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JOURNAL OF THE INDIAN SOCIETY OF
AGRICULTURAL STATISTICS 72(3) 2018 179–186

Calibration Approach based Chain Ratio-Product Type Estimator involving Two Auxiliary Variables in Two Phase Sampling

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Received 14 December 2016; Revised 04 August 2018; Accepted 06 August 2018

SUMMARY

In this paper, we propose a chain ratio-product type estimator of population total in two phase sampling when information on two auxiliary characters is available in different phases. It is assumed that complete information is available for one auxiliary variable while information is not available for other auxiliary variable and the double sampling approach is proposed accordingly. It is assumed that the known auxiliary variable is positively correlated with the study variable while the unknown auxiliary variables are negatively correlated with the study variable and we have applied the two step calibration technique due to Estavao *et al*. (2002). Expressions for the bias and the mean square error of proposed estimators have been obtained as also their estimators. It is shown, through empirical studies that the proposed estimators perform better than existing estimators in terms of the criteria of absolute Relative bias and Percentage relative efficiency.

Keywords: Ratio-product type estimator; two phase sampling; two step calibration, auxiliary information.

AMS Subject Classification: 62D05.

1. INTRODUCTION

Survey statisticians often use auxiliary information to increase the precision of estimators of population parameters such as population mean or population total. In 1930's Cochran introduced the theory of ratio and regression methods of estimation by utilizing the available auxiliary information. The ratio estimator is preferred when there is positive correlation between the study and the auxiliary variable and in case the study and the associated auxiliary variable are negatively correlated, the product estimator is useful. For example, a negative correlation generally exists between the age of individuals and hours of sleep. When the information on the auxiliary variable is not available at the population level, statisticians often select a large preliminary sample to observe the auxiliary variate and further a subsample is selected from the large preliminary sample to observe the character under study. The method of first selecting a large preliminary sample and then subsampling from that large sample is known as two-phase or double sampling. The double sampling technique is appropriate when the auxiliary variables are easily obtainable and it is economically feasible to collect information on auxiliary variables; see Hidiroglou and Sarndal (1998), Fuller (1998), and Hidiroglou (2001). Under the assumption that complete information on an auxiliary variable x is not available, the ratio estimator in double sampling of population mean was given by Sukhatme (1962) as

$$\bar{y}_{Rd} = \bar{y}_n \frac{\bar{x}'_n}{\bar{x}_n}$$

where $\bar{y}_n = \frac{1}{n}\sum_{k=1}^{n} y_k$ is sample mean of study variable y, $\bar{x}_{n'} = \frac{1}{n'}\sum_{k=1}^{n'} x_k$ and $\bar{x}_n = \frac{1}{n}\sum_{k=1}^{n} x_k$ are the sample means of auxiliary variable x for first phase and second phase sample respectively.

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Now let us assume the availability of two auxiliary variables x_1 and x_2. Further, suppose that the auxiliary variable x_1, closely related to the other auxiliary variable x_2 but compared to x_2 less related to the study variable y, exists. For example, in case of yield estimation, area of a crop is positively correlated with the yield while fertilizer is less related to the yield. Then $\hat{\hat{X}}_{2\_Rd} = \frac{\bar{x}'_{2n}}{\bar{x}'_{1n}}\bar{X}_1$ will estimate $\bar{X}_2$ more precisely than $\bar{x}'_{2n}$ if $\rho_{x_1 x_2} > \left(\frac{1}{2}\right)C_{x_1}/C_{x_2}$, where $C_{x_i} = \frac{S_{x_i}}{\bar{X}_i}$; $S^2_{x_i} = \frac{1}{N-1}\sum_{k=1}^{N}\left(x_{ik} - \bar{X}_i\right)^2$; $\bar{X}_i = \frac{1}{N}\sum_{k=1}^{N} x_{ik}$, i = 1, 2.

In this context, Chand (1975) introduced the chain ratio estimators. Modification of chain type ratio estimators has been made by several authors which includes Kiregyera (1980), Prasad *et al*. (1996), Singh and Upadhyay (1995), Farell and Singh (2002), Gupta and Shabbir (2008), Grover and Kaur (2011). Murthy (1964) was pioneer in introducing product estimators followed by Bahl and Tuteja (1991), Singh and Vishwakarma (2007). Estavao and Sarndal (2002) introduced the technique of two step calibration when auxiliary information is available at two levels. Singh (2004) improved the two-phase calibration methodology of Hidiroglou and Sarndal (1995, 1998). Sisodia and Dwivedi (1981) proposed a class of ratio cum product type estimator for estimating population mean with single auxiliary variate and Sisodia and Dwivedi (1982) also suggested a class of ratio cum product type estimator in double sampling. Singh and Espejo (2003) considered a ratio–product type estimator and under double sampling scheme, Singh and Espejo (2007) also suggested a ratio-product type estimator. Choudhury and Singh (2012) developed a class of chain ratio–product type estimators for estimating population mean in double sampling.

In this paper, the objective is to estimate the population total of study variable denoted by y. Further, two auxiliary variables correlated with the study variable considered. Information on one auxiliary variable, say x_2, is not available while complete are information on the other variable, say x_1, is available. In what follows, a chain ratio-product type estimator has been proposed in section 3 using the two step calibration approach suggested by Estavao and Särndal (2002) under the assumption that there is a positive correlation between y and x_1 while y and x_2 are negatively correlated. The expressions for the bias and the mean square error (MSE) of proposed estimator have been developed along with their estimators. Section 4 reports the empirical studies carried out using simulated data. Finally, major conclusions are summarized in section 5.

