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BTIB Designs and their Efficiencies for Comparing Test versus Control Treatments with Correlated Observations

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SUMMARY

The experimental design for comparing test treatments with one or more controls is an important part of scientific experimentation. All possible paired comparisons among treatments may not be of equal interest. The main interest is to compare test treatments with control treatments and variance-balanced designs may not be useful. Balanced Treatment Incomplete Block (BTIB) designs and some reinforced BIB designs are usually considered for comparing test with one or more control treatments. Experiments to compare certain test with control treatment were first considered by Hoblyn, *et al.* (1954). Most of the authors developed block designs with no correlation among the plots in a block. But, in agricultural experiments, correlation among the plots or neighbour plot effect is quite common. Thus, an attempt has been made to develop *BTIB* designs having one control treatment with correlated observations. The designs are developed from 1st order neighbour balanced (NN1) block designs in linear blocks. List of developed BTIB designs having one & two control treatments are presented with their corresponding efficiencies for rho values ($0 \leq \rho \leq 1$), where rho (ρ) is correlation coefficient among plots within a block.

Keywords: BTIB designs, Correlated observations, 1st order neighbour balanced (NN1) designs.

1. INTRODUCTION

Experiments are usually conducted to compare all possible pair of treatments. Comparing newly developed variety(s)/genotype(s) with ruling one (or control/check) is an additional part in many areas of scientific experimentation. In such situations, the interest is mainly in a sub-set of all possible paired comparisons. Cox (1958) suggested augmenting an incomplete block design in test treatments with one or more replications of the control in each block to obtain appropriate design. Bechhofer and Tamhane (1981) developed the theory of incomplete block designs for comparing several treatments with a control. Their developments led to the concept of Balanced Treatment Incomplete Block (BTIB) designs. Further, Majumdar & Notz (1983) developed certain types of optimal BTIB designs. Later, Kiefer & Wynn (1981), Wilkinson *et al.* (1983) and Gill & Shukla (1985) studied efficiency of such designs with correlated observation. Generally, these BTIB designs are not variance or efficiency balanced.

In 1985, Das & Ghosh, introduced the concept of 'General Efficiency Balanced' (GEB) designs which are developed through method of reinforcement by Kageyama & Mukerjee (1986) are similar to R- type BTIB designs under certain conditions. Literature survey reveals that the study on block designs for test *vs.* control treatments was mostly confined to block designs where the plots in a block were uncorrelated. But, in agricultural experiments, the presence of

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correlation in the form of neighbour effects among the adjacent plots is a well established fact. Even though, the experimentations with correlated observations are rarely used in field trials because of its analysis procedure and no clear cut structure on information matrices (like **C**). Therefore, the present study aims to construct and examine the characteristics of BTIB designs having correlated observations along with efficiency values (Canonical, A and D).

2. DEFINITION AND MODELS OF BLOCK DESIGNS WITH CORRELATED OBSERVATIONS

2.1 Balanced Treatment Incomplete Block (BTIB)

Bechhofer and Tamhane (1981) defined that BTIB designs are the incomplete block designs in which each test treatment with the control appears $\boldsymbol{\lambda}_0$ times and any pair of test treatment appears together $\boldsymbol{\lambda}_1$ times in the same block.

In the form of notations, BTIB design with v+1 treatments (0, 1, 2, …, v) by the relation $\Sigma_{j=1}^{b} n_{0j} n_{ij} = \lambda_0$ for i=1, 2, …v and $\Sigma_{j=1}^{b} n_{ij} n_{i^*j} = \lambda_1$ for i≠i*, i=1, 2, …v, where n_{ij} be the elements of incidence matrix of the BTIB design.

