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Some Transformed and Composite Chain Ratio-type Estimators using Two Auxiliary Variables

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SUMMARY

The present paper has dealt some transformed and composite chain ratio type estimators of population mean in finite population survey sampling using information on two auxiliary variables related to the variable under study. Their bias and MSE are derived. The relative efficiency of the proposed estimators have been examined in comparison of several existing estimators in the literature using real data. It has been found that the proposed estimators have outperformed the existing chain ratio type estimators and composite chain ratio type estimators.

Keywords: Two phase sampling, Auxiliary variables, Chain ratio type estimators, Composite chain estimators.

1. INTRODUCTION

Information on auxiliary variables is generally used in sample surveys to improve the efficiency of the estimators of population parameter of interest. Theoretically, it has been established that, in general, the regression estimator is more efficient than the ratio and product estimators. However when the regression line of the character under study on the auxiliary character passes through the origin, these are equally efficient. Nevertheless, due to the stronger intuitive appeal, statisticians are more inclined towards the use of ratio and product estimators. Perhaps that is why an extensive work has been done in the direction of improving the performance of these estimators.

Consider that finite population $U = (U_1, U_2, \ldots\ldots U_N)$ consists of N identifiable sampling units. Associated with the unit U_i is a pair of numbers (y_i, x_i), where y_i is the value of the study variate y and x_i is the value of an auxiliary variable x related to y. The objective is to estimate $\bar{Y} = \sum_{i=1}^{N} y_i \big/ N$

For estimating of $\bar{Y}$, let a sample of size n is drawn from U by simple random sampling without replacement (SRSWOR). Let $\bar{y}$ and $\bar{x}$ be simple sample mean of y and x, respectively. When $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} x_i$ is known, the usual ratio estimator of $\bar{Y}$ is given by

$$\bar{y}_r = \frac{\bar{y}}{\bar{x}}.\bar{X} \tag{1.1}$$

with bias and mean square error (MSE) as follows.

$$\mathrm{B}(\bar{y}_r) = .\bar{Y}\left(\frac{1}{n} - \frac{1}{N}\right)\left(C_x^2 - \rho_{yx} C_x C_y\right) \tag{1.2}$$

$$\mathrm{MSE}(\bar{y}_r) = \bar{Y}^2\left(\frac{1}{n} - \frac{1}{N}\right)\left[C_y^2 + C_x^2 - 2\rho_{yx} C_x C_y\right] \tag{1.3}$$

where $C_y^2 = S_y^2 \big/ \bar{Y}^2$, $C_x^2 = S_x^2 \big/ \bar{X}^2$, ρ_{yx} is correlation between x and y, and $S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)^2$, $S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^2$.

The usual regression estimator of $\bar{\mathrm{Y}}$ is given by

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$$\bar{y}_{lr} = \bar{y} + b_{yx}\left(\bar{X} - \bar{x}\right) \tag{1.4}$$

with bias and MSE as follows.

$$B\left(\bar{y}_r\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\beta\left[\frac{\mu_{30}}{S_x^2} - \frac{\mu_{21}}{S_{xy}}\right] \tag{1.5}$$

$$MSE\left(\bar{y}_r\right) = \left(\frac{1}{n} - \frac{1}{N}\right)S_y^2\left(\; - \; {}^{2}_{yx}\right) \tag{1.6}$$

where $S_{xy} = \frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \bar{X}\right)\left(y_i - \bar{Y}\right)$ and $\mu_{rs} = \frac{1}{N}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^r \left(x_i - \bar{X}\right)^s$.

When $\bar{X}$ is not known the two phase sampling (double sampling) techinque is resorted to first find out the estimate of $\bar{X}$. The technique involves two steps.