2. THEORETICAL FRAMEWORK

A finite population of size N denoted by $\Omega = \{1, 2, ..., k, ..., N\}$ is considered and we assumed that complete information on the auxiliary variable (x_1) is available. A large preliminary sample $s' = (s' \in \Omega)$ of size n' following a sampling design $p(\circ)$, is drawn from Ω to estimate (x_2) for which information on population total is not available. The k th unit sampling weight is denoted as $a_{1k} = 1/\pi_{1k}$ with $\pi_{1k} = P(k \in s')$, known first-phase inclusion probability for k th unit. Next, a second phase sample $s = (s \in s')$ of size n is drawn from s' with the corresponding sampling weight for k th unit given by $a_{2k} = 1/\pi_{2k}$, where $\pi_{2k} = P(k \in s | s')$ is the conditional inclusion probability for k th unit given s'. $a_k = a_{1k}a_{2k}$ is the total sampling weight of k th unit, also known as the design weight. The *k-th* unit of the study variable (y), y_k is observed at the second phase i.e. $\{y_k : (k \in s)\}$. Our objective is to estimate the population total $t_y = \sum_{k\in\Omega} y_k$. The *k-th* unit for both the auxiliary variates are given by x_{1k} and x_{2k} respectively. Further we assume that $\sum_{k\in\Omega} x_{1k}$ is known and x_{1k} and x_{2k} are known values for every $k \in s'$. The well-known calibration approach (Deville and Särndal 1992) is used to modify the basic sampling design weight $a_k = a_{1k}a_{2k}$ that appear in the two phase double expansion estimator $\hat{Y}_{DE} = \sum_{k\in s} a_{1k}a_{2k}y_k$. The calibration approach aims at modifying the design weight a_k with the specific objective of finding out new calibrated weight w_k based on a distance function and a set of constraints, also known as calibration constraints. The proposed estimator based on the revised calibrated weight is $\hat{T}_{CAL} = \sum_{k\in s} w_k y_k$.

3. THE PROPOSED CHAIN RATIO-PRODUCT TYPE ESTIMATOR

Following Estavao and Särndal (2002), we consider two-step calibration technique and in each of the two steps a distance function is minimized to compute the calibrated weights. An intermediate calibrated weight is obtained at the first step which involves calibration from s' to Ω and in the second step, the final calibrated weight is obtained through calibration from s to s'. In first step, we minimized the chi-squared type distance function $\sum_{k\in s'}\frac{(w_{1k}-a_{1k})^2}{a_{1k}q_{1k}}$ to obtain first phase calibrated weight w_{1k}. The minimization is performed subject to the calibration constraint $\sum_{k\in s'} w_{1k}x_{1k}=\sum_{k\in\Omega} x_{1k}=X_1$, where q_i's are suitably chosen constants and $X_1=\sum_{k=1}^{N} x_{1k}$. The Lagrange function (L_1) associated with the minimization problem is given by

$$L_1=\sum_{k\in s'}\frac{(w_{1k}-a_{1k})^2}{a_{1k}q_{1k}}-2\lambda\left[\sum_{k\in s'} w_{1k}x_{1k}-X_1\right].$$

On differentiating L_1 with respect to w_{1k} and equating to zero we have $w_{1k}=a_{1k}+\lambda a_{1k}q_{1k}x_{1k}$ and on simplification, we get $\lambda=\frac{X_1-\sum_{k\in s'} a_{1k}q_{1k}x_{1k}}{\sum_{k\in s'} a_{1k}q_{1k}x_{1k}^2}$. With $q_{1k}=1/x_{1k}$, we obtained the first phase calibrated weight w_{1k} as

$$w_{1k}=a_{1k}+\left(X_1-\sum_{k\in s'} a_{1k}x_{1k}\right)\frac{a_{1k}}{\sum_{k\in s'} a_{1k}x_{1k}}. \quad (1)$$

The first phase calibrated weight w_{1k} is used again in the second step (calibration from s to s') to obtain the final calibrated weight w_k. In what follows, a chi-squared type distance function $\sum_{k\in s}\frac{(w_k-a_{1k}a_{2k})^2}{a_{1k}a_{2k}q_{2k}}$ is minimized to obtain the second phase calibrated weight , subject to the calibration constraints

$$\sum_{k\in s} w_k x_{1k}=\sum_{k\in s'} w_{1k}x_{1k}=X_1$$

$$\sum_{k\in s} w_k\frac{1}{x_{2k}}=\sum_{k\in s'} w_{1k}\frac{1}{x_{2k}}=X_2^*.$$

The Lagrange function (L_2) corresponds to the minimization problem is given by

$$L_2=\sum_{k\in s}\frac{(w_k-a_{1k}a_{2k})^2}{a_{1k}a_{2k}q_{2k}}-2\lambda_1\left(\sum_{k\in s} w_k x_{1k}-X_1\right)-2\lambda_2\left(\sum_{k\in s} w_k\frac{1}{x_{2k}}-X_2^*\right).$$

Taking $q_{2k}=x_{2k}/x_{1k}$ and differentiating L_2 with respect to w_k and equating it to zero we have

$$w_k=a_{1k}a_{2k}+\lambda_1 a_{1k}a_{2k}x_{2k}+\lambda_2\frac{a_{1k}a_{2k}}{x_{1k}}.$$

On simplification, we get

$$\lambda_1\sum_{k\in s} a_{1k}a_{2k}x_{1k}x_{2k}+\lambda_2\sum_{k\in s} a_{1k}a_{2k}=X_1-\sum_{k\in s} a_{1k}a_{2k}x_{1k}$$

$$\lambda_1\sum_{k\in s} a_{1k}a_{2k}+\lambda_2\sum_{k\in s}\frac{a_{1k}a_{2k}}{x_{1k}x_{2k}}=X_2^*-\sum_{k\in s} a_{1k}a_{2k}\frac{1}{x_{2k}}$$

