.32, 12.79), (8.32, 16.22), (8.78, 18.67)

SET 4: (6.50, 11.27), (6.68, 11.69), (6.83, 11.89), (7.14, 12.36), (7.47, 13.13), (8.32, 16.22)

GROUP 3

SET 5: (6.12, 10.27), (6.29, 10.63), (6.42, 11.13), (6.82, 11.86), (6.83, 11.89), (7.25, 12.61)

SET6: (6.12, 10.27), (6.35, 10.98), (6.52, 11.32), (6.59, 11.43), (6.61, 11.58), (7.12, 12.31)

Thus, from the first group {11.16, 11.19}, from the second group $\{\frac{1}{2}(12.72+12.79), \frac{1}{2}(11.89+12.36)\}$ and from the third group, {11.89+11.58} are respectively selected for estimating the population mean of yield for Gala. Thus, from stratum 2 we have the sample:

$\{11.16, 11.19, \frac{1}{2}(12.72+12.79), \frac{1}{2}(11.89+12.36),$ $11.89+11.58\}$

Using the equation below to estimate the population mean,

$$\bar{X}_{SBGQRSSE} = \sum_{h=1}^{L} \frac{W_h}{m_h}\left[\sum_{i=1}^{k} X_{ih(q_1(m_h+1):6)} + \sum_{i=k+1}^{2k}\left(\frac{1}{2}\left[X_{ih(q_2(m_h):6)} + X_{ih(q_2(m_h+1):6)}\right]\right) + \sum_{i=2k+1}^{3k} X_{ih(q_3(m_h+1):6)}\right]$$

$$\bar{X}_{SBGQRSSE} = \frac{64}{135\times 6}\left(7.60+8.04+\frac{1}{2}(7.71+8.14)+\frac{1}{2}(7.94+7.99)+8.19+9.29\right) + \frac{71}{135\times 6}\left(11.16+11.19+\frac{1}{2}(12.72+12.79)+\frac{1}{2}(11.89+12.36)+11.89+11.58\right)$$

$$= 10.044$$

To investigate the performance of SBGQRSS in estimating the population mean w.r.t. SRS, the data on the yield and trunk cross-section area (TCA) of Gala RedLum high-density apple trees is considered where yield is the variable of main interest and the TCA is the auxiliary variable. Assuming that the population is partitioned into two strata, an estimate of the mean and mean square errors are computed. For each stratum, we assume that each stratum follows the same distribution. When the underlying distribution is symmetric, the efficiency of SBGQRSS relative to SRS is given by:

$$eff\left(\bar{X}_{SBGQRSS}, \bar{X}_{SRS}\right) = \frac{Var(X_{SRS})}{Var(\bar{X}_{SBGQRSS})},$$

and when the underlying distribution is asymmetric, the efficiency of SBGQRSS relative to SRS is given by:

$$eff\left(\bar{X}_{SBGQRSS}, \bar{X}_{SRS}\right) = \frac{MSE(X_{SRS})}{MSE(\bar{X}_{SBGQRSS})},$$

where $MSE(\bar{X}) = Var(\bar{X}) + [Bias(\bar{X})]^2$.

From Table 3, it is clear that using SBGQRSS will increase the relative precision compared to SRS and results in negligible mean square error as the sample size increases.

Table 3. Summary results of estimating the population mean of yield of High Density Gala Redlum Apple using SBGQRSS scheme with samples of size 6, 9,..., 18 and comparison with SRS

Sample Size (m)	Simple Random Sampling		Stratified Balanced Group Quartile Ranked Set Sampling		Efficiency
	Mean	*MSE*	*Mean*	*MSE*	
6	9.29	1.731	10.090	0.1830	*9.45*
9	9.49	1.110	9.970	0.0852	*13.02*
12	9.981	0.490	10.044	0.0426	*11.50*
15	9.478	0.797	10.170	0.0320	*24.90*
18	9.782	0.480	10.200	0.0061	*78.68*

6. CONCLUSION

Using the RSS method always ensures an increase in the relative precision in estimating the population mean compared to SRS. The suggested SBGQRSS increases the precision and ease of estimating the population mean firstly by stratifying the population under study using the auxiliary information and then using the quartiles to select the variable of interest. The suggested estimators are compared with their competitors in SRS method to prove the efficiency of estimating the population mean using the proposed scheme. Our simulation results indicated that the suggested empirical mean estimates are strongly better than their competitors in RSS and SRS designs for the same number of measured units in case of various non uniform distributions. Since RSS and its modifications are cost effective techniques, it is worthwhile investigating the performance of various goodness of fit tests under the proposed scheme since much work has not been done in this field.

