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An Application of Weibull Process in Reliability Theory

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SUMMARY

The importance of reliability theory has grown out of the demands of modern technology and particularly out of the experiences complex military systems. The present investigation focus on importance of application of Weibull distribution in reliability theory as it is frequently used based on the assumption of a special type of non-homogeneous Poisson process. Also discussed some of the important inferential statistics of Weibull process in application to reliability theory.

Keywords: Reliability theory, Weibull distribution, Compound distribution, Maximum likelihood estimation, Confidence interval.

1. INTRODUCTION

The mathematical theory of reliability has grown out of the demands of modern technology and particularly out of the experiences in World War–II with complex military systems. One of the first areas of reliability to be approached with any mathematical sophistication was the area of machine maintains Gertsbakh (2013). Reliability and survival analysis are interchangeable terms. These are considered different in the sense that the term reliability is used while performing experiments on man-made systems and the term survival analysis for experimenting on God made or natural systems. The system is defined as an arbitrary device performing an activity.

Suppose that we make a statement that a particular electrical component is reliable, then by this we mean that the component will behave in a manner that is expected of it. But if a particular component happens to fail unexpectedly, we accept it as a chance, failure. Thus the expected behavior under some assumed conditions forms the basis for defining the word reliability. For more discussion one may refer to Santosh *et al*. (2018), Wanga *et al*. (2016), Zhai and Lin (2015) and Cui *et al*. (2014).

Reliability is the probability of a device performing its purpose adequately for the period intended under the given operating conditions.

Mathematically, the definition of reliability $R(t)$ or survival function $S(t)$ can be put as

$$S(t) = R(t) = P[T > t]$$

$$= 1 - P[T \leq t]$$

$$1 - F(t);\; t > 0$$

Where $F(t)$ is the c.d.f. of the random variable t representing the life time.

Clearly, since

$$S(0) = \lim_{t \to 0} S(t) = 1$$

$$S(\infty) = \lim_{t \to \infty} S(t) = 0$$

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We conclude that S(t) is a decreasing function in t. In other words, "the reliability of a system is the probability that when operating under stated environmental Conditions, the system will perform its intended function adequately for a specified interval of time."

The latest task on modeling and analysis of repairable systems is based on the assumption of a special type of non-homogeneous Poisson process known as Weibull process [Baln and Englehhardt (1991)]. This model is also called a power law process in literature. The name Weibull process derives primarily from the resemblance of the intensity function of the process to the hazard function of a Weibull distribution.

Particularly the intensity function is given by

$$V(t) = \left(\frac{\beta}{\theta}\right)\left(\frac{t}{\theta}\right)^{\beta-1} \quad \ldots (1.1)$$

The notions of hazard rate, should not to be confused the intensity with one another in notions. The latter is a relative rate of failure for non-repairable systems, whereas the former is an absolute rate of failure for repairable system. Ascher and Feingold (1984, p.33) provided the further discussion on this point.

The mean value function of a Weibull process has the form

$$m(t) = \left(\frac{t}{\theta}\right)^{\beta} \quad \ldots (1.2)$$

here $\theta(>0)$ is scale parameter and $\beta(>0)$ is shape parameter. Another parameterization that is sometimes used is $m(t) = \lambda t^{\beta}$ $\left\{\text{taking } \lambda = \frac{1}{\theta^{\beta}}\right\}$

here λ is called the intensity parameter.

With parameterization

If $\beta = 1$, it yields an homogeneous poisson process.

If $\beta > 1$, it yields a deteriorating system,

if $\beta < 1$, it provides a model for reliability growth.

It is necessary to cease taking further observations at same point, in order to obtain data. Such action is usually referred to as truncation of the process. In general, the process is said to be failure truncated if it is observed until a fixed number of failures have ocurred, and it is said to be time truncated if it is observed for a fixed length of time. With failure truncation, the data consists simply of the set of observed failure times, whereas with time truncation the number of occurences in the interval of observation is also part of the data.

