



## **Power Computation based Performance Assessment of ARIMA Intervention Modeling**

**Mrinmoy Ray and Ramasubramanian V.**

*ICAR-Indian Agricultural Statistics Research Institute, New Delhi*

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### **SUMMARY**

In this article, a comparative study between ARIMA and ARIMA-Intervention models has been done using power computations under various situations. The study uncover that, as the magnitude of the impact parameter increases, the power to detect such a change also increases. Also, for a fixed value of impact parameter, the power of the test was high for ramp followed by step and then pulse intervention situations. Again, for fixed impact parameter value, when the slope parameters were increased, the power of the test to detect significant change also increased in both step and pulse types of interventions. When the impact value was increased, it has been observed that the power of the test is reasonably good even when there are only a few post-intervention observations. Thus it can be concluded that ARIMA intervention modelling can be successfully employed for forecasting purposes.

*Keywords:* ARIMA model, Intervention model, Intervention, Step, Ramp, Pulse, Power computation.

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### **1. INTRODUCTION**

A data set containing values on a single phenomenon observed at consecutive periods is called time-series. The most popular time series model is Auto-Regressive Integrated Moving Average (ARIMA) when the data under consideration is linear. However, when the linear time-series under study is disturbed by some external event known as intervention then the forecasting performance of ARIMA model may be affected. It can be improved by employing appropriate techniques such as ARIMA-Intervention modeling. There are three kinds of interventions viz. step, pulse and ramp. Step intervention occurs at a particular period of time and exists in the subsequent time periods. The effect of step intervention may remain constant over time or it may increase or decrease over time. In agriculture, such type of intervention occurs due to introduction of new variety, new economic policy etc. Pulse intervention occurs only at particular period of time but the effect of these type of intervention may exists for that particular time period only or may continue to exist in the subsequent time periods. In agriculture, if in specific

years, a flood or drought or pest/ disease epidemic occurs then these may be deemed as pulse intervention. Ramp intervention occurs at particular period of time and exists in the subsequent time periods with an increasing magnitude. The effect of ramp intervention will always increase over time. In agriculture one of the best example of this type of intervention is the price rise of agricultural commodity.

The intervention modeling and analysis are used to account for impact of any unprecedented events in the time series data. Initially, application of intervention models was done to study impact of air pollution controls, economic controls on the consumer price index (Box and Tiao, 1975). A good account on intervention modeling is given in Box *et al.* (1994), Madsen (2008), Yaffee and McGee (2000) etc. Shao (1997) applied Intervention model to quantify the impact of sales promotional data. Bianchi *et al.* (1998) analyzed existing and improved methods for forecasting incoming calls to telemarketing centers for the purposes of planning and budgeting. They found that

ARIMA models with intervention analysis performed better for the time series studied. Girard (2000) used ARIMA model with intervention in order to analyse the epidemiological situation of whooping-cough in England and Wales for the period of 1940-1990. Mcleod and Vingilis (2005) employed power function in intervention analysis to determine the probability that a proposed intervention analysis application will detect a meaningful change. Ismail *et al.* (2009) studied monthly data of five star hotels' occupancy in Bali city in the aftermath of occurrence of bombing in October, 2002 and have shown that intervention model is more appropriate for forecasting when compared to the conventional ARIMA model. Lam *et al.* (2009) used a time series intervention ARIMA model to measure the intervention effects and the asymptotic change in the simulation results of the business process reengineering that is based on the activity model analysis. Brakel and Roles (2010) employed Intervention model to study survey redesign. Ray *et al.* (2014) employed ARIMA Intervention model for modeling and forecasting cotton yield of India considering the introduction of Bt cotton as unprecedented technology. Ray *et al.* (2017) proposed a new technology forecasting tool viz. time series intervention model based trend impact analysis for envisioning crop yield scenarios.

As the data for intervention analysis for different types of situations may not be possible to collect or may not be available, the purpose of this article is only generating various types of intervention data and compares the performance of ARIMA-Intervention models with the conventional ARIMA models in these situations.

The rest of the article is organized in different sections. In section 2, overview of ARIMA, ARIMA-Intervention, simulation algorithm, statistics for evaluating the forecasting performance and power computation are given. Then in section 3, the comparison between ARIMA and ARIMA-Intervention has been numerically illustrated. Lastly, conclusions are given in section 4.

**2. MATERIALS AND METHODS**

**2.1 ARIMA model**

An ARIMA model is given by:

$$\phi(B)\Delta^d y_t = \theta(B)\varepsilon_t \tag{1}$$

where

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p \text{ (Autoregressive component)}$$

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q \text{ (Moving average component)}$$

$\varepsilon_t$  = white noise error term

d = differencing term

$$\Delta^d y_t = (1 - B)^d y_t$$

B = Backshift operator i.e.  $B^a y_t = y_{t-a}$ , where 'a' is the time lag

ARIMA methodology is carried out in three stages, viz., identification, estimation and diagnostic checking. Parameters of ARIMA model are tentatively selected at the identification stage and at the estimation stage, parameters are estimated using iterative least square techniques. The adequacy of the selected model is then tested at the diagnostic checking stage. If the model is found to be inadequate, the three stages are repeated until satisfactory ARIMA model is selected for the time-series under consideration.

