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Robust Block Designs for Comparing Test Treatment versus Control with Correlated Observations

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SUMMARY

Experiments for comparing the test treatments with one control treatment have special importance because in many situations it is necessary to compare test treatments against control with extra importance. In such situations, experimental designs for test treatments against control are used. Balanced Treatment Incomplete Block (BTIB) designs are usually used for the purpose. Kageyama and Mukerjee (1986) constructed a type of BTIB designs as Generalized Efficiency Balanced (GEB) block designs. The robustness properties of such designs are studied by several authors like, Srivastava *et al.* (1996), Singh *et al.* (2005), Shunmugathai and Srinivasan (2011) and others. All the authors mostly discussed the robustness of BTIB designs after missing of a single observation in a block. The presence of correlation in the form of neighbour effects among the adjacent plots in agricultural experiments is a well-established fact, Wilkinson *et al.*(1983), Kiefer and Wynn (1981), Gill and Shukla (1985) etc. The present paper develops the robustness criteria of BTIB designs for missing of a single test or control treatment from any block with correlated adjacent plots. A series of robust BTIB designs have been developed. The C matrices of the BTIB design and the residual BTIB design after removal of a single plot has also been presented having correlated observations. The efficiencies of the above designs for different values of plot to plot correlation coefficient (ρ) have been listed.

Keywords: Robustness of block designs; BTIB designs; GEB designs; Correlated observations.

1. INTRODUCTION

Designed experiments are generally conducted for making all the possible paired comparisons among the treatments. Comparing newly developed varieties with a standard one (called control) is an integral part of many areas of scientific experimentation. In such situations the interest is only in a subset of all possible paired comparisons. Balanced Treatment Incomplete Block (BTIB) (Bechhofer and Tamhane, 1981) designs andsome reinforced BIB designs are usually used for the purpose. However, the BTIB designs are not balanced (variance balance or efficiency balance). Das and Ghosh (1985) introduced the concept of 'General efficiency balanced' (GEB) designs as generalization of different types of balanced designs. Generalized Efficiency Balanced (GEB) block designs developed through method of reinforcement by Kageyama and Mukerjee (1986) are similar to R-type BTIB designs in which no extra replicate of the control treatment is added other than the obligatory replicate required for the construction of the design. The robustness of block designs for test treatments vs. control treatment against missing observations pertaining to test treatment has been investigated by several authors viz., Srivastava *et al.* (1996), Singh *et al.* (2005), Shunmugathai and Srinivasan (2011) and so on. Literature survey reveals that the study is mostly confined to design of experiments where the plots in a block are uncorrelated. But the presence of correlation in the form of neighbour effects among the adjacent plots in agricultural

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experiments is a well-established fact. Kumar *et al.* (2019) developed BTIB designs having one control treatment with correlated observations considering first order neighbour balanced (NN1) block designs in linear blocks.

In the present study an attempt has been made to examine the robustness property of block designs (test treatments vs. control) with correlated observations. The robustness of block designs of test treatments vs. control with correlated observations has been studied for the loss of a single observation. Due to loss of a single observation the resultant design may become disconnected as well as the efficiency of the resultant design may be reduced in comparison to the original design. In the present paper, both of these criteria have been considered for studying robustness. Again robustness has been investigated for R-type BTIB designs only to see the consequence of loss of a single observation in BTIB designs in general. Results for other BTIB designs can easily be derived from the results obtained in the present study. In the next Section some preliminaries have been discussed. In Section 3, condition of robustness as per connectedness criterion has been developed. Efficiency of the resultant designs has been discussed in Section 4.