2.2 Model and important results of block designs with correlated observations

Let us assume a class of BIB designs (**D**) with *v* treatments and *b* blocks of sizes *k* (<*v*). Fixed effects additive model is considered for analyzing a first order neighbour balanced (NN1) block design having correlated observations in linear blocks as given below:

$$\mathbf{Y} = \boldsymbol{\mu}\mathbf{1} + \mathbf{X}\boldsymbol{\tau} + \mathbf{Z}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \qquad (2.1)$$

Where, **Y** is a vector of observations of order (n × 1), μ is a general mean, **1** is a vector of ones of order (n × 1), **X** is incidence matrix of observations versus treatments having order (n × v), **τ** is a vector of treatment effects of order (v × 1), **Z** is incidence matrix of observations versus blocks of order (n × b), **β** is vector of block effects having order (b × 1) and **ε** is a vector of random errorsof order (n × 1). According to Gill and Shukla (1985), **ε** be the error terms independently and normally distributed with mean zero & variance & covariance matrix be **V**, such that $\mathbf{V}^{-1} = \sigma_{\varepsilon}^{-2}\mathbf{I_b} \otimes \mathrm{W}_{\mathbf{k}}$ [$\mathbf{I}_\mathrm{b}$ is an identity matrix of order b, ⊗ denotes the kronecker product and $\mathbf{W}_\mathrm{k}$ is the correlation matrix of k observations within a block (Majumder *et al.* 2015, Patil *et al.* 2016 & Manjunatha *et al.,* 2017].

$$\mathbf{W_k} = \begin{bmatrix} 1 & \rho & 0 & \cdots & 0 \\ \rho & 1 & \rho & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & \rho & 1 & \rho \\ 0 & 0 & \cdots & \rho & 1 \end{bmatrix} \qquad (2.2)$$

Where, ρ (-1≤ ρ ≤ +1) is the correlation coefficient between of neighbouring plots in a block.

The information matrix (**C**) for estimating the treatment effects having correlated observations estimated by generalized least squares is

$$\mathbf{C} = \mathrm{X'V^{-1}X - X'V^{-1}Z\,(Z'V^{-1}Z)^{-1}Z'V^{-1}X} \qquad (2.3)$$

The **C** matrix (2.3) for estimating the treatments effect in a block design is symmetric, non-negative definite with zero row and column sums.

3. NN1 BIB DESIGNS FOR CORRELATED OBSERVATIONS

3.1 Results on Neighbour Balanced BIB designs for correlated observations

Assuming the model (2.1) for a BIB design (D) with *v, b, r, k* and *λ*, the following relationships among the quantities can be defined.

Let e_i be the number of blocks in which treatment *i* occurs at an end plot, e_{Li} be the number of blocks for which treatment *i* occurs at a left end plot and e_{Ri} be the number of blocks for which treatment *i* occurs at a right end plot.

Here, $e_i = e_{Li} + e_{Ri}$.

Let *g (j, r)* be the treatment number of the r^{th} plot in the j^{th} block and A_i is the set of blocks in which treatment *i* occurs.

$e_{ii'}$ = # { j: j ∈ ($A_i \cap A_{i'}$), g(j,1) = i or g (j,k) = i} + # { j: j ∈ ($A_i \cap A_{i'}$), g (j, 1) = i' or g (j,k) = i'},

i.e. $e_{ii'}$ is the number of blocks in ($A_i \cap A_{i'}$), in which either i or i' occurs at an end and that blocks contain i' or i and a block will be counted twice when both i and i' are at the two end plots.

Now, $N_{ii'}$ = # { j: g(j,r) = i or g (j,s) = i', |r – s| = 1};

i.e. $N_{ii'}$ is the number of times *i* & *i'* are neighbour in a block and $\theta_{ii'}$ is the number of times when both *i* and *i'* treatments are at end plots in a block.

Provided, $\sum_{i'(\neq i)} e_{ii'} = 2r + (k-2)e_i$,

$\sum_{i'(\neq i)} N_{ii'} = 2r - e_i$ $\sum_{i'(\neq i)} N_{ii'} = 2r - e_i$,

$\sum_{i'(\neq i)} \theta_{ii'} = e_i$

When the neighbor effect exists among the adjacent treatments as neighbors belonging to same block, the model (2.1) is practical.