Solving both the equations for λ_1 and λ_2, replacing these values in w_k, we get the second phase calibrated weight w_k. On inserting the value of w_k in $\hat{T}_{CAL}$, we obtain the proposed chain ratio-product type estimator as

$$\hat{T}_{CAL-RP}=\sum_{k\in s} a_{1k}a_{2k}y_k+(X_1-\sum_{k\in s} a_{1k}a_{2k}x_{1k})\hat{\beta}_{1(RP)}+(X_2^*-\sum_{k\in s} a_{1k}a_{2k}\frac{1}{x_{2k}})\hat{\beta}_{2(RP)} \quad (3)$$

where $X_1=\sum_{k=1}^{N} x_{1k}$, $X_2^*=\sum_{k\in s'} w_{1k}\frac{1}{x_{2k}}$,

$$\hat{\beta}_{1(RP)}=\frac{\sum_{k\in s} a_{1k}a_{2k}y_k x_{2k}\sum_{k\in s} a_{1k}a_{2k}/x_{1k}x_{2k}-\sum_{k\in s} a_{1k}a_{2k}\sum_{k\in s} a_{1k}a_{2k}\,y_k/x_{1k}}{\sum_{k\in s} a_{1k}a_{2k}x_{1k}x_{2k}\sum_{k\in s} a_{1k}a_{2k}/x_{2k}x_{1k}-(\sum_{k\in s} a_{1k}a_{2k})^2}$$

$$\hat{\beta}_{2(RP)}=\frac{\sum_{k\in s} a_{1k}a_{2k}\,y_k/x_{1k}\sum_{k\in s} a_{1k}a_{2k}x_{1k}x_{2k}-\sum_{k\in s} a_{1k}a_{2k}\sum_{k\in s} a_{1k}a_{2k}y_k x_{2k}}{\sum_{k\in s} a_{1k}a_{2k}x_{1k}x_{2k}\sum_{k\in s} a_{1k}a_{2k}/x_{2k}x_{1k}-(\sum_{k\in s} a_{1k}a_{2k})^2}$$

Let us write $X_{2SRS}^*=X_1\frac{\overline{x}_{2n'}^*}{\overline{x}_{1n'}}$, $\overline{x}_{1n}=\frac{1}{n}\sum_{k=1}^{n} x_{1k}$, $\overline{x}_{2n}^{\oplus}=\frac{1}{n}\sum_{k=1}^{n}\frac{1}{x_{2k}}$, $\overline{y}_n=\frac{1}{n}\sum_{k=1}^{n} y_k$, $\overline{x}_{1n'}=\frac{1}{n'}\sum_{k=1}^{n'} x_{1k}$, $\overline{x}_{2n'}^*=\frac{1}{n'}\sum_{k=1}^{n'} 1/x_{2k}$. Then under simple random sampling without replacement (SRSWOR) design, the expression (3) reduces to

$$\hat{T}_{CAL-RP} = N\bar{y}_n + (X_1 - N\bar{x}_{1n})\hat{\beta}_{1(SRS)} + (X^*_{2SRS} - N\bar{x}^{\oplus}_{2n})\hat{\beta}_{2(SRS)} \tag{4}$$

where

$$\hat{\beta}_{1(SRS)} = \frac{\bar{l}_3\bar{l}_2 - \bar{l}_1}{\bar{l}_3\bar{l}_4 - 1},\ \hat{\beta}_{2(SRS)} = \frac{\bar{l}_4\bar{l}_1 - \bar{l}_2}{\bar{l}_3\bar{l}_4 - 1}, \text{ with}$$

$$\bar{l}_1 = \frac{1}{n}\sum_{k=1}^{n} y_k / x_{1k},\ \bar{l}_2 = \frac{1}{n}\sum_{k=1}^{n} y_k x_{2k},\ \bar{l}_3 = \frac{1}{n}\sum_{k=1}^{n} 1/x_{1k}x_{2k},$$

$$\bar{l}_4 = \frac{1}{n}\sum_{k=1}^{n} x_{1k}x_{2k}.$$

To obtain the Bias and MSE of $\hat{T}_{CAL-RP}$ in (4), we proceed as follows.

Let $\bar{l}_1 = \bar{L}_1 + \varepsilon_1$, $\bar{l}_2 = \bar{L}_2 + \varepsilon_2$, $\bar{l}_3 = \bar{L}_3 + \varepsilon_3$, $\bar{l}_4 = \bar{L}_4 + \varepsilon_4$, $\bar{x}_{1n} = \bar{X}_1 + \varepsilon_{x_1}$, $\bar{x}^{\oplus}_{2n} = \bar{X}^*_2 + \varepsilon_{x_2}$

$\bar{x}_{1n'} = \bar{X}_1 + \varepsilon'_{x_1}$, $\bar{x}^*_{2n'} = \bar{X}^*_2 + \varepsilon'_{x_2}$, $\bar{y}_n = \bar{Y}_1 + \varepsilon_y$ and

$$E(\varepsilon_1) = E(\varepsilon_2) = E(\varepsilon_3) = E(\varepsilon_4) = E(\varepsilon_{x_1}) = E(\varepsilon_{x_2}) = E(\varepsilon'_{x_1}) = E(\varepsilon'_{x_2}) = E(\varepsilon_y) = 0$$

$$E(\varepsilon_y^2) = \left(\frac{1}{n} - \frac{1}{N}\right)S_y^2;\ E(\varepsilon_{x_1}^2) = \left(\frac{1}{n} - \frac{1}{N}\right)S^2_{(x_1)};$$

$$E(\varepsilon_{x_2}^2) = \left(\frac{1}{n} - \frac{1}{N}\right)S^2_{(1/x_2)};\ E(\varepsilon'^2_{x_1}) = \left(\frac{1}{n'} - \frac{1}{N}\right)S^2_{(x_1)};$$

$$E(\varepsilon'^2_{x_2}) = \left(\frac{1}{n'} - \frac{1}{N}\right)S^2_{(1/x_2)};\ E(\varepsilon_y\varepsilon_{x_1}) = \left(\frac{1}{n} - \frac{1}{N}\right)S_{y(x_1)}$$