2. SOME PRELIMINARIES

Let us consider a block design (test treatment s vs. control) D having correlated observations with treatments labelled as 1, ..., v, v+1 with v+1 denoting the control treatment and 1, 2, ..., v denoting the test treatments. These treatments are arranged in b blocks of equal sizes (k+1). Let r = $(r_1, r_2, ..., r_v, r_{v+1})'$ be the replication vector with r_{v+1} denoting the replication number of control treatment and r_i denoting the replication number of i^{th} test treatment, i = 1, 2, ..., v. A Fixed effect additive model is considered for analysing a block design (test treatment vs. control) having correlated observations as

$$\mathbf{Y} = \mu\mathbf{1} + \mathbf{X}\boldsymbol{\tau} + \mathbf{Z}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \qquad (1)$$

where, $\mathbf{Y}$ is a n × 1 vector of observations, μ is a general mean, $\mathbf{1}$ is a n × 1 vector of ones, $\mathbf{X}$ is a n × (v+1) incidence matrix of observations versus treatments, $\boldsymbol{\tau}$ is a (v+1) × 1 vector of treatment effects, $\mathbf{Z}$ is a n × b incidence matrix of observations versus blocks, $\boldsymbol{\beta}$ is a b × 1 vector of block effects and $\boldsymbol{\varepsilon}$ is a n × 1 vector of random errors. According to Gill and Shukla (1985), the error $\boldsymbol{\varepsilon}$ is independently and normally distributed with mean zero and variance- covariance matrix $\mathbf{V}$, where $\mathbf{V} = \sigma^2\mathbf{I}_b \otimes \mathbf{W}_{k+1}$, $\mathbf{I}_b$ is an identity matrix of order b, ⊗ denotes the kronecker product and $\mathbf{W}_{k+1}$ is the correlation matrix of k+1 observations within a block).

In the present study we considered the correlation structure as considered by Keifer and Wynn (1981) considered the effects of correlation on the efficiency of BIB designs. Here we refer to the model of correlated observations as developed by Keifer and Wynn (1981) (see also Shah and Sinha (1989)). The covariance structure of a block of size k in a block design d ∈ D (v, b, k) with v treatments in b blocks of size k, is assumed as

$\text{Cov}(y_{ji}, y_{j'i'}) = \sigma^2$, if j = j' and i = i'

$= \rho\sigma^2$, if j = j' and |i – i'| = 1

= 0, otherwise.

where, y_{ji} is the observation of the i-th treatment in j-th block, i=1, 2, …, v; j =1, 2, ..., b.

This model of correlated observation is known as first order nearest neighbour (NN1) model in the literature. In the above NN1 model there is no correlation among the plots between the blocks and correlation structure between plots within a linear block are the same in each block. Thus the correlation matrix under the model (1) would be

$$\mathbf{W}_{k+1} = \begin{bmatrix} 1 & \rho & 0 & \cdots & 0 & 0 \\ \rho & 1 & \rho & \cdots & 0 & 0 \\ 0 & \rho & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & 0 \\ \vdots & \vdots & \vdots & & \vdots & 0 \\ 0 & 0 & 0 & \cdots & 1 & \rho \\ 0 & 0 & 0 & & \rho & 1 \end{bmatrix} \qquad (2)$$

where, ρ(-1≤ ρ ≤ +1) is the correlation coefficient between two neighbouring plots in a block.

Then, the information matrix (C matrix) for estimating the treatment effects in model (1) can be obtained by generalized least squares method as (Gill and Shukla, 1985)

$$\mathbf{C} = \mathbf{X}'\mathbf{V}^{-1}\mathbf{X} - \mathbf{X}'\mathbf{V}^{-1}\mathbf{Z}\,(\mathbf{Z}'\mathbf{V}^{-1}\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{V}^{-1}\mathbf{X} \qquad (3)$$

2.1 R-type BTIB designs

Let us add one extra treatment as (v+1)th treatment to each and every block of any BIB design (v, b, r, k & λ). Now the resultant design D with v+1 treatment in b blocks of size k+1 is a design with correlated observations as mentioned in (1). Also let the first v treatments be the test treatments and the (v+1)th treatment be the control treatment. Thus the replication number of the control treatment would be r* = b and let λ* be the numbers of times the control treatment appeared in the design with v test treatments as pair. The resultant design is also an R-type BTIB design. According to Theorem 2.1 (Majumder, *et al.*, 2013), the elements of **C**- Matrix (3) of design D with (v+1) treatments will be