(i) at first phase, draw a preliminary large sample of size n' from U by SRSWOR. Enumerate the sampled units for x. Let $\bar{x}' = \sum_{i=1}^{n'} x_i \Big/ n$ be sample mean based on n' units, and (ii)at second phase, draw a sub sample of size *n* from n' by SRSWOR. Enumerate the sampled unites for *y*. Let $\bar{y}$ and $\bar{x}$ be sample mean based on n units drawn from $'$ units. A double sampling ratio estimator of $\bar{Y}$ is given by

$$\bar{y}_{rd} = \frac{\bar{y}}{\bar{x}}\bar{x}' \tag{1.7}$$

with bias and MSE of $\bar{y}_{rd}$ as given below

$$B\left(\bar{y}_{rd}\right) = \bar{Y}\left(\frac{1}{n} - \frac{1}{n'}\right)\left(C_x^2 - \rho_{yx}C_xC_y\right) \tag{1.8}$$

$$MSE\left(\bar{y}_{rd}\right) = V(\bar{y}) + \bar{Y}^2\left(\frac{1}{n} - \frac{1}{n'}\right)\left[C_x^2 - 2\rho_{yx}C_yC_x\right],$$

$$\text{where } V(\bar{y}) = \left(\frac{1}{n} - \frac{1}{N}\right)S_y^2 \tag{1.9}$$

The usual double sampling regression estimator is given by

$$\bar{y}_1 = \bar{y} + b_{yx}\left(\bar{x}' - \bar{x}\right) \tag{1.10}$$

with bias and MSE as follows

$$B\left(\bar{y}_1\right) = \beta\left(\frac{1}{n} - \frac{1}{n'}\right)\left[\frac{\mu_{30}}{S_{xy}} - \frac{\mu_{21}}{S_{xy}}\right] \tag{1.11}$$

$$MSE\left(\bar{y}_1\right) = V(\bar{y}) - \bar{Y}^2\left(\frac{1}{n} - \frac{1}{n'}\right)\rho_{yx}^2C_y^2 \tag{1.12}$$

Chand (1975) proposed chain ratio-type estimator of $\bar{Y}$ when the population mean of an auxiliary variable *x* highly related with the study variable y is not known but the population mean of another auxiliary variable *z* which is less correlated with *y* but is known. The estimator he proposed is highly correlated with *x* as

$$T_1 = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)\left(\frac{\bar{Z}}{\bar{z}'}\right)$$

$$\text{where } \bar{Z} = \sum_{i=1}^{N} Z_i \Big/ N \text{ and } \bar{z}' = \sum_{i=1}^{n'} z_i \Big/ n' \tag{1.13}$$

and MSE of T_1 as follows

$$MSE\left(T_1\right) = V\left(\bar{y}\right) + \bar{Y}^2\left[\left(\frac{1}{n} - \frac{1}{n'}\right)C_x^2 + \left(\frac{1}{n'} - \frac{1}{N}\right)C_z^2 - 2\left(\frac{1}{n} - \frac{1}{n'}\right)\rho_{yx}C_yC_x - 2\left(\frac{1}{n'} - \frac{1}{N}\right)\rho_{yz}C_yC_z\right] \tag{1.14}$$

Kiregyera (1980) proposed chain ratio-cum regression estimator of $\bar{Y}$ as

$$T_{l2} = \bar{y} + b_{yx}\left(\bar{x}'\frac{\bar{Z}}{\bar{z}'} - \bar{x}\right) \tag{1.15}$$

where b_{yx} is estimated regression coefficient of *y* on *x* from *n* sampled units.

with MSE of T_{l2} of as follows

$$MSE\left(T_{l2}\right) = MSE\left(\bar{y}_1\right) + \bar{Y}^2\frac{f'}{n'}\left[\frac{\rho_{yx}^2C_y^2C_z^2}{C_x^2} - 2\frac{\rho_{yx}\rho_{yz}C_y^2C_z}{C_x}\right],$$

$$\text{where } f' = \left(1 - \frac{n'}{N}\right) \tag{1.16}$$

Kiregyera (1984) developed another estimator of $\bar{Y}$ along with its MSE as

$$T_{l3} = \bar{y} + b_{yx}\left[\bar{x}' + b_{xz}\left(\bar{z} - \bar{z}'\right) - \bar{x}\right] \tag{1.17}$$

