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Latin Square Designs with Neighbour Effects

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SUMMARY

The research work presented in this paper is motivated by a real life scenario in the context of agricultural experiments. It is believed that the neighboring plots in a Block Design or in a Latin Square Design tend to influence each other in terms of the mean yield through the 'effects of the treatments' applied in these plots. We contemplate a linear model and study its analysis in considerable details.

Keywords: Block designs, Latin square designs, Direct treatment effects, Neighbor treatment effects, Left neighbors, Right neighbors, Diagonal neighbors, Linear model, ANOVA.

1. INTRODUCTION

Let us consider the following typical real life scenario that is observed in an agricultural field featuring plots arranged in blocks or in rows and columns. Naturally, we are referring to a block design or to a row-column design. Specifically, we may concentrate on a Latin Square Design of order n [denoted as LSD_n so that in effect there are n^2 plots arranged in a n×n square and there are n treatments laid out in the form of a Latin Square of order n. The treatment effects are to be compared - in the presence of row effects and columns effects.

This calls for a standard Latin Square Design, its model specification and the data analysis. This is available in any standard text book in a very general form. We now contemplate a situation wherein there is a possibility of 'flow' of 'treatment effects to the neighbouring plots along 'row direction', 'column direction' as also possibly along 'diagonal direction'.

We will specifically concentrate on LSDs of order 4. According to Fisher and Yates Table (1938), there are 4 non- isomorphic LSDs of order 4 and these are listed below for ready reference. Our listing is different from what is given in the original table. We will fix the notations and model specifications as follows. Confining to LSD - I,

$$y(1,1;A) = \mu + \rho_1 + \gamma_1 + \tau_A + RN_B + RN_D + CN_B + CN_D + e(1,1;A)$$

In the above we are referring to a 'circular model' with obvious interpretations for the parameters involved in the model. In effect, we have inserted Row Neighbour and Column Neighbour Effects on the top of the (Direct) Effects (τ's) of the treatments. There are 16 observations - resulting in 15 df and we have the standard decomposition: Rows (3), Columns (3), Treatments (3), RNs (3) and LN (3). Once we are able to successfully identify these SS, we will end up with no df left for the errors. This suggests - we need to carefully study the model and underlying estimability issues.

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2. ESTIMABILITY ISSUES: STANDARD LSD

For an LSD of order n, it is well-known that the splitting of total df is accomplished by orthogonal decomposition into (i) Rows with n-1 df; (ii) Columns with n-1 df; (iii) Treatments with n-1 df and (iv) Errors with (n-1)(n-2) df. There are two aspects of this phenomenon. Decomposition of TSS into orthogonal components due to Rows/Columns/Treatments/Errors is a very standard exercise. Understanding of SS due to the first three [assignable] components in terms of orthogonal observational contrasts is also a routine task. However, in the same vein, it is a non-trivial task to characterize the ErrorSS in terms of orthogonal error functions - though SS itself is readily derivable through subtraction. In order to accomplish this fit of identification of error functions, we have to exercise the notion of 'Tetra-Differences' and use it twice [second order tetra-difference]. Vide Shah and Sinha (1996) for use of Tetra- Difference [TD] Technique in the context of Row-Column Designs.

For LSD - I of order 4, below we display the 6 error functions - built on use of second order tetra-differences. Once for all, we keep a record of the details.

Table 1. LSD - I : Latin Square Design of order 4

A	B	C	D
B	C	D	A
C	D	A	B
D	A	B	C

We now display the error functions in matrix notation LY where L is a matrix of order 6×16 having the following structure:

Remark 1. It may be noted that the last 3 error functions have a special behaviour in terms of model expectations. Whereas the former three use collective contributions of the two TDs to cancel out the parameters, each of the latter three provides nice simplification in each TD.

Remark 2. That the error functions are orthogonal to Row Contrasts is immediate from the above matrix representation of L-matrix. Orthogonality wrt Column Contrasts is also easy to verify. To ascertain orthogonality wrt Treatment Contrasts we have used the TDs tactfully.

Next we argue that these error functions are indeed linearly independent, though notorthogonal. For that we work out (LL') which is a 6×6 matrix given below.