$$E(\varepsilon_y\varepsilon_{x_2}) = \left(\frac{1}{n} - \frac{1}{N}\right)S_{y(1/x_2)};\ E(\varepsilon_y\varepsilon'_{x_1}) = \left(\frac{1}{n'} - \frac{1}{N}\right)S_{y(x_1)}$$

$$E(\varepsilon_y\varepsilon'_{x_2}) = \left(\frac{1}{n'} - \frac{1}{N}\right)S_{y(1/x_2)};\ E(\varepsilon_{x_1}\varepsilon'_{x_1}) = \left(\frac{1}{n'} - \frac{1}{N}\right)S^2_{(x_1)}$$

$$E(\varepsilon_{x_2}\varepsilon'_{x_2}) = \left(\frac{1}{n'} - \frac{1}{N}\right)S^2_{(1/x_2)};\ E(\varepsilon_{x_1}\varepsilon_{x_2}) = \left(\frac{1}{n} - \frac{1}{N}\right)S_{(x_1)(1/x_2)} \tag{5}$$

$$E(\varepsilon_{x_1}\varepsilon'_{x_2}) = E(\varepsilon_{x_2}\varepsilon'_{x_1}) = E(\varepsilon'_{x_1}\varepsilon'_{x_2}) = \left(\frac{1}{n'} - \frac{1}{N}\right)S_{(x_1)(1/x_2)}$$

$$E(\varepsilon_{x_1}\varepsilon_i) = \left(\frac{1}{n} - \frac{1}{N}\right)S_{(x_1)r_i},\ E(\varepsilon_{x_2}\varepsilon_i) = \left(\frac{1}{n} - \frac{1}{N}\right)S_{(1/x_2)r_i},\ i = 1,2,3,4$$

$$E(\varepsilon'_{x_1}\varepsilon_i) = \left(\frac{1}{n'} - \frac{1}{N}\right)S_{(x_1)r_i},\ E(\varepsilon'_{x_2}\varepsilon_i) = \left(\frac{1}{n'} - \frac{1}{N}\right)S_{(1/x_2)r_i},\ i = 1,2,3,4 \tag{6}$$

where $\bar{L}_i = \frac{1}{N}\sum_{k=1}^{N} l_{ik}$, $i = 1(1)4$; $\bar{X}_1 = \frac{1}{N}\sum_{k=1}^{N} x_{1k}$;

$$\bar{X}^*_2 = \frac{1}{N}\sum_{k=1}^{N} 1/x_{2k};\ \bar{Y} = \frac{1}{N}\sum_{k=1}^{N} y_k$$

$$S_y^2 = \frac{1}{N-1}\sum_{k=1}^{N}(y_k - \bar{Y})^2;\ S^2_{x_1} = \frac{1}{N-1}\sum_{k=1}^{N}(x_{1k} - \bar{X}_1)^2;$$

$$S^2_{(1/x_2)} = \frac{1}{N-1}\sum_{k=1}^{N}\left(\frac{1}{x_{2k}} - \bar{X}^*_2\right)^2$$

$$S_{x_1y} = \frac{1}{N-1}\sum_{k=1}^{N}(x_{1k} - \bar{X}_1)(y_k - \bar{Y});$$

$$S_{(1/x_2)y} = \frac{1}{N-1}\sum_{k=1}^{N}\left(\frac{1}{x_{2k}} - \bar{X}^*_2\right)(y_k - \bar{Y})$$

$$S_{(x_1)(1/x_2)} = \frac{1}{N-1}\sum_{k=1}^{N}(x_{1k} - \bar{X}_1)\left(\frac{1}{x_{2k}} - \bar{X}^*_2\right);$$

$$S_{(x_1)l_j} = \frac{1}{N-1}\sum_{k=1}^{N}(x_{1k} - \bar{X}_1)(l_{jk} - \bar{L}_j);\ j = 1(1)4$$

$$S_{(1/x_2)l_j} = \frac{1}{N-1}\sum_{k=1}^{N}\left(\frac{1}{x_{2k}} - \bar{X}^*_2\right)(l_{jk} - \bar{L}_j);\ j = 1(1)4.$$

Now expressing $\hat{T}_{CAL-RP}$ in (4) in terms of $\varepsilon's$ and expanding the right hand side by ignoring the terms involving $\varepsilon's$ in degrees greater than two, we have

$$\hat{T}_{CAL-RP} = N\left[\begin{aligned}&\bar{Y}\left(1 + \frac{\varepsilon_y}{\bar{Y}}\right) - \bar{X}_1\left(A_0 + A_1\frac{\varepsilon_1}{\bar{L}_1} + A_2\frac{\varepsilon_2}{\bar{L}_2} + A_3\frac{\varepsilon_3}{\bar{L}_3} + A_4\frac{\varepsilon_4}{\bar{L}_4}\right)\frac{\varepsilon_{x_1}}{\bar{X}_1}\\ &+\bar{X}^*_2\left(B_0 + B_1\frac{\varepsilon_1}{\bar{L}_1} + B_2\frac{\varepsilon_2}{\bar{L}_2} + B_3\frac{\varepsilon_3}{\bar{L}_3} + B_4\frac{\varepsilon_4}{\bar{L}_4}\right)\left(\frac{\varepsilon'_{x_2}}{\bar{X}^*_2} - \frac{\varepsilon'_{x_1}}{\bar{X}_1} - \frac{\varepsilon_{x_2}}{\bar{X}^*_2}\right)\end{aligned}\right]$$

where

$$A_0 = \bar{L}_1 - \bar{L}_2\bar{L}_4 + \bar{L}_1\bar{L}_3\bar{L}_4 - \bar{L}_2\bar{L}_3\bar{L}_4^2;\ A_1 = \bar{L}_1 + \bar{L}_1\bar{L}_3\bar{L}_4;$$
$$A_2 = \bar{L}_2\bar{L}_4 + \bar{L}_2\bar{L}_3\bar{L}_4^2;\ A_3 = \bar{L}_2\bar{L}_3\bar{L}_4^2 - \bar{L}_1\bar{L}_3\bar{L}_4;$$
$$A_4 = \bar{L}_2\bar{L}_4 + 2\bar{L}_2\bar{L}_3\bar{L}_4^2 - \bar{L}_1\bar{L}_3\bar{L}_4$$