$$MSE(T_{l3}) = \bar{Y}^2\left[\left(\frac{1}{n} - \frac{1}{N}\right)\left(C_y^2 - K_{yx}^2C_x^2\right) + \left(\frac{1}{n'} - \frac{1}{N}\right)\left(K_{yx}^2C_x^2 + K_{yx}K_{xz}C_z^2\left(K_{yx}K_{xz} - 2K_{yz}\right)\right)\right] \tag{1.18}$$

where $K_{yx} = \frac{\rho_{yx}C_y}{C_x}$, $K_{xz} = \frac{\rho_{xz}C_x}{C_z}$ and $K_{yz} = \frac{\rho_{yz}C_y}{C_z}$

and b_{xz} is estimated regression coefficient of *x* on *z* from n' sampled units.

Since then many research workers have developed exponential chain ratio type and regression type estimator of $\bar{Y}$ in the past. Some relevant works are briefly described below.

Singh and Choudhury (2012) proposed a exponential chain ratio estimator under double sampling as

$$\bar{Y}_{\mathrm{Re}}^{dc} = \bar{y}\exp\left(\frac{\frac{\bar{x}'\bar{z}}{\bar{z}'} - \bar{x}}{\frac{\bar{x}'\bar{z}}{\bar{z}'} + \bar{x}}\right) \quad (1.19)$$

Khare *et al.* (2013) developed a generalized chain ratio cum –regression type estimator of $\bar{Y}$ as

$$T_{l4} = \bar{y} + b_{yx}\left[\bar{x}'\left(\frac{\bar{Z}}{\bar{z}'}\right)^{\alpha} - \bar{x}\right] \quad (1.20)$$

where α is some scalar quantity.

MSE of T_{l4} for optimum value of as $\alpha$is given by

$$MSE\left(T_{l4}\right) = MSE\left(\bar{y}_{lrd}\right) - \bar{Y}^2\frac{f'}{n'}\rho_{yz}^2 C_y^2 \quad (1.21)$$

Singh *et al.* (2015) developed a composite type chain ratio estimator of $\bar{Y}$ as follows

$$\bar{Y}_{EC}^{RdR} = \left(\alpha I_1 + \left(1-\alpha\right)I_2\right) \quad (1.22)$$

where α is some constant and $I_1 = \bar{Y}_{\mathrm{Re}}^{dc}$ as given in (1.19), and $I_2 = \bar{Y}_{EdR}^{dc} = \bar{y}\exp\left(\frac{\frac{N\frac{\bar{x}'\bar{z}}{\bar{z}'} - n\bar{x}}{N-n} - \frac{\bar{x}'\bar{z}}{\bar{z}'}}{\frac{N\frac{\bar{x}'\bar{z}}{\bar{z}'} - n\bar{x}}{N-n} + \frac{\bar{x}'\bar{z}}{\bar{z}'}}\right)$

where I_2 is dual chain ratio estimator of $\bar{Y}$ following the dual to ratio estimator for Srivenkataramana (1977).

MSE of $\bar{Y}_{EC}^{RdR}$ for optimum value of α is given by

$$MSE(\bar{Y}_{EC}^{RdR})_{opt} = \bar{Y}^2\left[\frac{1-f}{n}C_y^2 + N_1 + N_2 - \frac{\left(N_3+N_4\right)^2}{N_5}\right] \quad (1.23)$$

where $N_1 = \frac{\left(1-f^*\right)}{n}\frac{g^2}{4}C_x^2\left(1-\frac{g}{4}K_{yx}\right)$,

$$N_2 = \frac{\left(1-f_1\right)}{n_1}\frac{g^2}{4}C_z^2\left(1-\frac{g}{4}K_{yz}\right),$$

$$N_3 = \frac{\left(1-f^*\right)}{n}\frac{g}{2}C_x^2\left(1-\frac{2}{g}K_{yx}\right),$$

$$N_4 = \frac{\left(1-f_1\right)}{n_1}\frac{g}{2}C_z^2\left(1-\frac{2}{g}K_{yz}\right),$$

$$N_5 = \frac{\left(1-f^*\right)}{n}C_x^2 + \frac{\left(1-f_1\right)}{n_1}C_z^2$$

and $g = \frac{n}{N-n}$, $f = \frac{n}{N}$, $f_1 = \frac{n'}{N}$, $f^* = \frac{n}{n'}$