Lemma 3.1: Assuming the above correlated structure and parameters of the BIB design in **D**, the elements of the C matrix of design **D** are

$$C_{ii} = \frac{r(k-1) + 2r\rho(k-3-2\rho) + \rho e_i(2+3\rho)}{k+2\rho(k-1)}$$ and

$$C_{ii'} = \frac{-\lambda(1+2\rho)^2 + \rho(e_{ii'} + kN_{ii'}) + \rho^2(2e_{ii'} + 2(k-1)N_{ii'} - \theta_{ii'})}{k+2\rho(k-1)}$$

(i ($\neq$i') = 1, 2, …,v)

Proof: It is straight forward, if it proceed with the model 2.1, the **C** matrix as 2.3. It is also examined that the **C** matrix is completely symmetric with row or column sums as zero.

Theorem 3.1: The BIB design will be neighbour balanced (NN1) BIB design with correlated observations in linear blocks if any of the following conditions are satisfied for any particular ρ.

C1. All $e_{ii'}$'s, $N_{ii'}$'s and $\theta_{ii'}$'s are individually constant for all i' (i' = ($\neq$i) 1, 2, .., v).

C2. The quantities ($e_{ii'}$ + k $N_{ii'}$) and (2$e_{ii'}$ + 2(k-1) $N_{ii'}$ - $\theta_{ii'}$) are individually constant for

all i' (i' = ($\neq$i) 1, 2, .., v).

Proof: Considering the lemma 3.1, the proof is straight forward.

3.2 Construction of NN1 BIB designs in linear blocks with correlated observations

Let us consider a First Order Neighbour Balanced (NN1) μ-resolvable BIB design **D** (Sahu & Majumder, 2012) having *p* initial blocks with parameters v, $b = pv$, $r = pk$, k, $\lambda = r(k-1)/(v-1)$, $\mu = k$, $m = v$, $t = p$ with $\lambda_1 = 2p(k-1)/(v-1)$, where λ_1 is an integer and it appears any pair of treatments as 1st order neighbors with $e_{Li} = e_{Ri}$ and $e_i = 2e_{Li} = 2e_{Ri}$ $\forall$ i (= 1,2,..,v). In **D**, all $e_{ii'}$'s, $N_{ii'}$'s and $\theta_{ii'}$'s are individually constant for all i' (i' = ($\neq$i) 1, 2, .., v), $e_{ii'}$ = 4λ/k; $N_{ii'}$ = 2λ/k and $\theta_{ii'}$ = 2p/(v-1) and also e_i = 2p $\forall$ i (= 1,2,..,v).

Illustration 3.1: Let D be a μ-resolvable BIB design having parameters v =7, b = 21, r =12, k = 4, λ = 6, μ = 4, m =7 and t = 3, the blocks of design D are

1	3	2	6	3	2	6	4	2	6	4	5
2	4	3	0	4	3	0	5	3	0	5	6
3	5	4	1	5	4	1	6	4	1	6	0
4	6	5	2	6	5	2	0	5	2	0	1
5	0	6	3	0	6	3	1	6	3	1	2
6	1	0	4	1	0	4	2	0	4	2	3
0	2	1	5	2	1	5	3	1	5	3	4

The illustration 3.1 shows a μ-resolvable NN1 BIB design. Thus each pair of treatments in the design which are immediately neighbor to each other is occurring 3 times *i.e.,* $\lambda 1$ =3. It is also noted that in the above design considering correlated error, the parameters of NN1 structure are $e_{ii'}$ = 6, $N_{ii'}$ = 3, $\theta_{ii'}$ = 1, $e_{Li} = e_{Ri}$ = 3 and e_i = 6, ($e_{ii'}$ + k $N_{ii'}$) = 18 and (2$e_{ii'}$ + 2(k-1)$N_{ii'}$ - $\theta_{ii'}$) = 29, $\forall$ i (= 1,2,..,7).

4. R-TYPE BTIB DESIGNS - CORRELATED OBSERVATIONS IN LINEAR BLOCKS

As per Bechhofer & Tamhane (1981), a new series of BTIB designs with correlated observations developed from NN1 BIB designs D (v, b, r, k, λ) which is shown (in Lemma 3.1 & Theorem 3.1).

Let one extra treatment as control is added to last plots of a NN1 BIB design **D** (*v, b, r, k* & *λ*). The new design **D*** (v*= v+1, b* = b, **r**= (r. $\mathbf{1}_V$, r*=b), k*=k+1, *λ* & λ*=r) will be a BTIB design with correlated observations.