2. MATHEMATICAL DEVELOPMENT OF WEIBULL PROCESS

Let $T_1, T_2, \ldots\ldots T_n$ be the n successive occurences of non-homogeneous Poisson process and $m(t) = \left(\frac{t}{\theta}\right)^{\beta}$ denote the mean function of the process and let $m(t)$ is continuous. If $Z_j = \left(\frac{Tj}{\theta}\right)^{\beta}$ then $z_1 < z_2 < \ldots\ldots < z_n$ are distributed as the first n successive occurence times of an homogeneous Poisson process with intensity $\lambda = 1$, i.e.

$$Z_j = \left(\frac{T_i}{\theta}\right)^{\beta} \sim \exp(1), \qquad 0 < z_1 < z_2 < \ldots\ldots < z_n < \infty$$

Note that Z_j's are not independent. If we let $Z_0 = 0$ and define

$$X_j = Z_j - Z_{j-1}, \qquad j = 1, 2, \ldots, n$$

Then by a well known property of HPP there differences are independently and identically distributed as exponential with mean unity. i.e. the joint density of $x_1, x_2, \ldots\ldots, x_n$ is

$$f(x_1, x_2, \ldots\ldots, x_n) = e^{-\sum_{j=1}^{n} x_1}, \qquad 0 < x_j < \infty \quad \ldots (2.1)$$

Now consider

$$z_1 = x_1$$

$$z_2 = x_2 + x_1$$

… … … … …

… … … … …

… … … … …

$$z_n = x_n + x_{n-1} + \ldots\ldots + x_1$$

so that the Jacobian of the transformation is

$$|J| = 1$$

then the joint density of $Z_1, Z_2, ..., Z_n$ becomes

$$f(z_1, z_2, ..., z_n) = e^{-\sum_{j=1}^{n}(z_j - z_{j-1})}.|J|$$

$$= e^{-z_n}, \qquad 0 < z_1 < ...... < z_n < \infty \qquad ...(2.2)$$

Now for the joint density of $T_1, T_2, ..., T_n$, consider

$$Z_1 = \left(\frac{T_1}{\theta}\right)^{\beta}$$

$$Z_2 = \left(\frac{T_2}{\theta}\right)^{\beta}$$

… … … … … …

… … … … … …

… … … … … …

$$Z_n = \left(\frac{T_n}{\theta}\right)^{\beta}$$

so that the jocabian of the transformation

$$|J| = \begin{vmatrix} \frac{\beta}{\theta}\left(\frac{T_1}{\theta}\right)^{\beta-1} & 0 & 0 & \cdots & 0 \\ 0 & \frac{\beta}{\theta}\left(\frac{T_2}{\theta}\right)^{\beta-1} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \frac{\beta}{\theta}\left(\frac{T_n}{\theta}\right)^{\beta-1} \end{vmatrix}$$

$$= \left(\frac{\beta}{\theta}\right)^n \prod_{j=1}^{n}\left(\frac{T_j}{\theta}\right)^{\beta-1}$$

Thus the joint density function of $T_1, T_2, ..., T_n$ is

$$f(t_1, t_2, ..., t_n) = e^{-\left(\frac{T_n}{\theta}\right)^{\beta}}.|J|$$

$$= \left(\frac{\beta}{\theta}\right)^n \left[\prod_{j=1}\left(\frac{t_j}{\theta}\right)\right]^{\beta-1}.\exp\left[-\left(\frac{t_n}{\theta}\right)^{\beta}\right], \quad 0 < t_1 < t_2 < ... < t_n < \infty \qquad ...(2.3)$$

This density seems to be quite simple. It denotes the liklihood of the sample $t_1, t_2, ..., t_n$. Now we proceed to obtain the maximum likelihood estimates of β and θ.

2.1 Maximum Likelihood Estimates of β and θ

Taking the logarithm of eq. (2.3) and differentiate w.r.t. β and θ, and then equating then to zero, we get,

$$\log f = n\log\left(\frac{\beta}{\theta}\right) + (\beta - 1)\sum_{j=1}^{n}\log\left(\frac{t_j}{\theta}\right) - \left(\frac{t_n}{\theta}\right)^{\beta}$$

$$\frac{\partial \log f}{\partial \beta} = \frac{n\theta}{\beta}.\frac{1}{\theta} + \Sigma\log\left(\frac{t_j}{\theta}\right) - \left(\frac{t_n}{\theta}\right)^{\beta}.\log\left(\frac{t_n}{\theta}\right) = 0 \qquad ...(2.1.1)$$