$$B_0 = \bar{L}_1\bar{L}_4 - \bar{L}_2 - \bar{L}_2\bar{L}_3\bar{L}_4 + \bar{L}_1\bar{L}_3\bar{L}_4^2;\ B_1 = \bar{L}_1\bar{L}_4 + \bar{L}_1\bar{L}_3\bar{L}_4^2;$$
$$B_2 = \bar{L}_2 + \bar{L}_2\bar{L}_3\bar{L}_4;\ B_3 = \bar{L}_1\bar{L}_3\bar{L}_4^2 - \bar{L}_2\bar{L}_3\bar{L}_4;$$
$$B_4 = \bar{L}_1\bar{L}_4 - \bar{L}_2\bar{L}_3\bar{L}_4 + 2\bar{L}_1\bar{L}_3\bar{L}_4^2$$

Now subtracting $N\bar{Y}$ from both sides, we get

$$\hat{T}_{CAL-RP} = N\left[\begin{aligned}&\bar{Y}\left(\frac{\varepsilon_y}{\bar{Y}}\right) - \bar{X}_1\left(A_0 + A_1\frac{\varepsilon_1}{\bar{L}_1} + A_2\frac{\varepsilon_2}{\bar{L}_2} + A_3\frac{\varepsilon_3}{\bar{L}_3} + A_4\frac{\varepsilon_4}{\bar{L}_4}\right)\frac{\varepsilon_{x_1}}{\bar{X}_1}\\ &+\bar{X}^*_2\left(B_0 + B_1\frac{\varepsilon_1}{\bar{L}_1} + B_2\frac{\varepsilon_2}{\bar{L}_2} + B_3\frac{\varepsilon_3}{\bar{L}_3} + B_4\frac{\varepsilon_4}{\bar{L}_4}\right)\left(\frac{\varepsilon'_{x_2}}{\bar{X}^*_2} - \frac{\varepsilon'_{x_1}}{\bar{X}_1} - \frac{\varepsilon_{x_2}}{\bar{X}^*_2}\right)\end{aligned}\right]$$

Taking expectations in both sides and using the result in (6), we get the bias of $\hat{T}_{CAL-RP}$ to the first degree of approximation as

$$Bias\left(\hat{T}_{CAL-RP}\right)=N\begin{bmatrix}\left(\frac{1}{n}-\frac{1}{N}\right)\left(\mathrm{A}_1C_{x_1l_1}-\mathrm{A}_2C_{x_1l_2}-\mathrm{A}_4C_{x_1l_4}-\mathrm{A}_3C_{x_1l_3}\right)\bar{X}_1\\+\left(\frac{1}{n}-\frac{1}{n'}\right)\left(\mathrm{B}_2C_{(1/x_2)l_2}-\mathrm{B}_1C_{(1/x_2)l_1}-\mathrm{B}_3C_{(1/x_2)l_3}-\mathrm{B}_4C_{(1/x_2)l_4}\right)\bar{X}_2^*\\+\left(\frac{1}{n'}-\frac{1}{N}\right)\left(\mathrm{B}_2C_{(x_1)l_2}-\mathrm{B}_1C_{(x_1)l_1}-\mathrm{B}_3C_{(1/x_1)l_3}-\mathrm{B}_4C_{(x_1)l_4}\right)\bar{X}_2^*\end{bmatrix}$$

where $C_{(x_1)l_j}=\frac{S_{(x_1)l_j}}{\bar{X}_1\bar{L}_j}\,;j=1(1)4$, $C_{(1/x_2)l_j}=\frac{S_{(1/x_2)l_j}}{\bar{X}_2^*\bar{L}_j}\,;j=1(1)4$

and corresponding estimator of the Bias of $\hat{T}_{CAL-RP}$ is given by

$$\hat{Bias}\left(\hat{T}_{CAL-RP}\right)=N\begin{bmatrix}\left(\frac{1}{n}-\frac{1}{N}\right)\left(a_1c_{x_1l_1}-a_2c_{x_1l_2}-a_4c_{x_1l_4}-a_3c_{x_1l_3}\right)\bar{X}_1\\+\left(\frac{1}{n}-\frac{1}{n'}\right)\left(b_2c_{(1/x_2)l_2}-b_1c_{(1/x_2)l_1}-b_3c_{(1/x_2)l_3}-b_4c_{(1/x_2)l_4}\right)\hat{\bar{X}}_2^*\\+\left(\frac{1}{n'}-\frac{1}{N}\right)\left(b_2c_{(x_1)l_2}-b_1c_{(x_1)l_1}-b_3c_{(1/x_1)l_3}-b_4c_{(x_1)l_4}\right)\hat{\bar{X}}_2^*\end{bmatrix}$$

where

$$\mathrm{a}_0=\bar{l}_1-\bar{l}_2\bar{l}_4+\bar{l}_1\bar{l}_3\bar{l}_4-\bar{l}_2\bar{l}_3\bar{l}_4^2\,;\ \mathrm{a}_1=\bar{l}_1+\bar{l}_1\bar{l}_3\bar{l}_4\,;$$
$$\mathrm{a}_2=\bar{l}_2\bar{l}_4+\bar{l}_2\bar{l}_3\bar{l}_4^2;\ \mathrm{a}_3=\bar{l}_1\bar{l}_3\bar{l}_4-\bar{l}_2\bar{l}_3^2\bar{l}_4\,;$$
$$\mathrm{a}_4=\bar{l}_1\bar{l}_3\bar{l}_4-\bar{l}_2\bar{l}_3-2\bar{l}_2\bar{l}_3^2\bar{l}_4\,;$$