**2.2 Intervention model**

The intervention model can be represented as follows:

$$y_t = \frac{\omega(B)}{1 - \delta(B)} B^b I_t + N_t \tag{2}$$

where,  $y_t$  is the dependent (time series) variable,  $I_t$  is the indicator variable coded according to the type of intervention,  $\delta(B) = 1 + \delta_1 B + \dots + \delta_r B^r$  i.e. slope parameter,  $\omega(B) = \omega_0 + \omega_1 B + \dots + \omega_s B^s$  i.e. impact parameter, b is delay parameter;  $N_t$  is the noise series, which represents the background observed series  $y_t$  but without intervention effects i.e.  $N_t$  is nothing but the  $y_t$  given in the ARIMA model equ. 1. The parameters of Intervention model are represented graphically in Fig. 1.

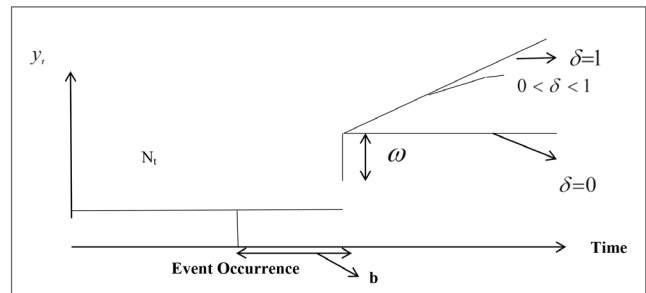


Fig. 1. Graphical representation of intervention model

In general, the values an intervention variable can take depends on the type of intervention. For step intervention

$$I_t = \begin{cases} 0 & t \neq T' \\ 1 & t \geq T' \end{cases} \forall t \quad (3)$$

with  $T'$  is time of intervention when it first occurred.

For pulse intervention

$$I_t = \begin{cases} 0 & t \neq T' \\ 1 & t = T' \end{cases} \forall t \quad (4)$$

For ramp

$$I_t = \begin{cases} 0 & t < T' \\ t - T' & t \geq T' \end{cases} \forall t \quad (5)$$

As with ARIMA model, fitting the intervention model consists of the usual three stages i.e. identification, estimation, diagnostic checking.

### 2.3 Algorithm for simulating data for various intervention situations

For simulation of ARIMA-Intervention process, the first step is to simulate ARIMA process. The ARIMA process has been simulated as given in Dunne (1992) which is as follows-

#### Simulation of Moving Average (MA) process:

A MA (q) model can be written as follows:

$$y_t = \varepsilon_t - \sum_{j=1}^q \theta_j \varepsilon_{t-j} \quad (6)$$

Firstly, generation of  $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n$  which are normally distributed and uncorrelated i.e.  $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$  are generated and averaged in an appropriate manner. For instance,  $\varepsilon_2 - \theta_1 \varepsilon_1, \varepsilon_3 - \theta_1 \varepsilon_2, \dots, \varepsilon_n - \theta_1 \varepsilon_{n-1}$  is MA(1) series. Similarly, MA(2) or higher order MA process can be generated.

#### Simulation of Auto Regressive (AR) process:

An AR (p) model can be written as follows:

$$y_t = \sum_{i=1}^p \phi_i \varepsilon_{t-i} + \varepsilon_t \quad (7)$$

AR series simulation begins with a single element of a white noise sequence and generates the AR series term by term. For instance, an AR(1) process can be generated as  $\varepsilon_1, \varepsilon_2 + \phi_1 \varepsilon_1, \varepsilon_3 + \phi_1 (\varepsilon_2 + \phi_1 \varepsilon_1), \dots$

Subsequently,

$$y_t = \phi_1 y_{t-1} + \varepsilon_t \quad (8)$$

Similarly, AR (2) or higher order AR processes can be generated.

#### Simulation of Auto Regressive Moving Average (ARMA) process:

An ARMA (p, q) model can be written as follows:

$$y_t - \sum_{i=1}^p \phi_i y_{t-i} = \varepsilon_t - \sum_{j=1}^q \theta_j \varepsilon_{t-j} \quad (9)$$

It can be considered as an AR (p) process which is then subjected to averaging in accordance with the MA (q) process, for simulating ARMA process.