The developed designs with correlated observations are also an incomplete block designs in which each test treatment with the control appears λ^*_0 times and any pair of test treatment appears λ^*_1 together in the same block. Formal parametric relations of BTIB designs (in section 2) are also applicable for the newly developed designs.

In the design **D***, the treatments are arranged in b*(= b) blocks of equal sizes k*(=k+1). The replication vector is **r** = (r. $\mathbf{1}_V$, r*=b)' with r^* denoting the control/added treatment replication and r denoting the replication number of i^{th} test treatment, i=1,2,..., v.

The fixed effects additive model assumed for analyzing the newly developed BTIB design with correlated observations (**D***) will be similar to the model 2.1 with v* = v+1 treatments and the correlation matrix of (k+1) observation within a block $\mathbf{W}_{k+1}$ is

$$\mathbf{W_{k+1}} = \begin{bmatrix} 1 & \rho & 0 & \cdots & 0 & 0 \\ \rho & 1 & \rho & \cdots & 0 & 0 \\ 0 & \rho & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & 0 \\ \vdots & \vdots & \vdots & & \vdots & 0 \\ 0 & 0 & 0 & \cdots & 1 & \rho \\ 0 & 0 & 0 & & \rho & 1 \end{bmatrix} \quad (4.1)$$

Where, ρ ($-1 \le \rho \le +1$) is the correlation between the neighbouring plots in a block.

4.1 Results on Neighbour Balanced BTIB designs for correlated observations

Let one extra treatment as control is added to last plots of a NN1 BIB design D (*v, b, r, k* and *λ*). The new design D*(v*= v+1, b* = b, **r** = (r. $\mathbf{1}_V$, r*=b), k*=k+1, *λ* and λ*=r) will be a BTIB design with correlated observations and must hold following relations.

$$e_i = \frac{b}{v}, \quad \sum_{i'=1(\neq i)}^{v} e_{ii'} = r - e_i + e_i(k-1) = r + e_i(k-2)$$, when all $e_{ii'}$ are equal, $e_{ii'} = \frac{r+e_i(k-2)}{v-1}$

$$\sum_{i=1(\neq i)}^{v} N_{ii'} = 2(r - 2e_i) + 2e_i = 2(r - e_i)$$, when all $N_{ii'}$ are equal, $N_{ii'} = \frac{2(r-e_i)}{v-1}$;

$$\theta_{ii'} = 0; \quad e_{ij'} = r - e_i + 2e_i = r + e_i;$$

$$N_{ij'} = e_i; \quad \theta_{ij'} = e_i; \quad e_{j'} = b$$

4.1.1 Method of construction of R-Type BTIB designs for comparing test and control treatment with correlated observations

Theorem 4.1.1: If there exists a First Order Neighbour Balanced (NN*1*) BIB design (D) then there will be a R- type BTIB design (D*) with parameters v*= v+1, b*= b, $r^{*\prime} = [r.1_v^{\prime}, r^{\prime}]$, k* = k+1with correlated errors. The elements of C- Matrix of D* will be

$$C_{ii} = \frac{rk + 2r\rho(k-2-2\rho) + \rho e_i(2+3\rho)}{1+k(1+2\rho)}; \ \forall \ (i=1,2,\ldots,v);$$

$$C_{ii'} = \frac{-\lambda(1+2\rho)^2 + \rho\left(e_{ii'}+(k+1)N_{ii'}\right) + \rho^2\left(2e_{ii'}+2kN_{ii'} - \theta_{ii'}\right)}{1+k(1+2\rho)};$$
$$\forall \ ((i,i'(i \neq i')=1,2,\ldots,v);$$

$$C_{ij'} = \frac{-\lambda^*(1+2\rho)^2 + \rho\left(e_{ij'}+(k+1)N_{ij'}\right) + \rho^2\left(2e_{ij'}+2kN_{ij'} - \theta_{ij'}\right)}{1+k(1+2\rho)};$$
$$\forall (i=1,2,\ldots,v \text{ and } j'=v+1) \text{ and}$$

$$C_{j'j'} = \frac{r^*k + 2r^*\rho(k-2-2\rho) + \rho e_{j'}(2+3\rho)}{1+k(1+2\rho)}; \ \forall (j'=v+1)$$