$$\mathrm{b}_0=\bar{l}_1\bar{l}_4-\bar{l}_2-\bar{l}_2\bar{l}_3\bar{l}_4+\bar{l}_1\bar{l}_3\bar{l}_4^2\,;\ \mathrm{b}_1=\bar{l}_1\bar{l}_4+\bar{l}_1\bar{l}_3\bar{l}_4^2\,;$$
$$\mathrm{b}_2=\bar{l}_2+\bar{l}_2\bar{l}_3\bar{l}_4\,;\ \mathrm{b}_3=\bar{l}_1\bar{l}_3\bar{l}_4^2-\bar{l}_2\bar{l}_3\bar{l}_4\,;$$
$$\mathrm{b}_4=\bar{l}_1\bar{l}_4-\bar{l}_2\bar{l}_3\bar{l}_4+2\bar{l}_1\bar{l}_3\bar{l}_4^2$$

$$c_{(x_1)l_j}=\frac{s_{(x_1)l_j}}{\bar{X}_1\bar{l}_j}\ ;\ j=1(1)4\,;\ c_{(1/x_2)l_j}=\frac{s_{(1/x_2)l_j}}{\hat{\bar{X}}_2^*\bar{l}_j}\ ;\ j=1(1)4\ ;$$

$$\hat{\bar{X}}_2^*=\frac{1}{n'}\sum_{k=1}^{n'}\frac{1}{x_{2k}}\,;$$

$$s_{(1/x_2)l_j}=\frac{1}{n-1}\sum_{k=1}^{n}\left(\frac{1}{x_{2k}}-\hat{\bar{X}}_2^*\right)\left(l_{jk}-\bar{l}_j\right);\ j=1(1)4\,;$$

$$\bar{l}_i=\frac{1}{n}\sum_{k=1}^{n}l_{ik}\ ,\ i=1(1)4\,,$$

$$s_{(x_1)l_j}=\frac{1}{n-1}\sum_{k=1}^{n}\left(x_{1k}-\bar{x}_1\right)\left(l_{jk}-\bar{l}_j\right);\ j=1(1)4\,.$$

Now to obtain the MSE of the estimator $\hat{T}_{CAL-RP}$, we express $\hat{T}_{CAL-RP}$ in (4) in terms of $\varepsilon's$ and expanding the right hand side by ignoring the terms involving $\varepsilon's$ in degrees greater than or equal to two, we have

$$\hat{T}_{CAL-RP}=N\left[\bar{Y}\left(1+\frac{\varepsilon_y}{\bar{Y}}\right)+A_0\bar{X}_1\left(\frac{\varepsilon_{x_1}}{\bar{X}_1}\right)+B_0\bar{X}_2^*\left(\frac{\varepsilon'_{x_2}}{\bar{X}_2^*}-\frac{\varepsilon'_{x_1}}{\bar{X}_1}-\frac{\varepsilon_{x_2}}{\bar{X}_2^*}\right)\right]$$

where $\mathrm{A}_0=\bar{L}_1-\bar{L}_2\bar{L}_4+\bar{L}_1\bar{L}_3\bar{L}_4-\bar{L}_2\bar{L}_3\bar{L}_4^2$ and $\mathrm{B}_0=\bar{L}_2-\bar{L}_1\bar{L}_3+\bar{L}_2\bar{L}_3\bar{L}_4-\bar{L}_1\bar{L}_3^2\bar{L}_4$.

Now subtracting $N\bar{Y}$ from both sides, we get

$$\hat{T}_{CAL-RP}=N\left[\bar{Y}\left(\frac{\varepsilon_y}{\bar{Y}}\right)+A_0\bar{X}_1\left(\frac{\varepsilon_{x_1}}{\bar{X}_1}\right)+B_0\bar{X}_2^*\left(\frac{\varepsilon'_{x_2}}{\bar{X}_2^*}-\frac{\varepsilon'_{x_1}}{\bar{X}_1}-\frac{\varepsilon_{x_2}}{\bar{X}_2^*}\right)\right].$$

Squaring both sides, taking expectations and using the result in (5) and (6), we get the MSE of $\hat{T}_{CAL-RP}$ to the first degree of approximation as

$$MSE\left(\hat{T}_{CAL-RP}\right)=N^2\left[\left(\frac{1}{n}-\frac{1}{N}\right)C_1+\left(\frac{1}{n}-\frac{1}{n'}\right)C_2+\left(\frac{1}{n'}-\frac{1}{N}\right)C_3\right]$$

where

$$C_1=C_y^2\bar{Y}^2+A_0^2\bar{X}_1^2C_{(x_1)}^2-2A_0\overline{YX}_1\rho_{(x_1)y}C_yC_{(x_1)}$$

$$C_2=B_0^2\bar{X}_2^{*2}C_{(1/x_2)}^2-2B_0\overline{YX}_2^*\rho_{(1/x_2)y}C_yC_{(1/x_2)}-2A_0B_0\bar{X}_1\bar{X}_2^*\rho_{(x_1)(1/x_2)}C_{(x_1)}C_{(1/x_2)}$$

$$C_3=B_0^2\bar{X}_2^{*2}C_{(x_1)}^2-2B_0\overline{YX}_2^*\rho_{(x_1)y}C_yC_{(x_1)}+2A_0B_0\bar{X}_1\bar{X}_2^*C_{(x_1)}^2$$

$$C_y=\frac{S_y}{\bar{Y}}\ ;\ C_{(x_1)}=\frac{S_{(x_1)}}{\bar{X}_1}\ ;\ C_{(1/x_2)}=\frac{S_{(1/x_2)}}{\bar{X}_2^*}\,,$$