#### Simulation of Auto Regressive Integrated Moving Average (ARIMA) process:

An ARIMA (p, d, q) process  $y'_t$  which follows the property that  $y_t = \Delta^d y'_t$  where  $\Delta$  is the difference operator (i.e.  $\Delta y'_t = y'_t - y'_{t-1}$ ) is a stationary ARMA (p, q) process. Hence, ARIMA (p, d, q) process can be simulated by starting with an ARMA (p, q) series and applying the inverse of difference operator  $d$  times. For instance, ARIMA (p, 1, q) process  $y'_t$ . Then,  $y_t$  is an ARMA (p, q) process as discussed above giving  $y_1, y_2, \dots, y_n$ . The ARIMA series can be generated from the ARMA series by taking partial sums. i.e.

$$y'_1 = y_1, y'_2 = y_1 + y_2, \dots, y'_n = \sum_{i=1}^n y_i$$

#### Proposed ARIMA-Intervention process simulation:

Let  $y'_1, y'_2, \dots, y'_{T'}, \dots, y'_n$  be the simulated values of ARIMA model

- Let at time point  $t = T'$ , an intervention has occurred.
- Let the intervention variable be  $I_t$ .
- The post intervention observations with impact parameter  $\omega$  can be generated as, say
 
$$z_1 = y'_1 + I_t \omega, z_2 = y'_2 + I_t \omega, \dots, z_n = y'_n + I_t \omega$$
- Now the slope parameter  $\delta$  can be introduced by setting:
 
$$k_t = z_t, k_{t+1} = \delta k_t + z_{t+1}, k_{t+2} = \delta k_{t+1} + z_{t+2}, \dots$$
- The delay parameter  $b$  can be introduced by replacing  $T'$  with  $T' + b$ .

This algorithm is adopted to generate intervention models for various orders with step, pulse and ramp functions.

## 2.4 Forecasting Performance

Forecasting performance of the model has been judged by computing Mean Absolute Percent Error (MAPE) and Mean Squared Error (MSE). The model with less MAPE and MSE is favored for forecasting purposes. The MAPE and MSE is computed as

$$MAPE = \frac{1}{n} \sum_{t=1}^n |y_t - \hat{y}_t| / y_t \times 100 \quad (10)$$

$$MSE = \frac{1}{n} \sum_{t=1}^n (y_t - \hat{y}_t)^2 \quad (11)$$

where  $n$  is the total number of forecast values.  $y_t$  is the actual value at period  $t$  and  $\hat{y}_t$  is the corresponding forecast value.

## 2.5 Power computation

In its simplest form, intervention analysis itself may be regarded as a generalization of the two samples problem (corresponding to pre and post intervention periods) to the case where the error or noise term is autocorrelated rather than independent. Moreover, in many intervention analysis applications, time series data may be expensive or otherwise difficult to collect. In such cases, ‘power functions’ are helpful, because they can be used to determine the probability that a proposed intervention analysis application will detect a meaningful change. Power is the statistical term used for the probability that a test will reject the null hypothesis of no change at a given level of significance for a prescribed change. McLeod and Vingilis (2005) have suggested power computation methods for use with time series analyses for certain cases of intervention analysis.

According to them, the impact parameter  $\omega$  is tested through Z-test. The null hypothesis is that there is no impact or change due to intervention and the alternative hypothesis is that there is change due to intervention. The test statistic is given by-

$$Z = \hat{\omega} / \hat{\sigma}_{\hat{\omega}} \quad (12)$$

where  $\hat{\sigma}_{\hat{\omega}}$  is the ARIMA-intervention model error variance.

The power of the test i.e. rejecting of null hypothesis when alternative hypothesis is true is computed by

$$\pi(\hat{\omega}) = \Phi(-Z_{1-\alpha/2} - \hat{\omega} / \hat{\sigma}_{\hat{\omega}}) + 1 - \Phi(Z_{1-\alpha/2} - \hat{\omega} / \hat{\sigma}_{\hat{\omega}}) \quad (13)$$

where  $\Phi(\cdot)$  is the cumulative distribution function of standard normal variate.

In this study, a SAS IML code has been developed for simulation which is given in Appendix.

## 3. ILLUSTRATION

### Simulated setup

To investigate the performance of ARIMA-Intervention model as compared to conventional ARIMA model eleven different situations have been considered. For this, time series datasets have been generated each with sample size 50 with 37<sup>th</sup> time point as the intervention point. In the simulation process, the autoregressive and moving average parameters were fixed as 0.71 and 0.11 respectively and differencing order as one as in case of all-India cotton yield during the period 1961 to 2009 from an earlier study (Ray *et al.*, 2014). From the same study, the impact parameter ( $\omega_0$ ) was fixed as 104 but the slope parameter ( $\delta$ ) instead of the value of 0.18 observed for all-India yield  $\delta$  values were varied as 0, 0.25, 0.5, 0.75 and 0.98 for step and pulse intervention types. As slope parameter is not necessary in case of ramp intervention, only one model for ramp intervention has been simulated. In each situation, delay parameter was considered as zero. For each dataset, out of 50 observations, last 10 observations were kept to judge the forecasting performance and first 40 observations were used for model fitting. ARIMA-Intervention model and by conventional ARIMA model were used for forecasting by fitting them on each datasets. The forecasting performance was judged by MAPE as well as MSE. The summary of the simulation results are given in Table1.

From fitted step intervention models of the simulated datasets, it has been observed that in all the situations, the MAPE of ARIMA-Intervention model is always less than the ARIMA model though in some cases these differences were small. Another important aspect which can be observed is that, the confidence intervals in case of ARIMA-intervention models are narrower as compared to ARIMA models. From the fitted pulse intervention model of the simulated datasets, it has been observed that in all the situations, the MAPE of ARIMA-Intervention model is always less than the ARIMA model. Another important aspect that

can be observed is that, even if the pulse intervention only has effect at a single time period, the forecasting performance of the ARIMA model is seriously distorted with the actual values not lying between the confidence intervals of the forecasted values. Ramp intervention also revealed that ARIMA-Intervention model is better

that ARIMA when there is a significant change due to intervention.