$$\frac{\partial \log f}{\partial \theta} = -\frac{n}{\theta} - (\beta - 1)\left(\frac{n}{\theta}\right) + \beta\left(\frac{t_n}{\theta}\right)^{\beta-1} = 0$$

$$\Rightarrow \; -\frac{\beta.n}{\theta} + \beta\left(\frac{t_n}{\theta}\right)^{\beta-1}.t_n = 0$$

$$\Rightarrow \; \frac{n}{\theta} = \left(\frac{t_n}{\theta}\right)^{\beta}.\frac{1}{\theta}$$

$$\Rightarrow \; \hat{\theta} = \frac{t_n}{n^{1/\hat{\beta}}} \qquad ...(2.1.2)$$

Putting the value of $\hat{\theta}$ from (4.7) in (4.6), we obtain

$$\frac{n}{\beta} + \Sigma\log\left(\frac{t_j}{\theta}\right) - \left(\frac{t_n}{\theta}\right)^{\beta}.\log\left(\frac{t_n}{\theta}\right) = 0$$

$$\Rightarrow \; \frac{n}{\beta} + \Sigma\log t_j\frac{n^{1/\hat{\beta}}}{t_n} - n\log n^{1/\hat{\beta}} = 0$$

$$\Rightarrow \; \frac{n}{\beta} - \sum_{j=1}^{n}\log\left(\frac{t_n}{t_j}\right) + \frac{n}{\beta}\log n - \frac{n}{\beta}\log n = 0$$

$$\Rightarrow \; \hat{\beta} = \frac{n}{\sum_{j=1}^{n}\log\left(\frac{t_n}{t_j}\right)}$$

$$\Rightarrow \; \hat{\beta} = \frac{n}{\sum_{j=1}^{n-1}\log\left(\frac{t_n}{t_j}\right)} \qquad \{\text{since } \log 1 = 0\} \; ... (2.1.3)$$

The equation (2.1.2) and (2.1.3) give the MLE's $\hat{\beta}$ and $\hat{\theta}$ of β and θ respectively.

2.2 SUFFICIENT STATISTIC

The density in equation (2.2) shows that $\left(t_n, \prod_{j-1}^{n} t_j\right)$ is a joint sufficient statistic for (θ, β). Note that the MLE's are one to one functions of the joint sufficient statistic. Therefore the m.l.e.'s are also sufficient statistics, and they also possess the same desirable and useful properties as enjoyed by the MLE's under ordinary random sampling. Some more remarks regarding $\hat{\beta}$ and $\hat{\theta}$ are given at the end of section (2.1).

3. DISTRIBUTION OF THE MAXIMUM LIKLIHOOD ESTIMATES

Consider

$$Z_n = \left(\frac{T_n}{\theta}\right)^{\beta}$$

$$\Rightarrow \quad T_n = \theta Z_n^{\frac{1}{\beta}}$$

$$= \theta\left[\sum_{j=1}^{n} X_j\right]^{\frac{1}{\beta}}$$

$$\Rightarrow \quad \sum_{j=1}^{n} X_j = \left(\frac{T_n}{\theta}\right)^{\beta}$$

But from the section (2.2), X_j's are i.i.d. exponential with mean unity. Therefore, ΣX_j is distributed as gamma with shape parameter as *n* and scale parameter as unity.

This implies that

$$U = 2\Sigma X_j = 2\left(\frac{T_n}{\theta}\right)^{\beta} \sim \chi^2_{2n} \quad \ldots (3.1)$$

Again make the transformations

$$W_{n-1} = \log\left(\frac{Z_n}{Z_1}\right)$$

$$W_{n-2} = \log\left(\frac{Z_n}{Z_2}\right)$$

… … … … … …

… … … … … …

… … … … … …

$$W_1 = \log\left(\frac{Z_n}{Z_{n-1}}\right)$$

$$W_n = Z_n$$

which yields

$$Z_1 = W_n.e^{-W_{n-1}}$$

$$Z_2 = W_n.e^{-W_{n-2}}$$

… … … … … …

… … … … … …

… … … … … …

$$Z_{n-1} = W_n.e^{-W_1}$$

$$Z_n = W_n$$

so that, the jacobian of the transformation is

$$|J| = \begin{vmatrix} 0 & 0 & \cdots & W_n.e^{-W_{n-1}} & & & e^{-W_{n-1}} \\ 0 & 0 & W_n.e^{-W_{n-2}} & & 0 & & e^{-W_{n-2}} \\ \vdots & & & & & & \\ 0 & W_n.e^{-W_1} & 0 & 0 & \cdots & 0 & e^{-W_2} \\ W_n.e^{-W_1} & 0 & 0 & 0 & \cdots & 0 & e^{-W_1} \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \end{vmatrix}$$