MSE of $\bar{Y}_{EdR}^{dc}$ is given by

$$MSE\left(\bar{Y}_{EdR}^{dc}\right) = \bar{Y}^2\left[\left(\frac{1-f}{n}\right)C_y^2 + N_1 + N_2\right] \quad (1.24)$$

In view of the above facts, some transformed and composite chain ratio type estimators of population mean $\bar{Y}$ are proposed using two auxiliary variables in the present paper. Their bias and MSE are derived. The relative efficiency of the proposed estimators as compared to relevant existing estimators of $\bar{Y}$ are examined with real data.

2. PROPOSED TRANSFORMED CHAIN RATIO TYPE ESTIMATOR OF POPULATION MEAN

A transformed chain ratio type estimator is proposed as

$$T_2 = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)^{\alpha}\left(\frac{\bar{Z}}{\bar{z}'}\right)^{\beta} \quad (2.1)$$

where α and β are some unknown scalar quantities. Obviously, the estimator T_2 is biased estimator as $E(T_2) \neq \bar{Y}$. Bias of the transformed chain ratio-type estimator is as follows

$$B\left(T_2\right) = E\left(T_2\right) - \bar{Y}$$

$$= E\left[\bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)^{\alpha}\left(\frac{\bar{Z}}{\bar{z}'}\right)^{\beta}\right] - \bar{Y}$$

We assume that

$\bar{y} = \bar{Y} + \in_0, \bar{x}' = \bar{X} + \in_1, \bar{x} = \bar{X} + \in_2, \bar{z}' = \bar{Z} + \in_3$ with $E\left(\in_0\right) = E\left(\in_1\right) = E\left(\in_2\right) = E\left(\in_3\right) = 0$

Under these assumptions and following the procedures given in Sukatme and Sukhatme (1970), an appropriate bias of T_2 upto first order of approximation is obtained as

$$B\left(T_2\right) = \bar{Y}\left[\left(\frac{1}{n} - \frac{1}{N}\right)(QC_x^2 - \alpha\rho_{yx}C_yC_x) + \left(\frac{1}{n'} - \frac{1}{N}\right)\left(PC_x^2 + RC_z^2 - \alpha^2C_x^2 + \alpha\rho_{yx}C_yC_x - \beta\rho_{yz}C_zC_y\right)\right] \quad (2.2)$$

where

$$P = \frac{\alpha\left(\alpha-1\right)}{2},\ Q = \frac{\alpha\left(\alpha+1\right)}{2} \text{ and } R = \frac{\beta\left(\beta+1\right)}{2}$$

If $\alpha = 1$ and $\beta = 1$, we find that $P = 0$, $Q = 1$ and $R = 1$, and in this case T_2 reduces to T_1 with bias of T_1 and as given below

$$B(T_1) = \bar{Y}\left[\left(\frac{1}{n} - \frac{1}{n'}\right)\left(C_x^2 - \rho_{yx} C_y C_x\right) + \left(\frac{1}{n'} - \frac{1}{N}\right)\left(C_z^2 - \rho_{yz} C_z C_y\right)\right] \tag{2.3}$$

Mean Square Error (MSE) of the proposed estimator T_2

$$MSE(T_2) = E\left(T_2 - \bar{Y}\right)^2$$

$$= E\left[\bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)^{\alpha}\left(\frac{\bar{Z}}{\bar{z}'}\right)^{\beta} - \bar{Y}\right]^2$$

$$= E\left[\left(Y + \epsilon_0\right)\left(\frac{\bar{X} + \epsilon_1}{\bar{X} + \epsilon_2}\right)^{\alpha}\left(\frac{\bar{Z}}{\bar{Z} + \epsilon_3}\right)^{\beta} - \bar{Y}\right]^2$$