**Power computation**

Power of 11 models considered were computed. The results are given in Table-2.

**Table 1.** Summary of simulation study ( $\phi_1 = 0.71, \theta_1 = 0.11, d=1, \omega = 104$ ) on forecasting performance of ARIMA Intervention Modeling

Intervention type	$\delta$	Time period t	Simulated	ARIMA			ARIMA-Intervention			
				Forecast	L95	U95	Forecast	L95	U95	
Step	0	1	359.83	316.08	257.64	374.52	295.18	258.70	361.67	
		2	340.74	340.58	267.45	413.70	332.74	281.35	384.14	
		3	312.56	327.22	233.91	420.53	312.08	244.28	379.87	
		4	342.81	350.43	247.08	453.78	335.17	263.26	407.09	
		5	346.16	338.32	220.06	456.58	318.56	235.11	402.01	
		6	350.93	360.33	233.84	486.82	337.98	250.67	425.29	
		7	326.73	349.37	210.61	488.14	324.70	228.33	421.08	
		8	336.64	370.26	224.29	516.24	341.10	240.99	441.20	
		9	299.87	360.39	203.81	516.97	330.57	222.94	438.19	
		10	325.37	380.24	217.13	543.35	344.47	233.19	455.74	
			<b>MAPE</b>		<b>7.80</b>			<b>5.24</b>		
		<b>MSE</b>		<b>1065.32</b>			<b>656.30</b>			
		0.25	1	284.39	302.68	248.54	356.82	274.21	227.83	320.59
			2	287.23	311.10	245.09	377.10	280.29	228.21	332.38
			3	304.90	312.33	227.56	397.11	279.11	210.79	347.44
			4	298.89	320.34	227.17	413.51	284.53	211.55	357.51
			5	273.29	321.96	215.08	428.84	283.97	199.54	368.40
			6	290.10	329.60	215.68	443.52	288.82	200.10	377.53
			7	295.93	331.57	206.50	456.64	288.77	191.04	386.51
			8	301.46	338.88	207.53	470.23	293.15	191.33	394.97
			9	293.87	341.15	200.26	482.05	293.53	184.20	402.86
			10	308.67	348.18	201.51	494.84	297.53	184.26	410.81
			<b>MAPE</b>		<b>10.91</b>			<b>3.25</b>		
			<b>MSE</b>		<b>1181.55</b>			<b>138.34</b>		
		0.5	1	522.20	466.11	378.38	553.83	509.87	450.68	569.06
			2	506.36	508.29	391.27	625.32	515.96	456.17	575.75
			3	481.49	484.45	338.20	630.70	523.65	449.41	597.90
			4	515.12	526.64	361.15	692.13	525.81	449.15	602.47
			5	521.92	502.79	315.49	690.10	529.45	444.51	614.38
			6	530.19	544.98	342.30	747.67	530.74	442.39	619.08
			7	509.58	521.14	300.28	741.99	532.96	438.68	627.23
			8	523.15	563.33	329.28	797.37	534.13	436.11	632.16
			9	490.11	539.48	289.54	789.42	535.83	432.99	638.67
	10		519.41	581.67	320.00	843.34	537.04	430.42	643.67	
			<b>MAPE</b>		<b>5.24</b>			<b>3.61</b>		
		<b>MSE</b>		<b>1193.78</b>			<b>526.13</b>			



