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Wavelet based Hybrid Approach for Forecasting Volatile Potato Price

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SUMMARY

Potato (*Solanumtuberosum*) is a solanaceous root vegetable which also serves as staple food in many countries. In India potato assumes the status of top most important vegetable, grown as a short duration rabi crop. But due to relatively high cost of cultivation, potato growing farmers often face problems regarding post-harvest handling and marketing of the produce. The arrival pattern also varies through the year as a consequence of seasonality of production and perishability feature of vegetable crop. In this context Wavelet based modeling and forecasting technique to deal with volatile potato price is elaborated as an alternative to the traditional forecasting models, such as, Autoregressive integrated moving average (ARIMA) and Generalized autoregressive conditional heteroscedastic (GARCH) model. Maximal overlap discrete wavelet transform (MODWT) is advocated to represent the series at multi-resolution level and combined with ARIMA or GARCH class of models in order to increase the forecast accuracy. To this end, formulae for out-of-sample prediction has been worked out for Wavelet-GARCH hybrid model. Monthly potato price data of three markets, namely Haldwani, Agra and Lucknow of Uttar Pradesh, India have been considered for the present investigation. The combinatory of Wavelet-GARCH hybrid model has been found to outperform the individual ARIMA and GARCH model. The R software package has been used for data analysis.

Keywords: ARIMA, GARCH, MODWT, Volatility, Wavelet.

1. INTRODUCTION

Time series analysis deals with observations that are frequently made sequentially over time. There are two dominant approaches in analyzing time series data; first approach is the time-domain approach which is most common and the second one being frequency domain or spectral analysis approach. In time domain approach time series observations measured in a sequence are exploited where autocorrelation function plays a crucial role. Box-Jenkins approach of most stated Autoregressive integrated moving average (ARIMA) structure is implemented in forecasting linear stochastic phenomenon. Whereas Autoregressive conditional heteroscedastic (ARCH) class of models as developed by Engle (1982) and Generalized ARCH (GARCH) model by Bollerslev (1986) are used in forecasting non-linear dynamics, capturing volatility structure, leptokurtosis, asymmetric pattern of financial time series. ARCH model allows the conditional variance to change over time as a function of squared past errors, leaving the unconditional variance constant. But the feature of ARCH to give satisfactory forecast only with large number of parameters has necessitated the emergence of more parsimonious version, which is GARCH model where conditional variance is also a linear function of its own lags. In literature, GARCH family of models have been used widely for forecasting volatility in many fields including agriculture (Paul *et al*. 2009 and Ghosh *et al*. 2010a, b). Paul *et al*. (2016) have explored the effectiveness of price forecasting techniques for describing asymmetric volatility for onion in different markets. But every model has its own limitations. There are many instances of post sample forecast employing hybrid model. Wang *et al*. (2005) proposed an ARMA-GARCH error model to capture the ARCH effect present in daily stream flow series. In 2013, Liu and Shi advocated ARMA-GARCH approaches to forecast short-term electricity prices. Liu *et al*. (2013) applied ARMA-GARCH in-mean for wind speed forecasting. Paul *et al*. (2014) have applied

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hybrid ARIMAX-GARCH model for forecasting volatile wheat yield data. But standard parametric or their combinatory hybrid structure possess obvious limitation of being data and distribution dependent.

In case of frequency domain approach original time series is decomposed into certain number of patterns which renders visual impression of time series properties. Recently, an extremely powerful methodology of "Wavelet analysis" is rapidly emerging (Antoniadis 1997, Vidakovic 1999, Percival and Walden 2000). Although, a number of research papers have been published dealing with various theoretical aspects of non-parametric wavelet technique, but their application to data is still a difficult task. Basic idea behind the wavelet approach is the decomposition of original series and capturing multi-scale information at different frequency level. Wavelet detail part can describe the local variation at multi-resolution level and approximate part depicts the overall variations or trend in the data. The wavelet decomposed series can further be utilized in combination with other parametric or non-parametric forecasting techniques to provide out-of-sample forecast more accurately. Paul *et al*. (2013) made an attempt for modeling and forecasting of Indian monsoon rainfall time-series through using MODWT which had improvement over ARIMA model. Paul (2015) applied ARIMAX-GARCH-Wavelet Model for forecasting volatile wheat yield data in Kanpur district of Uttar Pradesh, India. In the present investigation, an attempt has been made to forecast the price of potato in different markets of India based on hybrid models combining Wavelet and ARIMA/GARCH model.