$$C_{ii} = \frac{r[(k+1)k - 2(k+2)\rho] + 2(k+1)\,\rho e_i}{(k+1)^2}$$

$$= \bar{a}\ ;\ \forall\ (i = 1, 2, \ldots, v),$$

$$C_{ii'} = \frac{-\lambda[(k+1) + 2(k+2)\rho] + (k+1)\rho((k+1)N_{ii'} + e_{ii'})}{(k+1)^2}$$

$$= \bar{b}\ ; \forall (i, i'(i \neq i') = 1, 2, \ldots, v),$$

$$C_{ij'} = \frac{-\lambda^*[(k+1)+2(k+2)\rho]+(k+1)\rho((k+1)N_{ij'}+e_{ij'})}{(k+1)^2}$$

$$= \bar{c}; \forall (i = 1, 2, \ldots, v \text{ and } j' = v+1$$

$$C_{ij'} = \frac{-\lambda^*[(k+1)+2(k+2)\rho]+(k+1)\rho((k+1)N_{ij'}+e_{ij'})}{(k+1)^2}$$

$$= \bar{c}; \forall (i = 1, 2, \ldots, v \text{ and } j' = v+1 \quad \text{and}$$

$$C_{j'j'} = \frac{r^*[(k+1)k-2(k+2)\rho]+2(k+1)\,\rho e_{j'}}{(k+1)^2} = \bar{d};\ j' = v+1,$$

where, e_i= Number of blocks with treatment i at an end; N_{ij}= Number of blocks with treatments i and j in adjacent positions and e_{ij}= Number of blocks with treatments iand j such that either one of them occurs at an end position. Thus the C-matrix (3) can be written as

$$\mathbf{C} = \begin{bmatrix} M_d & \bar{c}1_v \\ \bar{c}1'_v & \bar{d} \end{bmatrix}, \quad (4)$$

where $\mathbf{M_d} = (\bar{a} - \bar{b})\left[I - \frac{J}{\left(\frac{\bar{a}-\bar{b}}{\bar{b}}\right)}\right]$, **I** is an identity matrix of order v and **J (=1**v**1'**v**)** is a matrix of all unit elements of order v. Since **M**d is positive definite and non-singular matrix of order v, a generalized inverse of **C** as given by (Srivastava, *et al.,* 1996) will be C^g, where

$$C^g = \begin{bmatrix} M_d^{-1} & 0 \\ 0' & 0 \end{bmatrix} \quad (5)$$

Example 1: Let us consider an example of a design of test treatments vs. control treatment (R- type BTIB or GEB) with (v+1 = 8, b = 21, k+1= 5) developed from a neighbour balanced (NN1) BIB design with (v = 7, b = 21, r = 12, k = 4). The blocks of the design are given below:

(1 3 6 4 **8**), (2 4 7 5 **8**), (3 5 1 6 **8**), (4 6 2 7 **8**), (5 7 3 1 **8**), (6 1 4 2 **8**), (7 2 5 3 **8**)

(3 2 4 5 **8**), (4 3 5 6 **8**), (5 4 6 7 **8**), (6 5 7 1 **8**), (7 6 1 2 **8**), (1 7 2 3 **8**), (2 1 3 4 **8**)

(2 6 5 1 **8**), (3 7 6 2 **8**), (4 1 7 3 **8**), (5 2 1 4 **8**), (6 3 2 5 **8**), (7 4 3 6 **8**), (1 5 4 7 **8**)

Here, e_{ij}=3, N_{ij}=3, $e_{ij'}$=15, $N_{ij'}$=3, e_i=3, $e_{j'}$=21, here $[e_{ij} + (k+1)\ N_{ij}] = 18$, where $i(\neq j) = 1, 2, \ldots, 7$, j' = 8.

3. ROBUSTNESS OF R-TYPE BTIB DESIGNS WITH CORRELATED OBSERVATIONS

In what follows, we study the robustness property of test treatments vs. control treatment block (R- type BTIB) designs having correlation structure as described in (2) against loss of a single observation pertaining to test a treatment. Consider a block design D with correlated observations as discussed in Section 2. Without loss of generality, let an observation corresponding to a treatment in the first block be missing in the design D. Let D_0 be the resultant design. Let the information matrix of D_0 be $\mathbf{C}_0$, then $\mathbf{C}_0$ can be written as,

$$\mathbf{C} - \mathbf{C}_0 = \mathbf{ww'},$$

where, the matrix **ww'** is an information matrix of order v+1 for the first treatment (the missing treatment) in a test treatments vs. control block design (D). It is a symmetric matrix with row and column sums as zero.