It is readily verified that the above matrix is non-singular. [Det:(LL') = 4096:] Hence we can proceed with Gram-Smith process of orthogonalization to obtain orthonormal observational contrasts attributed to errors with 6 df. Uniqueness justifies that these indeed add up to the so-called Error SS in the ANOVA Table - ascertained through the identity: TSS= Row SS + Col SS + Tr SS + Error SS.

3. ESTIMABILITY ISSUES: MODEL FOR LSD-I WITH RN AND CN EFFECTS

It is not difficult to verify that

(a) SS due to Rows based on Row Total Contrasts is unaffected by all other parameters under the above model;

(b) SS due to Columns based on Column Total Contrasts is also unaffected by all otherparameters under the above model;

(c) Treatment Total for $A\ [TT(A)] = [y(1,1;A) + y(2,4;A) + y(3,3;A) + y(4,2;A)]$ minus Treatment Total for C $[TT(C)]$ unbiasely estimates $\tau_A - \tau_C$; likewise $TT(B) - TT(D)$ unbiasedly estimates $\tau_B - \tau_D$.

(d) $TT(A) - TT(B) + TT(C) - TT(D)$ contributes to $\tau_A - \tau_B + \tau_C - \tau_D$ plus Contrasts $[RN_B + RN_D - RN_A - RN_C]$ plus Contrasts $[CN_B + CN_D - CN_A - CN_C]$.

These are valid statements under the above model in the presence of RN and CN Effects. Let us now look at the error functions under the new model. To start with, we already have characterized 6 error functions under the standard LSD model in the absence of RN and CN Effects. These are displayed above. Interestingly, it transpires that all these continue to be error functions under the new model as well!

Therefore, a logical and valid data analysis would result into an ANOVA Table with the following description under the new model.

Remark 3: In the above $Tr\ SS$ with 2 df (SS^*) is computed as

$$\frac{[Tr\ Total(A) - Tr\ Total\ (C)]^2}{8} + \frac{[Tr\ Total(B) - Tr\ Total\ (D)]^2}{8}$$

Remark 4: Further, Mixed Effects SS with 1df (SS^{**}) is computed as

$$\frac{[Tr\ Total(A) + Tr\ Total\ (C) - Tr\ Total(B) - Tr\ Total\ (D)]^2}{16}$$

Remark 5: Under the assumption that RN and CN Effects are negligible, Mixed Effects SS reduces to $Tr\ SS$ and it adds to the 2 df to produce 3 df as usual. Further to this, we may slightly relax this assumption and demand that the Total of RN [CN] Effects of B and D matches with that of RN [CN] Effects of A and C.

Remark 6: Interestingly enough, the error functions remain intact under the new model.

4. ESTIMABILITY ISSUES : MODEL FOR LSD-II WITH RN AND CN EFFECTS

For this design, our first task is to identify the error functions in matrix notation LY where L is a matrix of order 6×16 and this we do under the standard LSD model using TD technique. We display it below.

It transpires that the matrix L has full row- rank. [Det:(LL') = 16384]. Further, under the new model, LY continues to serve as a pool of error functions. This fact is verified below inconsiderable details. In other words, we have demonstrated that RN and CN Effects do not affect the nature of the error functions in the new model. Further, it can be verified thatthe properties listed under (a) - (d) for LSD - I continue to hold under this LSD as well. Therefore, ANOVA Table remains intact.

Table 2. LSD - II : Latin Square Design of order 4

A	B	C	D
B	A	D	C
C	D	A	B
D	C	B	A

5. ESTIMABILITY ISSUES: MODEL FOR LSD-III WITH RN AND CN EFFECTS

For this design, again the non-trivial task is to identify the error functions in matrix notation LY where L is a matrix of order 6×16 and, to start with, this is done under the standard LSD model. We display it below.

It transpires that the matrix L has full row-rank. [Det:(LL') = 65536]. Next, under the new model, we need to verify its status as to the collection of error functions. This is done below. Further to this, we observe as follows. Though row - contrasts and column -contrasts are unaffected by the presence of RN and CN effects, it is not the same as before for Treatment Contrasts. Of three Treatment Contrasts, only one viz., the one based on T(A)+T(D)−T(B)−T(C) is free from RN and CN effects. Therefore, in the ANOVA Table, only 1 dfs is attributed to 'pure' SS for Treatments. Two others based on T(A) - T(D) and T(B) - T(C) involve [RND - RNA; CND - CNA] and [RNC - RNB; CNC - CNB] respectively. In a way,under the assumptions that [RNA = RND; CNA = CND] and [RNB = RNC; CNB = CNC],we have 3 df for Treatment SS.