Step	0.75	1	542.16	481.08	392.32	569.84	525.35	466.25	584.46	
		2	532.19	523.53	404.70	642.35	534.38	474.67	594.10	
		3	513.58	500.19	351.87	648.51	542.52	468.23	616.80	
		4	553.88	542.64	374.59	710.69	546.00	469.32	622.69	
		5	567.79	519.30	329.26	709.35	549.77	464.74	634.81	
		6	583.63	561.75	355.94	767.57	551.64	463.21	640.07	
		7	571.07	538.42	314.28	762.55	553.89	459.47	648.30	
		8	593.19	580.86	343.21	818.52	555.32	457.16	653.48	
		9	569.25	557.53	303.84	811.22	557.01	454.00	660.02	
		10	608.22	599.98	334.27	865.68	558.34	451.55	665.13	
			<b>MAPE</b>	<b>4.11</b>				<b>3.22</b>		
		<b>MSE</b>	<b>836.47</b>				<b>690.22</b>			
	0.98	1	579.45	508.26	417.58	598.95	553.72	494.80	612.63	
		2	582.32	550.91	428.64	673.18	568.81	509.26	628.36	
		3	578.30	528.75	376.53	680.98	578.21	503.88	652.53	
		4	635.12	571.40	398.49	744.31	584.66	507.95	661.36	
		5	667.71	549.24	354.00	744.49	588.96	503.73	674.18	
		6	704.62	591.89	380.12	803.66	596.71	442.62	750.80	
		7	715.80	569.73	339.36	800.10	644.64	499.96	789.32	
		8	764.61	612.38	367.85	856.91	646.71	498.30	795.12	
		9	770.64	590.22	329.42	851.03	648.46	495.12	801.79	
		10	843.21	632.87	359.47	906.26	740.08	592.97	887.19	
		<b>MAPE</b>	<b>15.87</b>				<b>9.52</b>			
	<b>MSE</b>	<b>16061.79</b>				<b>6576.48</b>				
Pulse	0	1	209.84	207.36	150.49	264.23	156.59	101.51	211.68	
		2	190.74	211.27	154.30	268.24	181.31	130.25	232.36	
		3	162.57	213.85	156.88	270.81	160.77	93.68	227.85	
		4	192.81	216.34	159.37	273.31	182.12	110.38	253.85	
		5	196.17	218.83	161.87	275.80	164.69	81.48	247.90	
		6	200.93	221.33	164.36	278.29	183.16	95.78	270.54	
		7	176.73	223.82	166.85	280.79	168.40	71.87	264.93	
		8	186.64	226.31	169.34	283.28	184.39	83.96	284.82	
		9	149.87	228.80	171.83	285.77	171.93	63.81	280.04	
		10	175.37	231.29	174.33	288.26	185.80	73.96	297.65	
			<b>MAPE</b>	<b>20.98</b>				<b>8.84</b>		
		<b>MSE</b>	<b>1768.87</b>				<b>501.86</b>			
	0.25	1	210.42	209.40	153.43	265.36	155.34	99.03	211.64	
		2	190.89	213.55	157.46	269.63	181.61	129.10	234.12	
		3	162.60	216.20	160.11	272.29	159.08	90.01	228.15	
		4	192.82	218.76	162.67	274.85	182.54	108.70	256.38	
		5	196.17	221.30	165.21	277.40	163.24	77.49	248.99	
		6	200.93	223.85	167.76	279.94	183.87	93.88	273.86	
		7	176.73	226.40	170.31	282.49	167.27	67.75	266.79	
		8	186.64	228.95	172.86	285.04	185.39	81.92	288.86	
		9	149.87	231.50	175.41	287.59	171.13	59.63	282.63	
		10	175.37	234.05	177.95	290.14	187.07	71.81	302.33	
		<b>MAPE</b>	<b>22.17</b>				<b>9.07</b>			
	<b>MSE</b>	<b>1958.12</b>				<b>529.34</b>				

Pulse	0.5	1	219.21	189.10	129.32	248.87	164.95	108.38	221.51		
		2	195.43	202.96	127.77	278.16	187.04	134.56	239.51		
		3	164.91	193.52	97.82	289.22	161.94	92.84	231.04		
		4	193.98	206.45	100.20	312.70	183.74	110.02	257.45		
		5	196.75	197.91	76.53	319.29	163.76	78.12	249.39		
		6	201.22	209.97	79.93	340.01	183.57	93.79	273.35		
		7	176.88	202.26	59.80	344.73	167.01	67.73	266.29		
		8	186.72	213.52	63.46	363.59	184.55	81.38	287.72		
		9	149.91	206.59	45.80	367.37	170.61	59.46	281.77		
		10	175.39	217.11	49.43	384.78	186.01	71.13	300.89		
			<b>MAPE</b>	<b>13.66</b>				<b>8.82</b>			
		<b>MSE</b>	<b>833.08</b>				<b>517.15</b>				
	0.75	1	257.30	232.13	173.79	290.47	208.96	156.94	260.97		
		2	226.34	249.64	176.54	322.74	225.72	173.31	278.14		
		3	189.26	238.84	145.65	332.03	198.53	129.40	267.65		
		4	212.84	255.28	151.98	358.57	214.64	141.14	288.14		
		5	211.18	245.51	127.39	363.63	193.35	107.98	278.71		
		6	212.19	260.95	134.53	387.37	208.29	118.92	297.67		
		7	185.18	252.15	113.54	390.75	191.48	92.71	290.25		
		8	192.98	266.66	120.78	412.55	205.05	102.46	307.64		
		9	154.62	258.75	102.35	415.15	191.70	81.27	302.13		
		10	178.94	272.41	109.41	435.41	203.85	89.72	317.98		
			<b>MAPE</b>	<b>29.94</b>				<b>8.26</b>			
		<b>MSE</b>	<b>3848.53</b>				<b>494.02</b>				
	0.98	1	332.01	295.46	237.38	353.53	286.23	238.86	333.60		
		2	306.81	318.14	245.54	390.74	311.73	259.24	364.22		
		3	272.83	305.51	212.87	398.16	292.87	223.61	362.13		
		4	297.56	326.95	224.35	429.54	314.47	240.97	387.97		
		5	295.68	315.53	198.12	432.93	299.17	213.82	384.52		
		6	295.47	335.80	210.24	461.37	317.52	228.23	406.81		
		7	266.54	325.50	187.76	463.25	305.19	206.57	403.81		
		8	271.96	344.69	199.79	489.59	320.84	218.43	423.26		
		9	230.93	335.44	180.02	490.86	310.97	200.80	421.15		
10		252.38	353.62	191.72	515.53	324.38	210.51	438.25			
		<b>MAPE</b>	<b>19.11</b>				<b>13.27</b>				
	<b>MSE</b>	<b>3535.44</b>				<b>1877.94</b>					
Ramp	0	1	959.84	908.01	798.47	1017.55	930.40	873.03	987.77		
		2	1090.74	1035.25	851.77	1218.74	1112.00	1037.00	1187.00		
		3	1212.57	1162.49	902.04	1422.95	1262.00	1168.00	1356.00		
		4	1392.81	1289.73	947.17	1632.30	1443.00	1338.00	1549.00		
		5	1546.17	1416.98	986.82	1847.14	1594.00	1474.00	1714.00		
		6	1700.93	1544.22	1021.02	2067.42	1774.00	1645.00	1903.00		
		7	1826.73	1671.46	1049.94	2292.98	1926.00	1785.00	2067.00		
		8	1986.64	1798.70	1073.77	2523.63	2106.00	1957.00	2255.00		
		9	2099.87	1925.94	1092.70	2759.17	2258.00	2098.00	2417.00		
		10	2275.37	2053.18	1106.93	2999.43	2437.00	2270.00	2603.00		
			<b>MAPE</b>	<b>7.56</b>				<b>4.61</b>			
			<b>MSE</b>	<b>19919.73</b>				<b>8913.87</b>			