$$|J| = e^{-\sum_{j=1}^{n-1} W_j W_n^{n-1}}$$

The joint density of $Z_1, Z_2, \ldots\ldots, Z_n$ is from (2.2)

$$f(z_1, z_2, \ldots\ldots, z_n) = e^{-z_n}, \qquad 0 < z_1 < Z_2 \ldots\ldots < z_n < \infty$$

Therefore the joint density of $w_1, w_2, \ldots\ldots, w_n$ is

$$f(w_1, w_2, \ldots\ldots, w_n) = e^{-w_n}.|J|$$

$$= \left[(n-1)!e^{\sum_{j=1}^{n} w_j}\right].\frac{1}{\Gamma n} w_n^{n-1}.e^{-w_n} \quad \ldots (3.2)$$

$$0 < w_1 < w_2 < \ldots\ldots < w_{n-1} < \infty \text{ and } 0 < w_n < \infty$$

$$= g(w_1, w_2, \ldots, w_n).h(w_n)$$

where $g(w_1, w_2, ..., w_n) = \left[(n-1)! e^{-\sum_{j=1}^{n-1} w_j}\right]$ … (3.3)

$0 < w_1 < w_2 < ..... < w_{n-1} < \infty$

is the density of (n–1) exponential order static from a sample of size (n–1) and

$$h(w_n) = \frac{1}{\Gamma n} w_n^{n-1}.e^{-w_n}, \qquad 0 < w_n < \infty \quad \text{… (3.4)}$$

is the gamma density with shape parameter n. thus,

$2W_n \sim \chi^2_{2n}$

$\Rightarrow \; 2Z_n \sim \chi^2_{2n}$

$$\Rightarrow \; U = 2\left(\frac{T_n}{\theta}\right)^{\beta} \sim \chi^2_{2n} \quad \text{… (3.5)}$$

which is the same as (3.1).

Also w_n is independent of the set of variables $(W_1, W_2, ...W_{n-1})$.

We know that the sum of exponential order static is a gamma variate.

$\therefore \; \sum_{i=1}^{n-1} W_i \sim \Gamma(n-1)$

$$\Rightarrow \; 2\sum_{i=1}^{n-1} W_i \sim \chi^2_{2(n-1)} \quad \text{… (3.6)}$$

Now consider

$$V = \frac{2n\beta}{\hat{\beta}} = 2\beta\Sigma \log\left(\frac{T_n}{t_1}\right)$$

$$= 2\beta\Sigma \log \frac{\theta.Z_n^{\frac{1}{\beta}}}{\theta.Z_i^{\frac{1}{\beta}}} = 2\beta.\frac{1}{\beta}\Sigma \log \frac{Z_n}{Z_i}$$

$$= 2\Sigma w_i \sim \chi^2_{2(n-1)} \quad \text{\{from (3.6)\}}$$

Also note that U is a function of W_n only and V is a function of $W_1, W_2, ..., W_{n-1}$ and is free from W_n and W_n is independent of $(w_1, w_2, ..., w_{n-1})$. Thus V and U are independent.

Now note that

$$\hat{\beta}\log\left(\frac{\hat{\theta}}{\theta}\right) = \hat{\beta}\log\left[\frac{\left(T_n / n^{1/\hat{\beta}}\right)}{\theta}\right]$$

$$= \hat{\beta}\log\left[\frac{T_n}{\theta}\right] - \frac{1}{\hat{\beta}}.\hat{\beta}\log n$$

$$= \hat{\beta}.\frac{1}{\beta}\log\left[2.\left(\frac{T_n}{\theta}\right)^n \Big/ 2\right] - \log n$$

$$= 2n.\frac{1}{\left(2n\beta / \hat{\beta}\right)}.\log\left[2.\left(\frac{T_n}{\theta}\right)^2 \Big/ 2\right] - \log n$$