Proof:

Let us add one extra treatment to each block at end of NN*1 BIB* design D (*v, b, r, k* and *λ*) with $e_{Li} = e_{Ri}$ and $e_i = 2e_{Li} = 2e_{Ri}\ \forall$ i (= 1,2,..,v) then new design D*'s parameters are v*= v+1, b* = b, **r** = (r.$\mathbf{1}_V$, r*=b), k*=k+1, *λ* and λ*=r with the neighboring relations as shown in the section 4.1 and it is R- type BTIB design having v* (= v+1) treatments and b* (= b) blocks of size k* (= k+1). From the model 2.1 & C matrix 2.3, the elements of C matrix will be $m = C_{ii}, \forall (i = 1,2,\ldots,v)$; $n = C_{ii'}, \forall (i,i'(i \neq i') = 1,2,\ldots,v)$;
$p = C_{ij'}, \forall (i = 1,2,\ldots,v \ \& \ j' = v+1)$ and
$q = C_{j'j'}, (j' = v+1)$

$$C = \begin{bmatrix} m & n & n & \cdots & n & p \\ n & m & n & \cdots & n & p \\ n & n & m & \cdots & n & p \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ n & n & n & \cdots & m & p \\ p & p & p & & p & q \end{bmatrix} \quad (4.2)$$

Illustration 4.1.1: Let us add one extra treatment to each block of the design of illustration 3.1. The blocks of the new design **D*** are given below:

1	3	2	6	7	3	2	6	4	7	2	6	4	5	7
2	4	3	0	7	4	3	0	5	7	3	0	5	6	7
3	5	4	1	7	5	4	1	6	7	4	1	6	0	7
4	6	5	2	7	6	5	2	0	7	5	2	0	1	7
5	0	6	3	7	0	6	3	1	7	6	3	1	2	7
6	1	0	4	7	1	0	4	2	7	0	4	2	3	7
0	2	1	5	7	2	1	5	3	7	1	5	3	4	7

Here, v*= 8, b* = b = 21, $e_i = 3$; $e_{ii'} = \frac{r+e_i(k-2)}{v-1} = 3$;
$N_{ii'} = \frac{2(r-e_i)}{v-1} = 3$; $\theta_{ii'} = 0$; $e_{ij'} = r + e_i = 15$; $N_{ij'} = e_i = 3$;
$N_{ij'} = e_i = 3$; $N_{ij'} = e_i = 3$;

4.1.2 Conditions for R-Type BTIB designs to be GEB designs

Kageyama and Mukherjee (1986) had constructed Generalized Efficiency Balanced (GEB) designs through method of reinforcement of a BIB design with parameters (v, b, r, k & λ). They found that if one new treatment is added to each block of the BIB design then the resultant design will be a GEB design with v+1 treatments.

We know that the **C**- matrix of any GEB design for (v+t) treatments will be

$$C^* = \theta\left[\begin{bmatrix} sI_v & 0 \\ 0 & zI_t \end{bmatrix} - g^{-1}\begin{bmatrix} s1_v \\ z1_t \end{bmatrix}\begin{bmatrix} s\,1_v' & z\,1_t' \end{bmatrix}\right], \quad (4.3)$$

where g = vs + tz for any values of 'θ' and the current study interested only t = 1.

Theorem 4.1.2: The R- type BTIB design **D*** developed in Theorem 4.1.1 will be a GEB design with correlated observations for a particular value of ρ if the **C** matrix of **D*** (4.2) will be expressed in terms of 4.3.