and corresponding estimator of the MSE of $\hat{T}_{CAL-P}$ is given by

$$MS\hat{E}\left(\hat{T}_{CAL-RP}\right)=N^2\left[\left(\frac{1}{n}-\frac{1}{N}\right)C_{11}+\left(\frac{1}{n}-\frac{1}{n'}\right)C_{22}+\left(\frac{1}{n'}-\frac{1}{N}\right)C_{33}\right]$$

where

$$C_{11} = c_y^2 \bar{y}^2 + a_0^2 \bar{X}_1^2 c_{(x_1)}^2 - 2c_0 \bar{y} \bar{X}_1 \rho_{(x_1)y} c_y c_{(x_1)}$$

$$C_{22} = b_0^2 \hat{\bar{X}}_2^{*2} c_{(1/x_2)}^2 - 2b_0 \bar{y} \hat{\bar{X}}_2^{*} \rho_{(1/x_2)y} c_y c_{(1/x_2)} - 2a_0 b_0 \bar{X}_1 \hat{\bar{X}}_2^{*} \rho_{(x_1)(1/x_2)} c_{(x_1)} c_{(1/x_2)}$$

$$C_{33} = b_0^2 \hat{\bar{X}}_2^{*2} c_{(x_1)}^2 - 2b_0 \bar{y} \hat{\bar{X}}_2^{*} \rho_{(x_1)y} c_y c_{(x_1)} + 2a_0 b_0 \bar{X}_1 \hat{\bar{X}}_2^{*} c_{(x_1)}^2$$

$$c_y = \frac{s_y}{\bar{y}}; \quad c_{(x_1)} = \frac{s_{(x_1)}}{\bar{X}_1}; \quad c_{(1/x_2)} = \frac{s_{(1/x_2)}}{\hat{\bar{X}}_2^{*}};$$

$$\mathrm{a}_0 = \overline{l_1} - \overline{l_2 l_4} + \overline{l_1 l_3 l_4} - \overline{l_2 l_3 l_4^2} \;;$$

$$\mathrm{b}_0 = \overline{l_2} - \overline{l_1 l_3} + \overline{l_2 l_3 l_4} - \overline{l_1 l_3^2 l_4} \;;$$

$$s_{(x_1)}^2 = \frac{1}{n-1} \sum_{k=1}^{n} \left(x_{1k} - \bar{x}_1 \right)^2 \;;$$

$$s_{(1/x_2)}^2 = \frac{1}{n-1} \sum_{k=1}^{n} \left(\frac{1}{x_{2k}} - \hat{\bar{X}}_2^{*} \right)^2 .$$

4. EMPIRICAL EVALUATIONS

This section deals with the results from simulation studies to evaluate the performance of the proposed chain ratio-product type estimator with the existing ones. The empirical result uses model-based simulation to generate fixed finite population. To assess the performance of the proposed estimator, we consider the following existing estimators

$$\hat{T}_{CS} = N\bar{y}_n \left[k \frac{\bar{x}'_{2n}}{\bar{x}_{2n}} \frac{\bar{X}_1}{\bar{x}'_{1n}} + (1-k) \frac{\bar{x}'_{1n}}{\bar{x}'_{2n}} \frac{\bar{x}_{2n}}{\bar{X}_1} \right]$$

(Choudhury and Singh, 2012)

where

$$k = \frac{\left(\frac{1}{n} - \frac{1}{n'}\right) C_{x_2}^2 \left(1 + \frac{\rho_{yx_2} C_y C_{x_2}}{C_{x_2}^2}\right) + \left(\frac{1}{n'} - \frac{1}{N}\right) C_{x_1}^2 \left(1 + \frac{\rho_{yx_1} C_y C_{x_1}}{C_{x_1}^2}\right)}{2\left[\left(\frac{1}{n} - \frac{1}{n'}\right) C_{x_2}^2 + \left(\frac{1}{n'} - \frac{1}{N}\right) C_{x_1}^2\right]}$$

$$\hat{T}_{CR} = N\bar{y}_n \frac{\bar{x}_{2n}}{\bar{x}'_{2n}} \frac{\bar{X}_1}{\bar{x}'_{1n}}$$

(Chain Ratio cum Product estimator)

$\hat{T}_{mpu} = N\bar{y}_n$ (Mean per unit estimator).

Two criteria viz. absolute Relative Bias (RB) and Percentage Relative Efficiency (PRE) are chosen to assess the performance of various estimators.

$$\mathrm{RB} = \frac{1}{M} \sum_{k=1}^{M} \left| \frac{\hat{T}_k - T_k}{T_k} \right|$$

$$\mathrm{PRE} = \frac{\mathrm{MSE_{MC}}\left(\hat{T}_{mpu}\right)}{\mathrm{MSE_{MC}}\left(\hat{T}_k\right)} \times 100 ,$$

$$\mathrm{MSE_{MC}} \hat{T}_k = \frac{1}{M} \sum_{k=1}^{M} \left(\hat{T}_k - T_k \right)^2$$

where T_k denotes the actual value of the estimators $\hat{T}_{CAL-RP}$, $\hat{T}_{CS}$, $\hat{T}_{CR}$ and $\hat{T}_{mpu}$ for the *k*-th simulation run, with predicted value as $\hat{T}_k$ for all the estimators and M denotes the number of simulation run. The Monte Carlo simulation was run M = 2500 times.