Under the assumptions as mentioned above, MSE of T_2 upto first order of approximation is obtained as

$$MSE(T_2) = V(\bar{y}) + \bar{Y}^2\left[\alpha^2\left(\frac{1}{n} - \frac{1}{n'}\right)C_x^2 + \beta^2\left(\frac{1}{n'} - \frac{1}{N}\right)C_z^2 - 2\alpha\left(\frac{1}{n} - \frac{1}{n'}\right)\rho_{yx} C_y C_x - 2\beta\left(\frac{1}{n'} - \frac{1}{N}\right)\rho_{yz} C_y C_z\right] \tag{2.4}$$

Now, if $\alpha = 1$ and $\beta = 1$, we find that in this case T_2 reduces to T_1 with MSE of T_1 as given below

$$MSE(T_1) = V(\bar{y}) + \bar{Y}^2\left[\left(\frac{1}{n} - \frac{1}{n'}\right)C_x^2 + \left(\frac{1}{n'} - \frac{1}{N}\right)C_z^2 - 2\left(\frac{1}{n} - \frac{1}{n'}\right)\rho_{yx} C_y C_x - 2\left(\frac{1}{n'} - \frac{1}{N}\right)\rho_{yz} C_y C_z\right] \tag{2.5}$$

We shall find out the optimum value of α and β by minimizing the $MSE(T_2)$ given in (2.4) with respect to α and β.

The above expression of $MSE(T_2)$ in (2.4) can also be written as

$$MSE(T_2) = V(\bar{y}) + \bar{Y}^2\left[\alpha^2\theta_3 C_x^2 + \beta^2\theta_2 C_z^2 - 2\alpha\theta_3\rho_{yx} C_y C_x - 2\beta\theta_2\rho_{yz} C_y C_z\right] \tag{2.6}$$

where $\theta_1 = \left(\frac{1}{n} - \frac{1}{N}\right),, \theta_2 = \left(\frac{1}{n'} - \frac{1}{N}\right)\theta_3 = \left(\frac{1}{n} - \frac{1}{n'}\right)$

Differentiating MSE (T_2) with respect to α, we get

$$\frac{\partial MSE(T_2)}{\partial \alpha} = \bar{Y}^2\left[2\alpha\theta_3 C_X^2 - 2\theta_3\rho_{yx} C_y C_X\right]$$

Equating the above differential to zero and after solving it, we get the optimum value of α as

$$\alpha_{opt} = \rho_{yx}\frac{C_y}{C_x} \tag{2.7}$$

Differentiating $MSE(T_2)$ with respect to β, we get

$$\frac{\partial MSE(T_2)}{\partial \beta} = \bar{Y}^2\left[2\alpha\theta_3 C_x^2 - 2\theta_3\rho_{yx} C_y C_x\right]$$

Equating the above differential to zero and after solving it, we get the optimum value of β as

$$\beta_{opt} = \rho_{yz}\frac{C_y}{C_z} \tag{2.8}$$

Putting the optimum value of α and β in equation (2.4), we get optimum MSE after little simplification as

$$MSE(T_2)_{opt} = V(\bar{y}) - \bar{Y}^2\left[\theta_3\rho_{yx}^2 C_y^2 + \theta_2\rho_{yz}^2 C_y^2\right] \tag{2.9}$$

3. PROPOSED COMPOSITE CHAIN RATIO TYPE ESTIMATOR OF POPULATION MEAN

We propose a composite chain ratio type estimator of population $\bar{Y}$ as

$$T_3 = W\bar{y} + (1 - W)T_1 \tag{3.1}$$

where W is some unknown constants.