Proof: The **C** matrix of **D*** (in 4.2) rewritten as

$$C^* = \frac{-g}{n}\left[\begin{bmatrix} \frac{(m-n)n}{-g} I_v & \underset{\sim}{0} \\ \underset{\sim}{0}' & (q-\frac{p^2}{n})\frac{n}{-g} \end{bmatrix} - \frac{1}{g}\begin{bmatrix} n1_v \\ p \end{bmatrix}\begin{bmatrix} n1_v' & p \end{bmatrix}\right]$$

$$= \theta\left[\begin{bmatrix} nI_v & \underset{\sim}{0} \\ \underset{\sim}{0}' & p \end{bmatrix} - \frac{1}{g}\begin{bmatrix} n1_v \\ p \end{bmatrix}\begin{bmatrix} n1_v' & p \end{bmatrix}\right],$$

it is examined that elements of C matrix of design D* sums in a row or column is zero (4.3), thus, m+(v-1) n+p = 0 and vp + q = 0.

Here, θ = -g/n, s = n, g = (vn+p) and z = p.

5. OPTIMALITY PROPERTIES OF NEWLY DEVELOPED BTIB DESIGNS WITH CORRELATED OBSERVATIONS

Let us define **Dρ** (v* = v+1, b, k+1) to denote a class of connected block designs with correlated observations having v+1 treatments arranged in b blocks of size k+1 for a particular value of ρ.

According Theorem 3.1 of Jacroux (1983), the design **D*** as stated in Theorem 4.1.1will be E-optimal in the class of block designs with correlated observations **Dρ** (v* = v+s, b, k+1) for a particular ρ value and s = 1. Constantine (1983) and Jacroux (1984) proved that in the class of binary BTBD (Balanced Treatment Block Design) as defined by Bechhofer and Tamhane (1981) which is also an SR(1) (Standard treatment exactly occurs one time in each block of the design) is A optimal among the class of all available SR(1) designs. The design **D*** developed in Theorem 4.1 is also an binary BTIB as well as SR(1) design with correlated errors. It is obvious that the design **D*** is also A- optimal in the class of designs with correlated errors **Dρ** (v* = v+1, b, k+1) for a particular ρ value.

5.1 Efficiency of R- type BTIB designs with correlated observations

The canonical efficiency of a design is calculated as the harmonic mean of the (v*-1) non-zero eigen roots of the matrix $r^{-\delta}C$. Here, in case of **C** matrix as (4.2), (m-n)/r roots with multiplicity (v*-2) and the remaining root is $\frac{(v^*-1)m}{r} + \frac{q}{r^*} - [(v^*-2)\frac{m-n}{r}]$.

The designs will have different A-and D-efficiencies. The choice of an appropriate design can thus be based on the lower bound of the above efficiency values (A and D). These lower bounds are obtained on the lines of Cheng and Wu (1981). For a connected block design *d*, let $\theta_1, \theta_2, \ldots, \theta_{v-1}$ be the non-zero eigen values of **C**. Now define

$$\phi_A(d) = \sum_{i=1}^{v-1}\theta_i^{-1} \text{ and } \phi_D(d) = \prod_{i=1}^{v-1}\theta_i^{-1}$$

Then, a design is A- [D-] optimal if it minimizes the $\varnothing_A(d), \varnothing_D(d)$ over $\mathbf{D}_\rho$(v* = v+1, b, k+1). The A-efficiency { $e_A(d)$ } and D-efficiency { $e_D(d)$ } of any design *d* over $\mathbf{D}_\rho$(v* = v+1, b, k+1) is defined as $e_A(d) = \frac{\varnothing_A(d_A^*)}{\varnothing_A(d)}$ and $e_D(d) = \left[\frac{\varnothing_D(d_D^*)}{\varnothing_D(d)}\right]^{1/(v-1)}$ where, d_A^* and d_D^* are the hypothetical A-optimal and D-optimal design over $\mathbf{D}_\rho$(v* = v+1, b, k+1), respectively. The $\phi_A(d_A^*)$ and $\phi_D(d_D^*)$ values are calculated following the lines of Ponnuswamy and Santharam (1997). Let $\theta_1, \theta_2, \ldots, \theta_{v-1}$ be the non-zero eigenvalues of **C** matrix. The design is A (or D) optimal if all θ_i's are equal. However, such a design for a given set of parameters may not exist. The information matrix of the hypothetical A (or D) optimal design either d_A^* (or d_D^*) would have eigenvalue $\theta = (v-1)^{-1}(\theta_1 + \theta_2 + \ldots + \theta_{v-1})$ with multiplicity (v-1). Thus $\phi_A(d_A^*)$ of a hypothetical design d_A^* and $\phi_D(d_D^*)$ of a hypothetical design d_D^* has been computed as mentioned above. Following the lines of Cheng and Wu (1981), the lower bounds of A-efficiency $\{e_A(d)\}$