Here, first we generated unknown auxiliary variate x_2 using the model

$$x_{2k} = \beta_1 \left(1/x_{1k} \right) + e_k \; ; \; k = 1, \ldots N$$

where $x_{1k} \sim \chi^2(5)$, $e_k \sim N\left(0, \sigma_e^2\right)$ and the value of β_1 is chosen as 15.And then we generated a fixed finite population of size N = 1000 using the model

$$y_k = \beta_2 x_{1k} + \beta_3 \left(1/x_{2k} \right) + \varepsilon_k \; ; \; k = 1, \ldots N$$

where $\varepsilon_k \sim N\left(0, \sigma_\varepsilon^2\right)$ and the scale parameters are taken as $\beta_2 = 5$ and $\beta_3 = 0.5$ .We chose different values for both σ_e and σ_ε to generate twelve different datasets for the simulation studies. In particular, different values for parameters σ_e and σ_ε are given as (2.5, 5.5, 7.5) and (10, 15, 20, 25) respectively. This lead to twelve different population data sets with different values of correlation between *y*, x_1 and x_2. These are described in Table 1. For each fixed finite population, a first phase sample s' of size 500 units was selected by SRSWOR design while from s', a subsample *s* of size 100 units was drawn by SRSWOR design and estimation of population total was carried out. In particular, we drew M = 2500 samples from the fixed population and for each sample we calculated the estimates of population total. The related results are reported in Table 2 to Table 4.

Two things emerged from the Table 2 to Table 4. Firstly, the absolute relative bias decreases and percentage relative efficiency increases as the correlation between *y* and x_1 increases and the correlation between *y* and x_2 decreases, which is

Table 1. Description of Simulation Parameters

Parameter Set		σ_e	σ_ε	$\rho(y,x_1)$	$\rho(y,x_2)$	$\rho(x_1,x_2)$
Set-1	1a	2.5	10	0.83	-0.46	-0.54
	1b	2.5	15	0.71	-0.40	-0.54
	1c	2.5	20	0.60	-0.34	-0.54
	1d	2.5	25	0.51	-0.29	-0.54
Set-2	2a	5.5	10	0.83	-0.34	-0.43
	2b	5.5	15	0.70	-0.29	-0.43
	2c	5.5	20	0.59	-0.24	-0.43
	2d	5.5	25	0.51	0.20	-0.43
Set-3	3a	7.5	10	0.80	-0.28	-0.36
	3b	7.5	15	0.69	-0.24	-0.36
	3c	7.5	20	0.58	-0.20	-0.36
	3d	7.5	25	0.50	-0.16	-0.36

Table 2. Relative Bias (RB) and Percentage Relative Efficiency (PRE) of Different Estimators of Population Total in parameter set-1

	Set 1a		Set 1b		Set 1c		Set 1d	
	RB	PRE	RB	PRE	RB	PRE	RB	PRE
$\hat{T}_{CAL-RP}$	0.02	1253.84	0.02	998.24	0.03	775.23	0.03	537.46
$\hat{T}_{mpu}$	0.10	100.00	0.10	100.00	0.11	100.00	0.11	100.00
$\hat{T}_{CS}$	0.09	156.25	0.09	144.36	0.10	126.59	0.10	118.95
$\hat{T}_{CR}$	0.05	408.23	0.06	364.26	0.06	258.63	0.07	242.64

the basic aim of this paper. Secondly, the proposed estimator outperforms all the existing estimators in terms of the RB and PRE for small as well as the large correlation between y, x_1 and x_2. As expected the relative gain in both bias and the mean squared error for the proposed estimator is highest for the parameter set-1.

From Table 1, it is clear that, the highest positive correlation between y and x_1 and the highest negative correlation between y and x_2 appear in the parameter set 1. Hence the gain in efficiency is highest for parameter set 1 followed by parameter set 2 and 3. If we consider, with in parameter set evaluation, then from Table 2 to Table 4, it is evident that within a given parameter set, the bias and the efficiency increases as the correlation between y and x_1 increases and the correlation between y and x_2 decreases. Let us consider Table 4 which contains the bias and relative efficiency for parameter set 3. In this particular set,

Table 3. Relative Bias (RB) and Percentage Relative Efficiency (PRE) of Different Estimators of Population Total in parameter set-2

	Set 2a		Set 2b		Set 2c		Set 2d	
	RB	PRE	RB	PRE	RB	PRE	RB	PRE
$\hat{T}_{CAL-RP}$	0.04	1034.24	0.05	794.33	0.05	521.46	0.06	324.20
$\hat{T}_{mpu}$	0.13	100.00	0.13	100.00	0.14	100.00	0.15	100.00
$\hat{T}_{CS}$	0.13	128.35	0.13	122.14	0.14	117.86	0.15	109.45
$\hat{T}_{CR}$	0.09	312.53	0.09	285.42	0.09	267.24	0.10	243.13

Table 4. Relative Bias (RB) and Percentage Relative Efficiency (PRE) of Different Estimators of Population Total in parameter set-3

	Set 3a		Set 3b		Set 3c		Set 3d	
	RB	PRE	RB	PRE	RB	PRE	RB	PRE
$\hat{T}_{CAL-RP}$	0.05	887.82	0.07	500.13	0.09	332.76	0.11	244.02
$\hat{T}_{mpu}$	0.15	100.00	0.15	100.00	0.16	100.00	0.17	100.00
$\hat{T}_{CS}$	0.14	104.40	0.15	104.08	0.16	103.80	0.16	103.54
$\hat{T}_{CR}$	0.10	208.26	0.11	207.31	0.11	206.45	0.12	205.68

$\rho(y,x_1)$ ranges from 0.80 to 0.50 while $\rho(y,x_2)$ ranges from -0.28 to -0.16. As the $\rho(y,x_1)$ increases from 0.50 to 0.80 and $\rho(y,x_2)$ decreases from -0.16 to -0.28 gradually, the bias decreases as well as the relative efficiency increases.

5. CONCLUDING REMARKS

In this paper, we have been able to develop a chain ratio-product type estimator of population total based on two step calibration approach proposed by Estavao and Särndal (2002) that performs better than the existing estimators. We assumed that the study variable was positively correlated with one auxiliary character while there was a negative correlation between the study and the other auxiliary variate. We also assumed that the auxiliary variable which is known at the population level, should be positively correlated and the other auxiliary variable for which the population total is unknown, should be negatively correlated with the study variable. It may be noteworthy that the proposed chain ratio type estimator outperforms the existing estimators in terms of bias as well as relative efficiency.

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