The bias of T_3 is obtained as

$$B(T_3) = E(T_3) - \bar{Y}$$

$$= E\left[W\bar{y} + (1 - W)T_1\right] - \bar{Y}$$

$$= E\left[W\bar{y} + (1 - W)\bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)\left(\frac{\bar{Z}}{\bar{z}'}\right)\right] - \bar{Y}$$

$$= E\left[W\left(\bar{Y} + \epsilon_0\right) + (1 - W)\left(\bar{Y} + \epsilon_0\right)\left(\frac{\bar{X} + \epsilon_1}{\bar{X} + \epsilon_2}\right)\left(\frac{\bar{Z}}{\bar{Z} + \epsilon_3}\right)\right] - \bar{Y}$$

under the assumptions on ϵ_i 's, i=1,2,3 in Section-2, the bias of T_3, upto the first order of approximation, is obtained as

$$B(T_3) = (1 - W)\bar{Y}\left[\theta_3\left(C_x^2 - \rho_{yx} C_y C_x\right) + \theta_2\left(C_z^2 - \rho_{yz} C_y C_z\right)\right] \tag{3.2}$$

Mean square error of proposed estimator T_3 is derived as

$$MSE(T_3) = E(T_3 - \bar{Y})^2$$

$$= E\left[W\bar{y} + (1-W)T_1 - \bar{Y}\right]^2$$

$$= E\left[W\bar{y} + (1-W)\bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)\left(\frac{\bar{Z}}{\bar{z}'}\right) - \bar{Y}\right]^2$$

$$= E\left[W(\bar{Y} + \epsilon_0) + (1-W)(\bar{Y} + \epsilon_0)\left(\frac{\bar{X} + \epsilon_1}{\bar{X} + \epsilon_2}\right)\left(\frac{\bar{Z}}{\bar{Z} + \epsilon_3}\right) - \bar{Y}\right]^2$$

Upto the first order approximation, $MSE(T_3)$ is obtained as

$$MSE(T_3) = W^2\theta_1\bar{Y}^2C_y^2 + (1-W)^2\bar{Y}^2[A] + 2(1-W)\bar{Y}^2[B] \quad (3.3)$$

where

$$A = \theta_1C_y^2 + \theta_2C_z^2 + \theta_3C_x^2 - 2\theta_3\rho_{yx}C_xC_y - 2\theta_2\rho_{yz}C_yC_z$$

and $B = \theta_1C_y^2 - \theta_3\rho_{yx}C_yC_x - \theta_2\rho_{yz}C_yC_z$

Differentiating the $MSE(T_3)$ given in (3.2) with respect to W, we get

$$\frac{\partial MSE(T_3)}{\partial W} = 2W\theta_1\bar{Y}^2C_y^2 - 2(1-W)\bar{Y}^2A + 1(1-2W)\bar{Y}^2B$$

Equating the above differential to zero and after solving it, we get the optimum value of W as

$$W_{opt} = \frac{A-B}{\theta_1C_y^2 + A - 2B} \quad (3.4)$$

Putting the value of W in equation no. (3.3), we get optimum MSE as follows

$$MSE(T_3)_{opt} = \bar{Y}^2\left[\frac{(A-B)^2\theta_1C_y^2 + (\theta_1C_y^2 - B)^2A + 2(A-B)(\theta_1C_y^2 - B)B}{(\theta_1C_y^2 + A - 2B)^2}\right] \quad (3.5)$$

4. EMPIRICAL ILLUSTRATION FOR RELATIVE EFFICIENCY OF THE PROPOSED ESTIMATORS

We have considered four real populations for the purpose of investigation of the relative efficiency of the estimators which descriptions are given below

Table 1. Description of the population

S.N.	Population size	Source of Data	Y	X	Z
1	34	Singh & Chaudhary *Theory and Analysis of Sample Survey Designs*, First Edition, 1986, pp- 177	Area Under Wheat in 1974	Area Under Wheat in 1973	Total Cultivated Area in 1971
2	34	Sukhatme & Sukhatme *Sampling theory of surveys with application*, 1970, pp -185	Area Under Wheat in 1937	Area Under Wheat in 1936	Total Cultivated Area in 1931
3	200	Sukhatme & Chand (1977)	Apple trees of bearing age in 1964	Bushels of apples harvested in 1964	Bushels of apples harvested in 1959
4	34	B.K.Singh, W. W. Chanu and Manoj Kumar (2015) *Journal of Statistics Applications & Probability* 4, No. 1, 37-51	Area under wheat in 1964	Area under wheat in 1963	Cultivated area in 1961