Now we take up the issue of error functions in the new model. We go through the same exercise as before.

Remark 7. It is interesting to note that for LSD-III, the 6 error functions listed above,are linearly independent but not all of these possess the property of 'zero expectation'. We find only 4 satisfying this 'zero expectation' property. Hence in the ANOVA Table, we will have 4 error df in the presence of RN and CN effects. On the other hand, under the assumption that [RNA = RND; CNA = CND] and [RNB = RNC; CNB = CNC], we not only have full 6 error df, we can also have Treatment SS with 3 df as usual.

Table 3. LSD - III : Latin Square Design of order 4

A	B	C	D
B	D	A	C
C	A	D	B
D	C	B	A

6. ESTIMABILITY ISSUES: MODEL FOR LSD-IV WITH RN AND CN EFFECTS

For this design, again the non-trivial task is to identify the error functions in matrix notation LY where L is a matrix of order 6×16 and, to start with, this has to be done under the standard LSD model first. We display it below.

Next we study the properties of the L - matrix in the presence of RN and CN Effects. We proceed as before.

It transpires that the matrix L has full row-rank. [Det:(LL') = 65536]. Further, under the new model, only the three error functions e_3; e_5; e_6 continue to serve as pure errors. For this design, again we have only 1 'pure' df for Treatment Contrasts viz., one based on TA + TB - TC - TD which is free from both RN and CN effects. The other two are based on TA - TB and TC - TD. This time we need the assumptions [RNA = RNB; CNA = CNB] and [RNC = RND; CNC = CND] so that we can revive the 2 error df for Treatment SS in the ANOVA Table. Under the same assumptions, it transpires that the other three error functions viz., e_1;e_2;e_4 also contribute to pure error.

Table 4. LSD - IV : Latin Square Design of order 4

A	B	C	D
B	A	D	C
C	D	B	A
D	C	A	B

7. CONCLUDING REMARKS

In this study we have noticed one interesting feature in the behaviour of the so-called error functions and Treatment SS in LSDs of order 4 - in the presence of RN and CN effects.

Taking clue from Fisher and Yates Table, we have focussed our study on 4 LSDs of order4 and we have introduced RN and CN effects in the standard model for an LSD. We are able to identify explicitly the 6 error functions underlying each LSD in the standard model without RN and CN effects and demonstrate that some of these continue to serve as error functions in models with RN and CN effects. Further to this, in the context of data analysis,while we examine the ANOVA Table, we observe that the so-called Treatment SS with 3 df is no longer available in the model with RN and CN effects - unless some parametric relations involving such effects are brought into the picture. In other words, behaviours of the ANOVA Table components for 4 LSDs are explicitly examined - with special emphasis on Treatment SS and Error SS. The results are summarized in the following Table.

In this study we did not include Diagonal Effects of Treatments in the linear model. That aspect has to be carefully brought into the picture. We might take it up in a future study. The analysis is likely to be hopelessly complicated.

One might argue that it is straightforward to find out 3 error functions by referring to

4 treatment groups arising out of a Latin Square of order 4. However, the other three may not always be easy to identify. We have applied TD technique to sort out all the 6 error functions.

For LSD-II, we show below how the 3 error functions based on 4 group formations maybe determined from the 6×16 L-matrix. For completeness, the groups are shown below.

Next, we find that

(i) (G1 - G2) is identified as error function 6;

(ii) (G1 + G2 - G3 - G4) is identified as (error function 1 + error function 3);

(iii) (G3-G4) can also be identified as L'X where X is the solution to (LL')X = L[G3-G4].

That is, X = $(LL')^{-1}$ [0; –4; 0; 0; 4; 0], leading to X = [-1; 0; -1; 0; 2; -1]. Therefore, (G3 - G4) can be viewed as (2 error function 5 - error function 1 - error function 3 – errorfunction 6).