2. WAVELET

Wavelets are analogous to the trigonometric sine and cosine functions. As with a sine or cosine wave, a wavelet function oscillates about zero. This oscillating property makes the function a wave. If $\psi(.)$ is a real-valued function defined over the real axis and satisfies two basic properties:

a) Integral of $\psi(.)$ is zero. $\int_{-\infty}^{\infty} \psi(u)\,du = 0$

b) Square of $\psi(.)$ integrates to unity. $\int_{-\infty}^{\infty} \psi^2(u)\,du = 1$

Then, the function $\psi(.)$ is called a wave.

2.1 Wavelet Transform

The Maximal overlap discrete wavelet transform (MODWT) is a linear filtering operation that transforms a series into coefficients related to variations over a set of scales. It is similar to DWT, in that, both are linear filtering operations producing a set of time-dependent wavelet and scaling coefficients. Both have basis vectors associated with a location *t* and a unit less scale $\tau_j = 2^j$ -1 for each decomposition level $j = 1,..,J_0$ and are suitable for performing ANOVA. MODWT is well defined for all sample sizes *N*, whereas for a complete decomposition of *J* levels, DWT requires *N* to be a multiple of 2^J. MODWT also differs from DWT in the sense that it is a highly redundant, non orthogonal transform. DWT pyramid algorithm is applied to time series ***X***, whereas MODWT coefficients are obtained by applying DWT pyramid algorithm once to ***X*** and another to the circularly shifted vector *T**X***. Hence, the first application yields the usual DWT (***W***) of the time series vector **X** computed as $\boldsymbol{W} = P\boldsymbol{X}$ and the second application consists of substituting *T***X** for **X** obtained as

$\boldsymbol{W} = PT\boldsymbol{X}$. Where, ***W*** and *P* can be written as $\boldsymbol{W} = [\boldsymbol{W}_1 \boldsymbol{W}_2 ... \boldsymbol{W}_J \boldsymbol{V}_J]'$, and $P = [P_1 P_2 ... P_J Q_J]$

For a time series **X** with arbitrary sample size *N*, the j^{th} level MODWT wavelet ($\boldsymbol{W}_j$) and scaling ($\boldsymbol{V}_j$) coefficients are defined as

$$\tilde{W}_{j,t} \equiv \sum_{l=0}^{L_j-1} \tilde{h}_{j,l} X_{t-l \bmod N} \quad \text{and} \quad \tilde{V}_{j,t} \equiv \sum_{l=0}^{L_j-1} \tilde{g}_{j,l} X_{t-l \bmod N}$$

where $h_{j,l}$ is the j^{th} level MODWT wavelet filter and $g_{j,l}$ is the j^{th} level MODWT scaling filter. For a time series ***X*** with *N* samples, MODWT yields an additive decomposition or MRA given as

$$\mathbf{X} = \sum_{j=1}^{j_0} \tilde{\mathbf{D}}_j + \tilde{\mathbf{S}}_{j_0}$$

$$\tilde{D}_{j,t} = \sum_{l=0}^{N-1} \tilde{u}_{j,l} \ \tilde{W}_{j,t+l \bmod N} \quad \text{and}$$

$$\tilde{S}_{j,t} = \sum_{l=0}^{N-1} \tilde{v}_{j,l} \ \tilde{V}_{j,t+l \bmod N}$$

where $\tilde{u}_{j,l}$ and $\tilde{v}_{j,l}$ being the filters obtained by periodizing $\tilde{h}_{j,l}$ and $\tilde{g}_{j,l}$. At a scale *j*, a set of coefficients $\{\tilde{\mathbf{D}}_j\}$ are called wavelet "details" and capture local fluctuations over whole period of a time series at each scale. Set of values $\tilde{\mathbf{S}}_{J_0}$ provide a "smooth" or overall "trend" of the original signal and adding $\tilde{\mathbf{D}}_j$ to $\tilde{\mathbf{S}}_{J_0}$, for j = 1, 2, …, J_0, gives an increasingly more accurate approximation for it. A time series can be completely or partially decomposed into a number of levels $J_0 \leq log_2$ (N).