REFERENCES

Al-Omari, A.I., and Jaber, K.H. (2008). Percentile Double quartile ranked set samples. *Journal of Mathematics and Statistics,* **4(1)**, 60-64.

Al-Omari, A.I., Syam, M., and Ibrahim, K. (2011). Investigating the use of stratified percentile ranked set sampling method for estimating the populationmean. *Proyecciones Journal of Mathematics*, **30(3)**, 351-368.

Al-Omari, A.I., and Zamanzade, E. (2017). Goodness of-fit-tests for Laplace distribution in ranked set sampling. *RevistaInvestigacionOperacional*, **38(4)**, 366-276.

Al-Omari, A.I., and Haq, A. (2019).Novel entropy estimators of a continuous random variable. *International Journal of Modelling, Simulation, and Scientific Computing,* **10(2)**, 195004.

David, H. A., Nagaraja, H. N. (2003). Order Statistics 3rd Edition. Wiley Series in Probability and Statistics. DOI:10.1002/0471722162

Haq, A, Brown J., Moltchanova, E., and Al-Omari, A.I. (2014). Mixed ranked set sampling design. *J. App. Statist.,***41(10)**, 2141-2156.

Haq, A., Brown, J., Moltchanova, E., and Al-Omari, A.I. (2016). Paired doubleranked set sampling. *Communications in Statistics-Theory and Methods*, **45(1)**, 2873-2889.

Ibrahim, K. (2011). On comparison of some variation of Ranked Set Sampling. *Sains Malaysiana*, **40**(4), 397–401

Jemain, A.A., and Al-Omari, A.I. (2006). Multistage median ranked set samples for estimating the population mean. *Pakistan Journal of Statistics,* **22(3)**, 195-207.

Jemain, A.A., Al-Omari, A.I., and. Ibrahim, K. (2007). Multistage median ranked set sampling for estimating the population median. *Journal of Mathematics and Statistics,* **3(2)**, 58-64

Jemain, A.A., Al-Omari, A.I., and. Ibrahim, K. (2008). Some variations of Ranked Set Sampling. *Electronic Journal of Applied Statistical Analysis*, **1**, 1-15

Mahdizadeh, M., and Zamanzade, E. (2017). Reliability estimation in multistage ranked set sampling. *REVSTAT: A Statistical Journal*, **15(4)**, 565-581

McIntyre, G.A. (1952). A Method for Unbiased Selective Sampling Using Ranked Sets. *Aust. J. Agril. Res.*, **3**, 385-390.

Muttlak, H.A. (1997). Median ranked set sampling. *Journal of Applied Statistical Science,* **6**, 245-255.

Muttlak, H.A. (2003). Investigating the use of quartile ranked set samples for estimating the population mean. *Journal of Applied Mathematics and Computation,* **146,** 437-443

Samawi, H.M., Ahmed, M.S., and Abu-Dayyeh, W. (1996). Estimating the population mean using extreme ranked set sampling. *Biometrical Journal,* **38(5)**, 577-586.

Syam, M., Ibrahim, K., and Al-Omari, A.I. (2012). The efficiency of stratifiedquartile ranked set sample in estimating the population mean. *Tamsui Oxford Journal of Information and Mathematical Sciences, (TOJMS)*, **28(2)**, 175-190.

Syam, M., Ibrahim, K., and Al-Omari, A.I. (2013). The efficiency of stratifieddouble percentile ranked set sample for estimating the population mean. *Far EastJournal of Mathematical Sciences (FJMS)*, **73(1)**, 157-177.

Takahasi, K., and Wakimoto, K. (1968). On the unbiased estimates of the population mean based on the sample stratified by means of ordering. *Ann. Inst. Statist. Math*, **20**, 1-31.

Zamanzade E., Al-Omari, A.I. (2016). New ranked set sampling for estimating the population mean and variance. *Hacettepe Journal of Mathematics and Statistics*, **46(6)**, 1891-1905

Zamanzade, E. (2019). EDF-based tests of exponentiality in pair ranked set sampling. *Statistical Papers*, **60(6)**, 2141-2159.