Now depending upon the position of the missing observation, the structure of would be as follows:

Structure of ww' for missing observation occurring at the end position in a block i.e., 1st **or (k+1)**th

Let an observation corresponding to first(or last) position of the 1st block of D be missed. Suppose the missing treatment be p and it has only one immediate neighbour (right or left) is q as the blocks are linear in nature, where (p(≠q) = 1, 2, …, v+1). In p^{th} row/column of **ww'** matrix, (p, p) cell, (p, q) cell, (p, k+1) cell and rest cells have the elements **a**, **b**, **d** and **c** respectively. In the qth row/column of (q, p), (q, q), (q, k+1) and (q, rest) cells will have the elements **b**, **e**, **h**and**g** respectively. Similarly, in $(k+1)^{th}$ row /column, the cells like (k+1, p), (k+1, q) and (k+1, k+1) have the elements **d**, **h** and **m** respectively and rest of the elements are **j**. The (k-2) rows/columns (of the k-2 remaining treatments in the first block) are identical. In each of these rows/columns, any cells with p, q and k+1 will be **c**, **g** and **j**, respectively. Whereas, the other cells are equal and the value will be **i**. Among v+1 rows/columns of **ww'** matrix, all cells of remaining (v-k) rows/columns will be zero. The matrix **ww'** will be divided by a common scalar element **n**.

For easy understanding, let us consider a design with v+1 = 8 and k+1 = 5 and the initial block of the design be (1 3 6 4 8). Let a single observation pertaining to the treatment 1be missing from the initial block. Then the structure of **ww'** would be as follows:

$$ww' = \frac{1}{n}\begin{bmatrix} a & 0 & b & c & 0 & c & 0 & d \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ b & 0 & e & g & 0 & g & 0 & h \\ c & 0 & g & i & 0 & i & 0 & j \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ c & 0 & g & i & 0 & i & 0 & j \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ d & 0 & h & j & 0 & j & 0 & m \end{bmatrix}$$

where,

$n = [k+1+2\rho k]\,[k+2\rho k-2\rho];$

$a = [k-\rho^2-2\rho+2\rho k]\,[k+2\rho k-2\rho];$

$b = [k\rho-2\rho-1-2\rho^2+2\rho^2 k]\,[k+2\rho k-2\rho]$

$c = -[1+3\rho+2\rho^2]\,[k+2\rho k-2\rho];$

$d = -[1+2\rho+\rho^2]\,[k+2\rho k-2\rho];$

$e = 1-2\rho(k-2)-\rho^2(7k-9)-2\rho^3(3k-4);$

$g = 1-\rho(k-5)-2\rho^2(2k-5)-4\rho^3(k-2);$

$h = 1-\rho(k-4)-\rho^2(3k-7)-2\rho^3(k-2);$

$i = 1+6\rho+12\rho^2+8\rho^3;$

$j = 1+5\rho+8\rho^2+4\rho^3$ and $m = 1+4\rho+5\rho^2+2\rho^3.$

Structure of ww' for missing observation occurring at 2nd **or k**th **positionin a block**

Let 2^{nd} or k^{th} position be missed in a particular block of design D (with v+1 = 8 and k+1 = 5), then the simplified form of **ww'** is given below.

$$ww' = \frac{1}{q'}\begin{bmatrix} a' & 0 & b' & c' & 0 & d' & 0 & e' \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ b' & 0 & f' & g' & 0 & h' & 0 & i' \\ c' & 0 & g' & j' & 0 & k' & 0 & l' \\ 0 & 0 & 0' & 0 & 0 & 0 & 0 & 0 \\ d' & 0 & h' & k' & 0 & m' & 0 & n' \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ e' & 0 & i' & l' & 0 & n' & 0 & p' \end{bmatrix}$$