$$= 2n.\frac{\log(U/2)}{V} - \log n \quad \text{… (3.7)}$$

where $U \sim \chi^2_{2n}$ and $V \sim \chi^2_{2(n-1)}$

$\therefore$ U and V are independent. It is also useful to consider the distribution of $\hat{\beta}\log\left(\frac{\hat{\theta}}{\theta}\right)$. Bain and Englehardt (1991) show that the asymptotic distribution of

$$Z = \frac{\Gamma n\hat{\beta}\log\left(\frac{\hat{\theta}}{\theta}\right)}{\log n} \quad \text{… (3.8)}$$

is standard normal. They have also tabulated the percentage q_k such that

$$P\left[\frac{\Gamma n\hat{\beta}\log\left(\frac{\hat{\theta}}{\theta}\right)}{\log n} \leq q_{\alpha}\right] = \alpha \quad \text{… (3.9)}$$

for various values of α and n.

Since $w_j = \log\frac{T_n}{T_j}$ is also distributed independently of θ, the distribution of the ratios $\frac{T_n}{T_j}$ is also independent of θ. For fixed β, T_n is a complete sufficient statistic for θ. Thus T_n and such ratios are stochastically independent. In particular T_n and $\hat{\beta}$ are independent.

4. SOME MORE INFERENCES ON β AND θ

UNBIASEDNESS OF $\hat{\beta}$:

$$E\left(\hat{\beta}\right)=2n\beta E\left(\frac{1}{\chi^2_{2(n-1)}}\right)$$

$$=\frac{2n\beta}{2(n-2)}=\frac{n}{n-1}.\beta$$

Thus the bias of $\hat{\beta}$ is

$$B\left(\hat{\beta}\right)=E\left(\hat{\beta}\right)-\beta$$

$$=\frac{n}{n-2}\beta-\beta=\frac{2}{n-2}\beta>0$$

$\therefore$ $\hat{\beta}$ is slightly positively biased.

Also note that $\frac{n-2}{n}\hat{\beta}$ is unbiased for β.

4.1 TESTING AND CONFIDENCE INTERVAL OF β

To test $H_0:\beta\le\beta_0$ against $H_1:\beta>\beta_0$ the test statistic is

$$\frac{2n\beta_0}{\hat{\beta}}=2\beta_0\sum_{j=1}^{n-1}\log\left(\frac{T_n}{T_j}\right)$$

which follows the chi–square distribution with $2(n-1)$ degrees of freedom. Thus to test the above hypothesis the size critical region is given by

$$P\left[2\beta_0\sum_{j=1}^{n-1}\log\left(\frac{T_n}{T_j}\right)\le\chi_{\alpha^{2(n-1)}}\right]=\alpha \qquad \dots (4.1.1)$$

where $\chi_{\alpha^{2(n-1)}}$ is the lower α. 100% point of χ^2 distribution with $2(n–1)$ degrees of freedom.

Using the statistic a lower (1–α).100% confidence interval for β is

$$\left[0,\frac{\hat{\beta}}{2n},\chi^2_{\alpha^{2(n-1)}}\right].$$

4.2 TESTING AND CONFIDENCE INTERVAL FOR θ

To test $H_0:\theta\le\theta_0$ against $H_1:\theta>\theta_0$, the test statistic is

$$\sqrt{n\hat{\beta}}\frac{\log\left(\hat{\theta}/\theta_0\right)}{\log n}\ge q_{1-\alpha}$$

Using this statistic a lower (1–α) 100% confidence interval for θ is

$$\left[0,\hat{\theta}\exp\left\{-q_{1-\alpha}\log n\Big/\sqrt{n\hat{\beta}}\right\}\right]$$

Hence $q_{1-\alpha}$ may be obtained from table 1 in Bain and Engelhardt (1991; p. 419).

5. COMPOUND WEIBULL PROCESSES

An alternative to the standard Poisson distribution for count data is a compound or mixed Poisson distribution. Such a compound distribution, which has a negative binomial form occurs when the population consists of components with Poisson distribution failure times but with intensities that vary from component to component according to a gamma distribution. For example, the intensities can differ from one component to the next in a population of repairable components by the reason of fluctuations in manufacturing or by some other reasons.