The details of parameters of the populations are given below

Table 2. Description of population parameters

S.N.	Parameter	Population I	Population II	Population III	Population IV
1	N	34	34	200	34
2	n′	10	10	30	10
3	n	4	4	20	7
4	C_x	0.72	0.76	2.02	.72
5	C_y	0.75	0.75	1.59	.75
6	C_z	0.85	0.62	1.44	.59
7	ρ_{yx}	0.98	0.93	0.93	.98
8	ρ_{yz}	0.44	0.89	0.77	.90
9	ρ_{zx}	0.45	0.83	0.84	.91
10	$\bar{X}$	208.88	218.41	2934.58	208.89
11	$\bar{Y}$	199.44	201.41	1031.82	199.44
12	$\bar{Z}$	856.41	765.35	3651.49	747.59

The relative efficiency of estimators is determined against the simple sample mean per unit $\bar{y}$ as

$$E_i = \frac{V(\bar{y})}{MSE(t_i)} \times 100$$

Table 3. The variance and relative efficiency (RE%) of the estimators as compared to simple sample mean $(\bar{y})$ in four different populations

Estimator	Population I		Population II		Population III		Population IV	
	Variance	RE (%)	Variance	RE(%)	Variance	RE (%)	Variance	RE (%)
$\bar{y}$	4977.47	100.00	5094.069	100.00	122168.87	100.00	2558.88	100.00
$\bar{y}_{rd}$	3446.36	144.42	3704.29	137.50	63017.20	193.80	1630.84	156.90
$\bar{y}_1$	3445.48	144.46	3692.49	137.90	81974.55	149.00	1629.91	156.99
T_1	2229.69	223.23	2411.50	211.24	60180.54	203.00	350.18	730.72
T_2	1415.42	351.66	793.64	641.86	41478.02	294.53	328.22	779.60
T_3	1578.98	315.23	884.68	575.80	40974.91	298.15	334.68	764.60
T_{l2}	3978.51	125.10	3260.18	156.66	41832.11	292.05	998.69	256.22
T_{l3}	3225.62	154.31	1848.91	275.51	49500.80	246.80	525.84	486.62
T_{l4}	3148.63	158.08	2402.59	212.59	36904.56	292.69	338.21	756.57
$\bar{Y}_{EC}^{RdR}$	2453.80	202.84	1416.61	359.59	51561.1	236.94	713.18	358.79
$\bar{Y}_{EdR}^{dc}$	4957.32	100.40	5050.92	100.85	121498.14	100.55	2588.24	98.86

where t_i's are various existing and proposed estimators. Explicitly, they are denoted as $t_1 = \bar{y}_{rd}$, $t_2 = \bar{y}_{lrd}$, $t_3 = T_1$, $t_4 = T_2$, $t_5 = T_3$, $t_6 = T_{l2}$, $t_7 = T_{l3}$, $t_8 = T_{l4}$, $t_9 = \bar{Y}_{EC}^{RdR}$ and $t_{10} = \bar{Y}_{EdR}^{dc}$

These relative efficiencies have been worked out for three populations which are described in Table 3.

It can be observed from the results of the Table 3 that the proposed estimator, i.e Transformed chain ratio type estimator has out-performed other estimators in all the populations except in population III where transformed chain ratio-type and composite chain ratio-type estimators are almost at par . For population III, transformed chain ratio-type, composite chain ratio-type, T_{l2} due to Kiregyera (1980) and T_{l4} due to Khare *et al.* (2013) are almost equally efficient.

In general, the proposed transformed chain ratio type estimator can be recommended for estimator of population mean ($\bar{Y}$) using information on two auxiliary variables. However, some more composite chain ratio cum-regression estimators can be envisaged, which is under investigation by authors.

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