3. WAVELET-GARCH HYBRID MODEL

For WAVELET-GARCH hybrid model prediction, first we test for the ARCH effects in each sub series and employ ARIMA or component model only for those series which does not show significant conditional heteroscedasticity. For the remaining series ARIMA-GARCH or AR-GARCH model is used as we have to forecast for mean model. The schematic representation of the methodology is depicted in Fig. 1. For the time series vector $\boldsymbol{X}$ ($X = \tilde{V}_j + \sum_{j=1}^{j} \tilde{W}_j$) in Haar wavelet transform, prediction equation of X_{N+1}, when $X_1, X_2, \ldots, X_N$ observations are given is

$$\hat{X}_{N+1} = \hat{\hat{V}}_{J,N+1} + \sum_{j=1}^{J} \hat{\hat{W}}_{j,N+1} \tag{1}$$

Considering AR (A_j)-GARCH(1,1) process at all the resolution levels, $\hat{\hat{W}}_{j,N+1}$ and $\hat{\hat{V}}_{J,N+1}$ are derived respectively as follows,

$$\hat{\hat{W}}_{j,N+1} = \sum_{k=1}^{A_j} a_{j,k} \hat{\hat{W}}_{j,N-2^J(K-1)} + \varepsilon_{j,N+1} \text{ and} \tag{2}$$

$$\hat{\hat{V}}_{J,N+1} = \sum_{k=1}^{A_{J+1}} a_{J+1,k} \hat{\hat{V}}_{j,N-2^J(K-1)} + \varepsilon_{J+1,N+1} \tag{3}$$

where,

$$\varepsilon_{j,N+1} = \eta_{j,N+1,} \sqrt{h_{j,N+1}} \tag{4}$$

and conditional variance formula for detail part is obtained as

$$h_{j,N+1} = \alpha_j \varepsilon^2{}_{j,N} + \beta_j h_{j,N} \tag{5}$$

$\alpha_j > 0$, $\beta_j > 0$ for all j = 1, 2, …, J is the sufficient condition or non-negativity and finite conditional variance. For GARCH (1,1) process to be weakly stationary sufficient condition is $\alpha_j + \beta_j < 1$.

For smooth part, at J^{th} resolution level, we can write

$$\varepsilon_{J+1,N+1} = \eta_{J+1,N+1,} \sqrt{h_{J+1,N+1}} \tag{6}$$

and conditional variance formula can be obtained by

$$h_{J+1,N+1} = \alpha_{J+1} \varepsilon^2{}_{J+1,N} + \beta_{J+1} h_{J+1,N} \tag{7}$$

In general for AR(A_j)-GARCH(p,q) process at all the resolution levels $\hat{\hat{W}}_{j,N+1}$, $\hat{\hat{V}}_{J,N+1}$ and $_{j,N\ 1}$ are respectively computed by equation (2), (3) and (4).

Proceeding in the similar fashion, the conditional variance formula for detail part is obtained as

$$h_{j,N+1} = \sum_{l=1}^{p} \alpha_{j,l} \varepsilon^2{}_{j,N} + \sum_{s=1}^{q} \beta_{j,s} h_{j,N} \beta_{j,s} h_{j,N} \tag{8}$$

$\alpha_{j,l} > 0$, $\beta_{j,s} > 0$ for all j=1,2,…,J and l=1,2,…,p and s=1,2,…,q is the sufficient condition or non-negativity and finite conditional variance. For GARCH (p,q) process to be weakly stationary sufficient condition is $\sum_{l=1}^{p} \alpha_{j,l} + \sum_{s=1}^{q} \beta_{j,s} < 1$.