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Table 5. Error Functions : Component TDs in Latin Square Design [LSD - I] of order 4

Error Function 1	TD-1:A(1,1)+C(2,2)-B(2,1)-B(1,2)	TD-2:A(3,3)+C(4,4)-B(3,4)-B(4,3)
Error Function 2	TD-1:A(1,1)+A(2,4)-B(2,1)-D(1,4)	TD-2: A(3,3)+C(4,2)-B(4,3)-D(3,2)
Error Function 3	TD-1:A(2,4)+C(1,3)-D(1,4)-D(2,3)	TD-2:A(4,2)+C(3,1)-D(3,2)-D(4,1)
Error Function 4	TD-1:A(1,1)+D(3,2)-B(1,2)-C(3,1)	TD-2:A(4,2)+D(2,3)-B(4,3)-C(2,2)
Error Function 5	TD-1:A(1,1)+A(3,3)-C(1,3)-C(3,1)	TD-2:A(2,4)+A(4,2)-C(2,2)-C(4,4)
Error Function 6	TD-1:A(1,1)+B(3,4)-C(3,1)-D(1,4)	TD-2:A(1,1)+B(4,3)-C(1,3)-D(4,1)

Table 6. Error Functions as linear contrasts of observations : Matrix L for LSD – I

Error functions	**(1,1)**	**(1,2)**	**(1,3)**	**(1,4)**	**(2,1)**	**(2,2)**	**(2,3)**	**(2,4)**	**(3,1)**	**(3,2)**	**(3,3)**	**(3,4)**	**(4,1)**	**(4,2)**	**(4,3)**	**(4,4)**
Error function1	1	-1	0	0	-1	1	0	0	0	0	-1	1	0	0	1	-1
Error function2	1	0	0	-1	-1	0	0	1	0	1	-1	0	0	-1	1	0
Error function3	0	0	1	-1	0	0	-1	1	-1	1	0	0	1	-1	0	0
Error function4	1	-1	0	0	0	1	-1	0	-1	1	0	0	0	-1	1	0
Error function5	1	0	-1	0	0	1	0	-1	-1	0	1	0	0	-1	0	1
Error function6	0	0	1	-1	0	0	0	0	-1	0	0	1	1	0	-1	0

Table 7. Matrix LL'

8	4	0	4	0	0
	8	4	4	0	0
		8	4	0	4
			8	4	0
				8	0
					6

Table 8. LSD - I. Coefficients of Error functions : RN Effects and their totals

error	RN_A	RN_B	RN_C	RN_D	**Total**
1	-1,1,1,-1	-1,-1,1,1	-1,1,1,-1	-1,-1,1,1	0
2	-1,1,1,-1	-1,-1,1,1	-1,1,1,-1	-,1,1 ,-1,-1,1,1 -1,1	0
3	-1,1,1,-1	-1,-1,1,1	-1,1,1,-1	-1,-1,1,1	0
4	-1,1,1,-1	-1,-1,1,1	-1,1,1,-1	-1,-1,1,1	0
5		-1,-1,1,-1,1,-1,1,1		-1,-1,1,-1,1,-1,1,1	0
6	-1,-1,1,1	-1,1	-1,-1,1,1	-1,1	0

Table 9. LSD - I : Coefficients of Error functions : CN Effects and their totals

error	CN_A	CN_B	CN_C	CN_D	Total
1	-1,1,1,-1	-1,-1,1,1	-1,1,1,-1	-1,-1,1,1	0
2	-1,1,1,-1	-1,-1,1,1	-1,1,1,-1	-1,-1,1,1 ,-1,-1,1,1	0
3	-1,1,1,-1	-1,-1,1,1	-1,1,1,-1	-1,-1,1,1	0
4	-1,1,1,-1	-1,-1,1,1	-1,1,1,-1	-1,-1,1,1	0
5		-1,-1,1,-1,1,-1,1,1		-1,-1,1,-1,1,-1,1,1	0
6	-1,-1,1,1	-1,1	-1,-1,1,1	-1,1	0