where,

$q' = [k+1+2\rho k]\,[k+2\rho k-4\rho];$

$a' = 1-2\rho(k-2)-\rho^2(5k-8)-2\rho^3(k-2);$

$b' = -[1+\rho(k-2)+2\rho^2(k-1)]\,[k+2\rho k-4\rho];$

c' $= 1-\rho(k-6)-4\rho^2(k-3)-4\rho^3(k-2);$

$d' = 1-\rho(2k-5)-6\rho^2(k-2)-4\rho^3(k-2);$

$e' = 1-\rho(k-5)-\rho^2(3k-8)-2\rho^3(k-2);$

$f' = [k-4\rho^2-4\rho+2\rho k]\,[k+2\rho k-4\rho];$

$g' = -[1+4\rho+4\rho^2]\,[k+2\rho k-4\rho];$

$h' = [-1-\rho(k-3)-2\rho^2(k-2)]\,[k+2\rho k-4\rho];$

$i' = -[1+3\rho+2\rho^2]\,[k+2\rho k-4\rho];$

$j' = 1+8\rho+20\rho^2+16\rho^3;$

$k' = 1-\rho(k-7)-\rho^2(4k-18)-4\rho^3(k-4);$

$l' = 1+7\rho+14\rho^2+8\rho^3;$

$m' = 1-2\rho(k-3)-\rho^2(7k-17)-\rho^3(6k-16)$ and

$n' = 1-\rho(k-6)-\rho^2(3k-13)-2\rho^3(k-4).$

Structure of ww' for missing observation occurring at 3rd **or (k-1)**th **position in a block**

Suppose missed plot be 3^{rd} or $(k-1)^{th}$ position in a particular block of design D(with v+1 = 8 and k+1 = 5), then obtained **ww'** is an information matrix of a missed plot for the design D.

$$ww' = \frac{1}{q'}\begin{bmatrix} p' & 0 & n' & n' & 0 & i' & 0 & p' \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ n' & 0 & m' & m' & 0 & h' & 0 & n' \\ n' & 0 & m' & m' & 0 & h' & 0 & n' \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ i' & 0 & h' & h' & 0 & f' & 0 & i' \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p' & 0 & n' & n' & 0 & i' & 0 & p' \end{bmatrix}$$

3.1 Conditions of robustness

If $\rho \neq 0$, we cannot develop the **w** as a vector. But when $\rho = 0$, **w** vector can be developed from the matrix **ww'**, where **w'** = $\{k(k+1)\}^{-1/2}$[k: **-f'**: 1], (k+1) is the block size and **f'** is a vector with (0, 1) elements representing the incidence of the (v) unaffected treatments in the first block which contains the missing treatment. Otherwise we can always write **C**0 = **C** - **ww'**. Now we give the following result.

Theorem 1: An R-type BTIB designs with correlated observations design D is robust as per connectedness criterion against the loss of a single observation.

Proof: The design D is robust as per connectedness criterion against loss of a single observation, if the rank of its C-matrix is equal to the rank of the c-matrix of the resultant design, i.e., if Rank of **C** = Rank of **C**0. Following Theorem 1 of Dey (1993), design D is robust if f [**I**v+1 – **C**g**ww'**] is a positive definite, where **I**v+1 is an Identity matrix of order v+1 and **C**g is ag-inverse of **C**.

Now in matrix **C** of design D,

$$\mathbf{M_d} = (\bar{a} - \bar{b})\left[I - \frac{J}{\left(\frac{\bar{a}-\bar{b}}{\bar{b}}\right)}\right] \text{ and } \mathbf{M_d}^{-1} = \frac{1}{(\bar{a}-\bar{b})}\left[I - \frac{J}{\left(\frac{\bar{a}-\bar{b}}{\bar{b}}\right)}\right].$$

Thus, **Cgww'** = $\begin{bmatrix} M_d^{-1} & 0 \\ 0' & 0 \end{bmatrix}\begin{bmatrix} M_{d1} & c^* \\ c^{*\prime} & m \end{bmatrix}$. Again, {**X'** [**I**v+1 – **Cgww'**] **X**} >0 for any non-null vector **X.** Thus [**I**v+1 – **Cgww'**] is positive definite. Hence the proof.