Engelhardt and Bain (1987) derived a compound Weibull process with intensity parameter λ and shape parameter β

$$\nu(I)=\lambda\beta t^{\beta-1} \quad : \quad I>0 \qquad \dots (5.1)$$

A Weibull process with fixed values at the parameters in an appropriate model when data are obtained from, a single system and inferences are made only for that system. A mixed model is more suitable when the intensity parameter varies from system to system. Consider the assumptions–

the failures of each system are distributed according to a Poisson process because the population is hetrogeneous, but with intensity functions it differs from system to system.

the counting process for the number of failures in time t for each individual system has density at the form θ

$$f(n \mid \lambda) = \frac{(\lambda t^{\beta})^n \exp(-\lambda t^{\beta})}{n!}. \quad n = 0,1,2,... \quad \ldots (5.2)$$

In the population each system has the same parameter β, but the intensity parameter λ varies according to a gamma density

$$f(n) = \int_0^{\infty} f(n,\lambda)\, d\lambda$$

$$= \int_0^{\infty} f(n \mid \lambda).g(\lambda)\, d\lambda$$

$$= \int_0^{\infty} \frac{(\lambda t^{\beta})^n \exp(-\lambda t)}{n!} . \frac{\lambda^{K-1} \exp\left(-\frac{\lambda}{\gamma}\right)}{\Gamma K \gamma^K} d\lambda \quad \lambda > 0$$

$$= \binom{n+K-1}{n} \frac{(\gamma t^{\beta})^n}{(1+\gamma t^{\beta})^{n+K}}, \quad n = 0,1... \quad \ldots (5.3)$$

If $\beta = 1$, then (5.3) is same as the equation $= \binom{x+\kappa-1}{x} \frac{(l\gamma)^x}{(1+l\gamma)^{x+\kappa}}, \quad x = 0,1,2,...,\ 0 < \kappa < \infty,\ 0 < \gamma < \infty$, with λ as ν and t as I.

In this case the mean and variance are given by

$$E[N(t)] = k\gamma t^{\beta}$$

$$\text{var}[N(t)] = K\gamma t^{\beta}(1+\gamma t^{\beta})$$

The distribution given (5.3) is a special form of Negative binomial distribution with parameters K and $P = \frac{1}{(1+\gamma t^{\beta})}$.

6. CONCLUSION

In the comparison of Poisson distribution, the Weibull distribution is greater flexible because it is a two parameter model and thus may be a useful model for consideration whenever the Poisson distribution is inadequate. However, it is clear that the compound Weibull development is an alternative to the standard Poisson distribution for count data is a compound or mixed Poisson distribution. In some compound Poisson situations a distribution other than the gamma may be more appropriate for the density of failure rates.

REFERENCES

Bain, L. J. and Engelhardt, M. (1991). *Statistical analysis of reliability and life testing Models*. Marcel Dekker, New York.

Cui, L., Li, H. , Xu, S. H. (2014). Stochastic methods in reliability and risk management, *Annals Operations Res.*, **212(1)**, 1-2.

Gertsbakh, I. (2013). *Reliability Theory: With Applications to Preventive Maintenance* springer-verlag berlin heidelverg, New York

Mann, N. (1968). Results on statistical estimation and hypothesis testing with applications to the Weibull and extreme value distribution. *Aerospace Research Laboratories*, Wright-Patterson Air Force Base, Ohio.

Santhosh T.V., Gopika , V., Ghosh, A.K. and Fernandes, B.G. (2018). .An approach for reliability prediction of instrumentation & control cables by artificial neural networks and Weibull theory for probabilistic safety assessment of NPPs, *Reliability Engineering & System Safety,* **170**, 31-44.

Wanga, Q., Yuana, X., Zuob, J., Zhangc, J., Hongc, J., Lin, Chen. (2016). Optimization of Ecological Industrial Chain design based on reliability theory – a case study, *J. Cleaner Prod.*, **124**, 175-182

Weibull, W. (1939). *A Statistical theory of the strength of materials*. Ing. Vetenskps. Akad. HandI.

Weibull, W. (1951). A Statistical distribution function of wide applicability. *J. Appl. Mechh.* **18**, 293–297.

Zhai, S., Lin, Z.S. (2015). Bayesian Networks Application in Multi-State System Reliability, *Analysis Applied Mechanics and Materials*, **347-350**, 2590-2595.