and D-efficiency $\{e_D(d)\}$ of any design d over $\mathbf{D}_\rho$ (v, b, k) are given by

$$e'_A(d) = \frac{(v-1)^2}{\{b(k-1)\emptyset_A(d)\}} \text{ and } e'_D(d) = \frac{(v-1)}{[b(k-1)\{\emptyset_D(d)\}^{1/v-1}]}.$$

The efficiency values (Canonical, A and D) for different values of ρ (0 ≤ ρ ≤ 1) of different BTIB designs (**D***) with correlated observations having one control treatment are presented in Table 1.

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Table 1. Efficiency values (Canonical, A and D) of different R-type BTIB designs with one control treatment for correlated observations (0 ≤ρ ≤1)

BIB					BTIB				
v	*b*	*r*	*k*	*λ*	*v**	*b**	*r**	*k**	*λ**
7	**21**	**9**	**3**	**3**	**8**	**21**	**9,21**	**4**	**3,9**
ρ	**CA**		**Ea**		**EaLB**		**Ed**		**EdLB**
0.0	0.854		0.909		0.909		0.944		0.944
0.1	0.821		0.899		0.853		0.938		0.890
0.2	0.786		0.888		0.797		0.930		0.834
0.3	0.749		0.874		0.739		0.920		0.778
0.4	0.708		0.859		0.680		0.910		0.720
0.5	0.665		0.841		0.621		0.896		0.662
0.6	0.618		0.820		0.561		0.881		0.602
0.7	0.568		0.794		0.500		0.861		0.542
0.8	0.514		0.763		0.439		0.837		0.482
0.9	0.456		0.724		0.377		0.805		0.420
1.0	0.393		0.675		0.315		0.764		0.357
7	**21**	**12**	**4**	**6**	**8**	**21**	**12,21**	**5**	**6,12**
ρ	**CA**		**Ea**		**EaLB**		**Ed**		**EdLB**
0.0	0.913		0.960		0.960		0.977		0.977
0.1	0.893		0.954		0.916		0.974		0.934
0.2	0.872		0.948		0.870		0.970		0.890
0.3	0.848		0.941		0.825		0.965		0.846
0.4	0.823		0.933		0.778		0.960		0.801
0.5	0.796		0.924		0.731		0.954		0.755
0.6	0.767		0.913		0.684		0.947		0.709
0.7	0.736		0.901		0.636		0.939		0.663
0.8	0.702		0.886		0.588		0.929		0.616
0.9	0.666		0.869		0.539		0.917		0.568
1	0.626		0.849		0.490		0.902		0.521
9	**36**	**12**	**3**	**3**	**10**	**36**	**12,36**	**4**	**3,12**
ρ	**CA**		**Ea**		**EaLB**		**Ed**		**EdLB**
0.0	0.830		0.878		0.878		0.921		0.921
0.1	0.780		0.868		0.824		0.913		0.867
0.2	0.730		0.856		0.769		0.904		0.812
0.3	0.678		0.843		0.712		0.894		0.756
0.4	0.626		0.828		0.655		0.883		0.699
0.5	0.573		0.810		0.598		0.869		0.641
0.6	0.519		0.789		0.540		0.852		0.583
0.7	0.464		0.764		0.482		0.832		0.524
0.8	0.409		0.734		0.423		0.807		0.464