**Table 2. Power for computation for fixed  $\omega$  and varying  $\delta$**

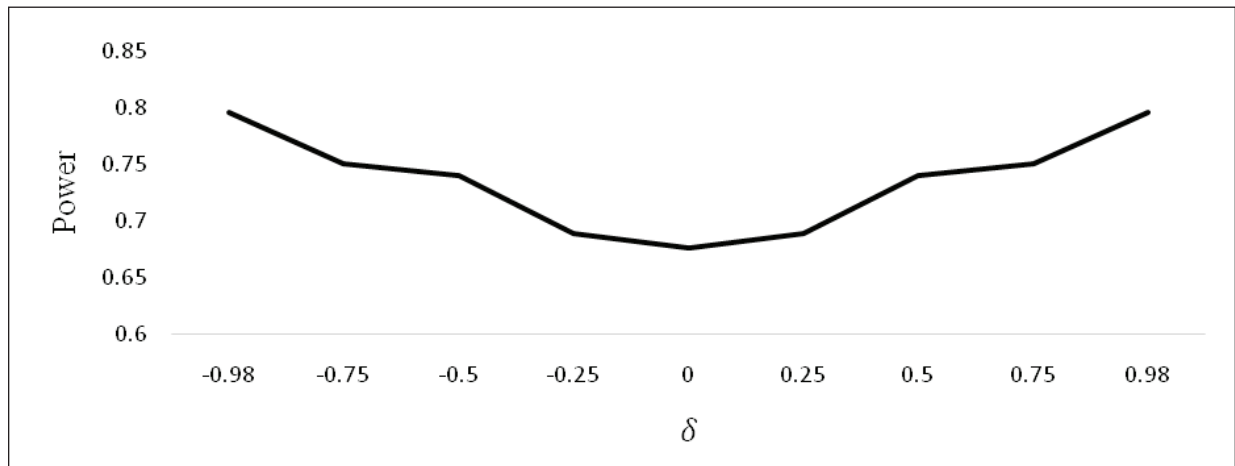
$\delta / \omega = 104$	Power		
	Step	Pulse	Ramp
0.00	0.67703	0.60615	0.84177
0.25	0.68994	0.62885	-
0.50	0.74112	0.68994	-
0.75	0.75077	0.72676	-
0.98	0.79642	0.76045	-

It can be concluded from the above table that for a fixed value of impact parameter, the power of the test was high for ramp followed by step and then pulse intervention. For fixed value of impact parameter, when the slope parameters were increased, the power of the test to detect significant change also increased in both step and pulse types of interventions. As the intervention is symmetric in nature because power

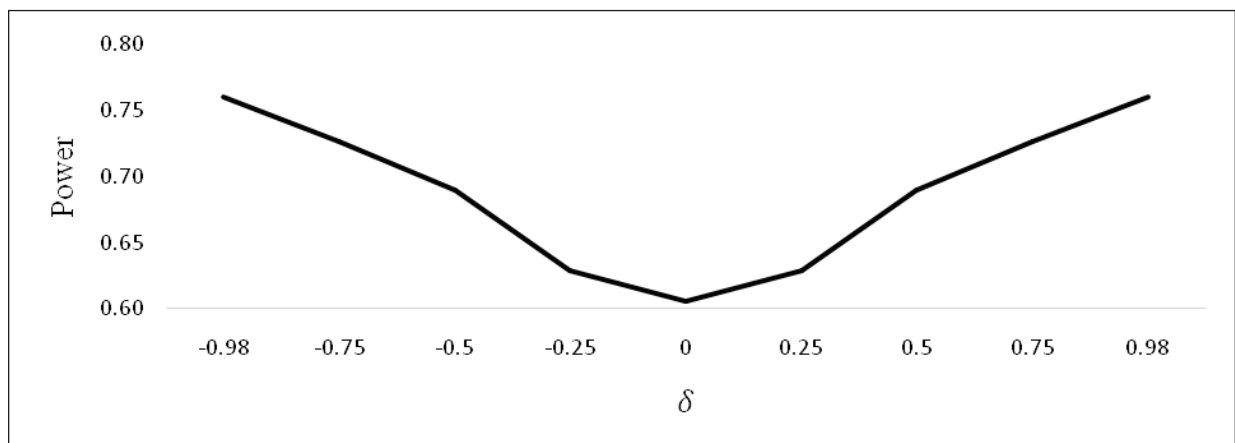
depends only on the magnitude of model parameters i.e. positive and negative parameters with same value will have same power. The graphical representation of the above result is given subsequently.