Table 10. ANOVA Table

Source of variation	df	SS	MSS	F-ratio
Rows	3	usual	usual	$F_{3,6}$
Columns	3	usual	usual	$F_{3,6}$
Treatment	2	(SS^*)	usual	$F_{2,6}$
Mixed RN,CN, Tr effects	1	(SS^{**})	-	-
Errors	6	as in original model	usual	-

Table 11. Error Functions as linear contrasts in the observations : Matrix L for LSD – II

Error functions	(1,1)	(1,2)	(1,3)	(1,4)	(2,1)	(2,2)	(2,3)	(2,4)	(3,1)	(3,2)	(3,3)	(3,4)	(4,1)	(4,2)	(4,3)	(4,4)
Error function1	1	-1	0	0	-1	1	0	0	0	0	-1	1	0	0	1	-1
Error function2	1	0	-1	0	-1	0	1	0	0	-1	0	1	0	1	0	-1
Error function3	0	1	-1	0	-1	0	0	1	1	0	0	-1	0	-1	1	0
Error function4	1	0	0	-1	0	1	-1	0	0	-1	1	0	-1	0	0	1
Error function5	1	0	-1	0	0	-1	0	1	-1	0	1	0	0	1	0	-1
Error function6	0	1	0	-1	-1	0	1	0	0	-1	0	1	1	0	-1	0

Table 12. LSD - II: Coefficients of Error functions : RN Effects and their totals

error	RN_A	RN_B	RN_C	RN_D	Total
1	-1,-1,1,1	1,1,-1,-1	-1,-1,1,1	1,1,-1,-1	0
2	___	1,1,-1,-1, 1,1,-1,-1	___	1,1,-1,-1, 1,1,-1,-1	0
3	-1,-1,1,1	1,1,-1,-1	-1,-1,1,1	1,1,-1,-1	0
4	-1,1,1,-1	1,-1,-1,1	-1,1,1,-1	1,-1,-1,1	0
5	-1,-1,1,1	1,1,-1,-1	-1,-1,1,1	1,1,-1,-1	0
6	1,-1,1,-1	1,-1,1,-1	1,-1,1,-1	1,-1,1,-1	0

Table 13. LSD - II : Coefficients of Error functions : CN Effects and their totals

Table 14. Error Functions as linear contrasts in the observations : Matrix L for LSD – III

error	CN_A	CN_B	CN_C	CN_D	Total
1	-1,-1,1,1	1,1,-1,-1	-1,-1,1,1	1,1,-1,-1	0
2	___	1,-1,1,-1, 1,-1,1,-1	___	1,-1,1,-1, 1,-1,1,-1	0
3	-1,-1,1,1	1,1,-1,-1	-1,-1,1,1	1,1,-1,-1	0
4	-1,1,1,-1	1,-1,-1,1	-1,1,1,-1	1,-1,-1,1	0

error	CN_A	CN_B	CN_C	CN_D	Total
5	-1,-1,1,1	1,1,-1,-1	-1,-1,1,1	1,1,-1,-1	0
6	1,1,-1,-1	1,-1,1,-1	1,-1,1,-1	1,-1,1,-1	0

Table 15. LSD - III: Coefficients of Error functions : RN Effects and their totals

error	RN_A	RN_B	RN_C	RN_D	Total
1	-1,1,-1,1	1,1,-1,-1	-1,-1,1,1	1,-1,1,-1	0
2	___	1,-1, 1,-1	1,-1,1,-1	1,-1,1,-1, 1,-1,1,-1	0
3	1,1,1,1	-1,1,-1,1	1,-1,-1,1	-1,-1,-1,-1	$4(RN_A - RN_D)$
4	-1,1,1,-1	1,1,1,1	-1,-1,-1,-1	1,-1,-1,1	$4(RN_B - RN_C)$
5	1,-1,1,-1	1,-1,1,-1, 1,-1,1,-1	____	1,-1,1,-1	0
6	1,-1,1,-1	_____	1,-1,1,-1, 1,-1,1,-1	1,-1,1,-1	0