0.9	0.353		0.698		0.364		0.775		0.404
1.0	0.297		0.652		0.304		0.735		0.343
9	**36**	**16**	**4**	**6**	**10**	**36**	**16,36**	**5**	**6,16**
ρ	**CA**		**Ea**		**EaLB**		**Ed**		**EdLB**
0.0	0.887		0.933		0.933		0.959		0.959
0.1	0.847		0.927		0.889		0.955		0.916
0.2	0.806		0.920		0.845		0.950		0.872
0.3	0.764		0.912		0.799		0.945		0.828
0.4	0.722		0.903		0.753		0.939		0.783
0.5	0.679		0.893		0.707		0.932		0.737
0.6	0.636		0.882		0.661		0.923		0.692
0.7	0.592		0.870		0.614		0.914		0.645
0.8	0.548		0.855		0.567		0.903		0.599
0.9	0.503		0.838		0.520		0.890		0.552
1.0	0.458		0.818		0.472		0.875		0.505
11	**55**	**15**	**3**	**3**	**12**	**55**	**15,55**	**4**	**3,15**
ρ	**CA**		**Ea**		**EaLB**		**Ed**		**EdLB**
0.0	0.815		0.857		0.857		0.902		0.902
0.1	0.777		0.847		0.804		0.894		0.849
0.2	0.736		0.835		0.750		0.885		0.794
0.3	0.693		0.822		0.695		0.875		0.739
0.4	0.649		0.807		0.639		0.862		0.683
0.5	0.602		0.790		0.583		0.848		0.626
0.6	0.553		0.769		0.526		0.831		0.569
0.7	0.503		0.745		0.470		0.810		0.511
0.8	0.450		0.716		0.412		0.785		0.452
0.9	0.394		0.681		0.355		0.754		0.393
1.0	0.336		0.637		0.297		0.714		0.333
11	**55**	**20**	**4**	**6**	**12**	**55**	**20,55**	**5**	**6,20**
ρ	**CA**		**Ea**		**EaLB**		**Ed**		**EdLB**
0.0	0.871		0.913		0.913		0.944		0.944
0.1	0.843		0.907		0.870		0.939		0.901
0.2	0.814		0.899		0.826		0.934		0.858
0.3	0.784		0.891		0.781		0.928		0.813
0.4	0.751		0.882		0.736		0.921		0.769
0.5	0.718		0.872		0.691		0.914		0.723
0.6	0.683		0.861		0.645		0.905		0.678
0.7	0.646		0.848		0.599		0.895		0.632
0.8	0.608		0.834		0.553		0.884		0.586
0.9	0.568		0.817		0.507		0.870		0.540
1.0	0.526		0.798		0.460		0.854		0.493

13	**78**	**18**	**3**	**3**	**14**	**78**	**18, 78**	**4**	**3,18**
ρ	**CA**		**Ea**		**EaLB**		**Ed**		**EdLB**
0.0	0.805		0.842		0.842		0.888		0.888
0.1	0.765		0.832		0.789		0.879		0.835
0.2	0.723		0.820		0.736		0.870		0.781
0.3	0.679		0.807		0.682		0.859		0.726
0.4	0.634		0.792		0.627		0.847		0.670
0.5	0.587		0.775		0.572		0.832		0.614
0.6	0.538		0.755		0.517		0.815		0.558
0.7	0.487		0.732		0.461		0.794		0.500
0.8	0.434		0.704		0.405		0.769		0.443
0.9	0.380		0.669		0.349		0.738		0.385
1	0.324		0.627		0.293		0.698		0.326
13	**78**	**24**	**4**	**6**	**14**	**78**	**24, 78**	**4**	**6,24**
ρ	**CA**		**Ea**		**EaLB**		**Ed**		**EdLB**
0.0	0.860		0.898		0.898		0.932		0.932
0.1	0.830		0.891		0.855		0.926		0.889
0.2	0.799		0.884		0.812		0.921		0.845
0.3	0.767		0.876		0.768		0.915		0.801
0.4	0.734		0.867		0.723		0.908		0.757
0.5	0.699		0.857		0.678		0.900		0.712
0.6	0.662		0.846		0.633		0.891		0.667
0.7	0.625		0.833		0.588		0.880		0.622
0.8	0.586		0.819		0.543		0.869		0.576
0.9	0.545		0.802		0.497		0.855		0.530
1	0.504		0.783		0.452		0.839		0.484

Note: CA= Canonical Efficiency, Ea = A Efficiency, Ed = D Efficiency, LB= Lower Bound

Note: CA= Canonical Efficiency, Ea = A Efficiency, Ed = D Efficiency, LB= Lower Bound