Now the impact parameter  $\omega$  vary from 54-354 with an interval of 50 without considering the slope parameter. Here two cases have been considered for power computation. In first case, a sample size of  $n=40$  was generated with  $T'=37$  time of intervention and  $m=4$  the number of post-intervention observations. In another case, a sample size of  $n=60$  was generated with  $T'=37$  and  $m=24$ . The results are given Table-3.

From Table 3, it can be concluded that as the magnitude of the impact parameter  $\omega$  increases, the power of detecting the change increases. Simultaneously it has been observed when the magnitude of impact parameter increases the power of detection of change



**Fig. 2.** For fixed  $\omega$  and varying  $\delta$  in step intervention



**Fig. 3.** For fixed  $\omega$  and varying  $\delta$  in pulse intervention



**Table 3. Power Computation for varied  $\omega$  and sample size**

$\omega$	Power	
	n= 40, T=37, m=4	n= 60, T=37, m=24
54	0.60497	0.71966
104	0.67703	0.82415
154	0.79643	0.90595
204	0.91067	0.96045
254	0.96212	0.98415
304	0.99115	0.99619
354	0.99797	0.99864

is high even with few post-intervention observations as can be seen by comparing the values of power for  $m=4$  and  $m=24$ . The power curve for the above result is in Fig 3.

#### 4. CONCLUSIONS

To sum up, it can be inferred that ARIMA-Intervention always superior than conventional ARIMA model whenever there is significant impact due to intervention. In addition, using power computations, it has been observed that, as the magnitude of the impact parameter increases, the power to detect such a change also increase. Also, for a fixed value of impact parameter, the power of the test was high for ramp followed by step and then pulse intervention situations. For fixed value of impact parameter, when the slope parameters were increased, the power of the test to detect significant change also increased in both step and pulse types of interventions. When the impact value was increased, it was also shown that the power

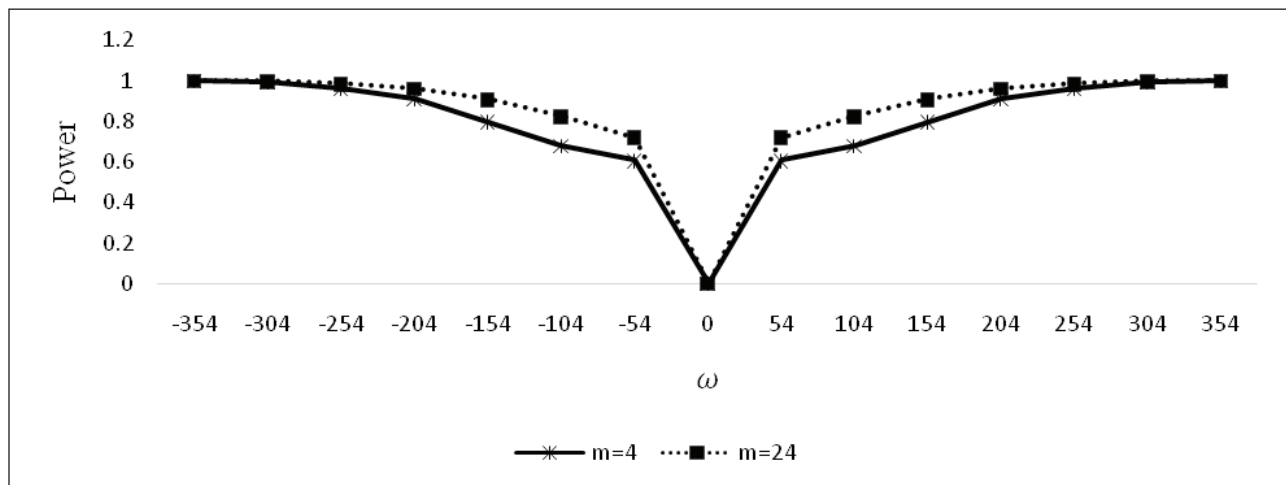
of the test is reasonably good even when there are only a few post-intervention observations.