Table 16. LSD - III : Coefficients of Error functions : CN Effects and their totals

error	CN_A	CN_B	CN_C	CN_D	Total
1	1,-1,1,-1	1,1,-1,-1	-1,-1,1,1	1,-1,1,-1	0
2	-1,-1,1,1	1,-1, 1,-1	1,-1,1,-1	1,1,-1,-1	0
3	-1,-1,-1,-1	-1,1,-1,1	1,-1,-1,1	1,1,1,1	$4(CN_D - CN_A)$
4	-1,1,1,-1	1,1,1,1	-1,-1,-1,-1	1,-1,-1,1	$4(CN_B - CN_C)$
5	-1,-1,1,1	1,-1,1,-1, 1,-1,1,-1	____	1,1,-1,-1	0
6	-1,-1,1,-1	_____	1,-1,1,-1, 1,-1,1,-1	1,1,-1,-1	0

Table 17. Error Functions as linear contrasts in the observations : Matrix L for LSD – IV

Error functions	(1,1)	(1,2)	(1,3)	(1,4)	(2,1)	(2,2)	(2,3)	(2,4)	(3,1)	(3,2)	(3,3)	(3,4)	(4,1)	(4,2)	(4,3)	(4,4)
Error function1	1	-1	0	0	-1	1	0	0	0	0	1	-1	0	0	-1	1
Error function2	0	0	1	-1	0	0	-1	1	-1	1	0	0	1	-1	0	0
Error function3	1	0	0	-1	0	-1	1	0	-1	0	0	1	0	1	-1	0
Error function4	1	0	0	-1	-1	0	0	1	0	1	0	-1	0	-1	0	1
Error function5	0	-1	1	0	1	0	0	-1	0	1	-1	0	-1	0	0	1
Error function6	0	1	0	-1	-1	0	1	0	1	0	-1	1	0	-1	0	1

Table 18. LSD - IV: Coefficients of Error functions : RN Effects and their totals

error	RN_A	RN_B	RN_C	RN_D	Total
1	-1,1,-1,1	1,1,-1,-1	-1,-1,1,1	1,-1,1,-1	0
2	___	1,-1, 1,-1	1,-1,1,-1	1,-1,1,-1, 1,-1,1,-1	0
3	1,1,1,1	-1,1,-1,1	1,-1,-1,1	-1,-1,-1,-1	$4(RN_A - RN_D)$
4	-1,1,1,-1	1,1,1,1	-1,-1,-1,-1	1,-1,-1,1	$4(RN_B - RN_C)$
5	1,-1,1,-1	1,-1,1,-1, 1,-1,1,-1	____	1,-1,1,-1	0
6	1,-1,1,-1	_____	1,-1,1,-1, 1,-1,1,-1	1,-1,1,-1	0

Table 19. LSD - IV : Coefficients of Error functions : CN Effects and their totals

error	CN_A	CN_B	CN_C	CN_D	Total
1	1,-1,1,-1	1,1,-1,-1	-1,-1,1,1	1,-1,1,-1	0
2	-1,-1,1,1	1,-1, 1,-1	1,-1,1,-1	1,1,-1,-1	0
3	-1,-1,-1,-1	-1,1,-1,1	1,-1,-1,1	1,1,1,1	$4(CN_D - CN_A)$
4	-1,1,1,-1	1,1,1,1	-1,-1,-1,-1	1,-1,-1,1	$4(CN_B - CN_C)$
5	-1,-1,1,1	1,-1,1,-1, 1,-1,1,-1	____	1,1,-1,-1	0
6	-1,-1,1,-1	_____	1,-1,1,-1, 1,-1,1,-1	1,1,-1,-1	0

Table 20. 4 groups of treatment combinations in LSD – II

Group	Treatment from Row 1	Treatment from Row 2	Treatment from Row 3	Treatment from Row 4
G1	A(1,1)	C(2,4)	D(3,2)	B(4,3)
G2	C(1,3)	A(2,2)	B(3,4)	D(4,1)
G3	B(1,2)	D(2,3)	C(3,1)	A(4,4)
G4	D(1,4)	B(2,1)	A(3,3)	C(4,2)

Table 21. Properties of LSDs in the presence of RN and CN Effects

LSD Type	number of error functions	number of estimable treatment contrasts
I	6	2(3 with additional assumptions)
II	6	2(3 with additional assumptions)
III	4(6 with additional assumptions)	1(3 with additional assumptions)
IV	3(with additional assumptions)	1(3 with additional assumptions)