Remark1: The condition for robustness as given in Theorem 4 (or Corollary 2) of Dey (1993) for a block design is also satisfied for $\rho = 0$.

4. EFFICIENCY OF R-TYPE BTIB DESIGNS WITH CORRELATED OBSERVATIONS FOR A LOSS OF A SINGLE OBSERVATION

The efficiency of an R-type BTIB designs with correlated observations for a loss of a single observation can be calculated as (Criterion 2 of Dey (1993).

$$E = \frac{\text{sum of reciprocals of non zero eigenvalues of C}}{\text{sum of reciprocals of non zero eigenvalues of } C_0} = \frac{tr[C^g]}{tr(C_0^g)}.$$

Table 1. Robust efficiency values of the BTIB design for different values of ρ (-1 to +1).For second design, (k-1=2), so, it limits up to E2.

Designs	Rho	E1	E2	E3
v + 1 = 8, b = 21, r = 12(21), k + 1 = 5, λ = 6(12)	-1	0.989	0.999	0.981
	-0.9	0.989	0.934	0.980
	-0.8	0.990	0.963	0.980
	-0.7	0.997	0.970	0.978
	-0.6	0.998	0.998	0.998
	-0.5	0.987	0.983	0.985
	-0.4	0.987	0.983	0.985
	-0.3	0.987	0.984	0.984
	-0.2	0.986	0.984	0.985
	-0.1	0.986	0.985	0.985
	0	0.985	0.985	0.985
	0.1	0.985	0.986	0.986
	0.2	0.984	0.986	0.986
	0.3	0.983	0.986	0.987
	0.4	0.982	0.986	0.987
	0.5	0.980	0.986	0.988
	0.6	0.978	0.985	0.988
	0.7	0.976	0.984	0.989
	0.8	0.972	0.982	0.990
	0.9	0.968	0.979	0.990
	1	0.961	0.975	0.990
v+1 = 8, b=21, r =9(21), k+1 =4, λ =3(9)	-1	0.985	0.962	
	-0.9	0.987	0.964	
	-0.8	0.995	0.965	
	-0.7	0.955	0.961	
	-0.6	0.984	0.975	
	-0.5	0.981	0.974	
	-0.4	0.981	0.974	
	-0.3	0.980	0.975	
	-0.2	0.980	0.976	
	-0.1	0.979	0.977	
	0	0.979	0.979	
	0.1	0.978	0.980	
	0.2	0.976	0.982	
	0.3	0.975	0.983	
	0.4	0.972	0.985	
	0.5	0.969	0.988	
	0.6	0.965	0.990	
	0.7	0.958	0.993	
	0.8	0.948	0.997	
	0.9	0.930	0.997	
	1	0.897	0.997	

Note: E1, E2 and E3 indicates efficiency values for 1^{st} or $(k+1)^{th}$ plot, 2^{nd} or k^{th} plot and 3^{rd} or $(k-1)^{th}$ plot, respectively, be missed in a block of Design D.

This efficiency has been calculated for many R-type BTIB designs derivable from BIB designs for various values of ρ. For an example, these values are given for two designs in Table1. These values were calculated for all possible situations of occurrence of missing observations as described above. The average efficiency E1, E2 and E3 for first design with standard error was found to be 0.983(±0.008), 0.981(±0.013) and 0.986(±0.004) respectively. The average efficiency E1 and E2 for second design was recorded as 0.969(±0.022) and 0.979(±0.011) respectively. It has been found the efficiency remain very high for any value of ρ and under any situation of occurrence of missing observation. Thus these designs are robust as per efficiency criterion. The efficiencies of designs have been calculated by in SAS IML code.

5. CONCLUSIONS

The robustness criteria of BTIB design for missing of a single test or control treatment from any block with correlated adjacent plots has been discussed as there exits correlation in the form of neighbour effects among the adjacent plots in agricultural experiments. Also, a series of robust BTIB designs have been developed. The C matrices of the BTIB design and the residual BTIB design after removal of a single plot has also been evaluated in case of correlated observations. The robust efficiencies of the above designs for different values of plot to plot correlation coefficient (ρ) were estimated.

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