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#### REFERENCES

- Bianchi, L., Jarrett, J. and Hanumara, R.C. (1998). Improving forecasting for telemarketing centres by ARIMA modeling with intervention. *Int. J. Forecast.*, **14**, 497-504.
- Box, G.E.P. and Tiao, G.C. (1975). Intervention Analysis with Application to Economic and Environment Problems. *J. Amer. Statist. Assoc.*, **70**, 70-79.
- Box, G.E.P., Jenkins, G.M. and Reinsel, G.C. (1994). *Time Series Analysis: Forecasting and Control (3rd ed.)*. Holden-Day, San Francisco.
- Brakel, J.V.D. and Roels, J. (2010). Intervention analysis with state-space models to estimate discontinuities due to a survey redesign. *Ann. Appl. Stat.*, **4**, 1105-1138.
- Dunne, A. (1992). Time Series Simulation. *J. R. Stat. Soc. Ser. D Stat. Soc.*, **41**, 3-8.
- Girard, D.Z. (2000). Intervention time series analysis of pertussis vaccination in England and Wales. *Health Policy*, **54**, 13-25.
- Ismail, Z., Suhartono, Yahaya, A. and Efendi, R. (2009). Intervention Model for Analyzing the Impact of Terrorism to Tourism Industry. *J. Math. Stat.*, **4**, 322-329.
- Lam, C.Y., Ip, W.H. and Lau, C.W. (2009). A business process activity model and performance measurement using a time series ARIMA intervention analysis. *Expert Syst. Appl.*, **36**, 925-932.
- Madsen, H. (2008). *Time Series Analysis*. Boca Raton: Chapman and Hall.

**Fig. 4.** Power curve for varied  $\omega$  and sample size

- McLeod, A.I. and Vingilis, E.R. (2005). Power computations for intervention analysis. *Technometrics*, **47**, 174-181.
- Ray, M., Ramasubramanian, V., Kumar, A. and Rai, A. (2014). Application of time series intervention modelling for modelling and forecasting cotton yield. *Stat. Appl.*, **12**, 61-70.
- Ray, M., Rai, A., Singh, K.N., Ramasubramanian, V. and Kumar, A. (2017). Technology forecasting using time series intervention based trend impact analysis for wheat yield scenario in India. *Technol. Forecast. Soc. Change*, **118**, 128-133.
- Shao, Y.E. (1997). Multiple intervention analysis with applications to sales promotional data. *J. Appl. Stat.*, **24**, 181-192.
- Yaffee, R. and McGee, M. (2000). *An Introduction to Time Series Analysis and Forecasting: With Applications of SAS and SPSS (1st ed.)*. New York: Elsevier.

## APPENDIX

**SAS IML code for simulation of data following ARIMA-Intervention model with given parameters: impact ( $\omega$ ), slope ( $\delta$ ) and delay (b) and given AR and MA parameters and degree of differencing for any type of intervention (step/ pulse/ ramp):**

```
proc iml;
  phi={1 0.71}; /*autoregressive parameter*/
  theta={1 0.11}; /*moving average parameter*/
  n=50; /*total sample size*/
  y=armasim(phi, theta, mean,variance,n,seed); /*arma simulation*/
  y1=cusum(y);
  y2=cusum(y1);
  d=1; /*give value of differencing order d from user's side; here the user has
  given it as 1*/
  if d=0 then do;
    a=y;
    print a;
    end;
  if d=1 then do;
    a=y1+11;
    print a;
    end;
  if d=2 then do;
    a=y2;
    print a;
    end;
  run; /*arima simulated value*/
  start intfunc(1,n,t,b,func);
  I=j(n,1.);
  if func=1 then do;
    y1=j(t+b-1,1,0);
    y2=j(1,1,1);
```

```
  y3=y1//y2;
  y4=j(n-t-b,1,0);
  I=y3//y4;
  print I;
  end;
  if func=2 then do;
    s1=j(t+b-1,1,0);
    s2=j(n-t-b+1,1);
    I=s1//s2;
    print I;
    end;
  if func=3 then do;
    s1=j(t+b-1,1,0);
    s2=j(n-t-b+1,1);
    s3=cusum(s2);
    I=s1//s3;
    print I;
    end;
  finish;
  n=50; /*total sample size*/
  t=37; /*point of intervention*/
  b=0; /*delay parameter*/
  func=2; /*func=1(pulse) func=2(step) func=3(ramp)*/
  run intfunc(1,n,t,b,func);
  w1=150; /*value of impact parameter*/
  w2=0;
  w3=0;
  z1=w1#I;
  k1=b+1;
  run intfunc(1,n,t,k1,func);
  z2=w2#I;
  k2=k1+1;
  run intfunc(1,n,t,k2,func);
  z3=w3#I;
  z=z1+z2+z3;
  ins=a+z; /*simulated value of intervention model only with impact
  parameter*/
  p1=j(t+b,1,0);
  %let k=n-t-b;
  d=1.05; /*value of slope parameter*/
  p2=j(&k,1,0);
  do t=1 to &k;
    p2[t,]=d##t;
  end;
  p3=p1//p2;
  p=w1#p3;
  print p;
  insa=ins+p;
  run;
  print insa; /* Final simulated value*/
```