For smooth part unconditional variance will be same as equation (6) and conditional variance formula is,

$$h_{J+1,N+1} = \sum_{l=1}^{p} \alpha_{J+1,l} \varepsilon^2{}_{J+1,N} + \sum_{s=1}^{q} \beta_{J+1,s} h_{J+1,N} \tag{9}$$

Finally, inverse wavelet transform yields the reconstruction of original series as presented in Fig 1. MODWT increases the prediction accuracy of GARCH model and can effectively describes the heteroscedasticity, volatility clustering, asymmetric property and nonlinearity property of volatility series

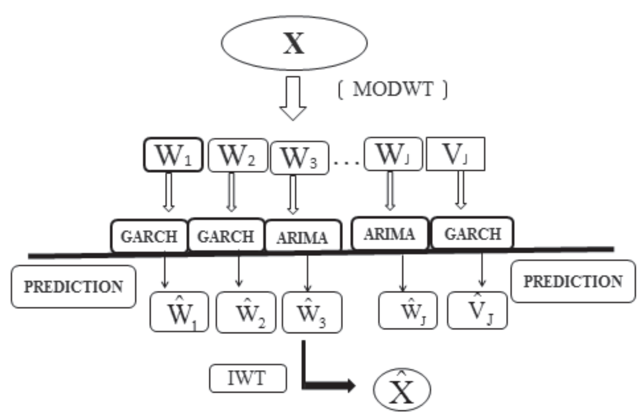

Fig 1: Structural schema of MODWT-GARCH hybrid

4. EMPIRICAL ILLUSTRATION

4.1 Data Set

For the present investigation potato price data series belong to Haldwani, Agra and Lucknow market for the period January, 2005 to December, 2015, collected from National Horticulture Research and Development Foundation (NHRDF) (the website: http://nhrdf.org/en-us/) are used. Last 6 observations i.e. from July, 2015 to December, 2015 constitute the validation set in each case.

4.2 Descriptive Statistics and Seasonal Indices

Table 1 briefs the descriptive statistics of the selected markets. Average potato prices are around 5Rs/Kg, 6Rs/Kg and 7 Rs/Kg respectively for the three markets. Quite high value of coefficient of variance (CV) entails a good degree of instability or volatility in the original data set. Highest CV is found in Haldwani market followed by Agra market and then Lucknow.

Table 1. Descriptive statistics of potato prices in different markets

Statistics	Haldwani	Agra	Lucknow
Observations	132.00	132.00	132.00
Mean (Rs/Q)	539.54	636.90	703.80
Minimum	156.00	175.00	189.05
Maximum	1783.00	2107.06	2059.04
Standard Deviation	333.33	364.72	366.27
CV	61.78	57.28	52.04
Kurtosis	2.97	3.50	2.22
Skewness	1.60	1.71	1.47

Table 2. Seasonal factors for potato prices in the different markets

Months	Haldwani	Agra	Lucknow
January	0.645	0.703	0.556
February	0.623	0.680	0.668
March	0.704	0.800	0.745
April	0.744	0.884	0.844
May	0.935	0.998	1.005
June	1.209	1.042	0.997
July	1.365	1.106	1.117
August	1.379	1.122	1.181
September	1.213	1.209	1.198
October	1.251	1.388	1.395
November	1.107	1.266	1.297
December	0.822	0.806	1.004

Skewness coefficient is indicating asymmetry in the data. Original data is seasonally adjusted to eliminate the influence of seasonality in price. Table 2 shows the seasonal index values. Relatively higher values of seasonal indices are found from June to November. Potato being a rabi crop, planting time is 15 September - 15 October and fresh arrival starts to reach the market by the end of November onwards.

4.3 Testing Stationarity and ARCH Effects

Augmented-dickey-Fuller (ADF) tests have been performed to see the presence of non-seasonal unit root in the seasonally adjusted series. It was seen that the seasonally adjusted series were non-stationary for all the three markets. Stationarity test in first order differenced series revealed the presence of further no more unit root.

ARCH effects have been tested to the squared residuals of best fitted ARIMA model and reported in Table 3. Significant value of Q test statistic determines the presence of autocorrelation in the squared residual series and significant value of Lagrange multiplier (LM) test indicates the existence of conditional heteroscedasticity. A perusal of Table 3 reveals the presence of conditional heteroscedasticity in the seasonally adjusted series of the selected markets. Therefore, GARCH model can be fitted to all the data.

4.4 Employing the Forecasting Models

ARIMA, GARCH and hybrid Wavelet-GARCH models are considered for modeling purpose. The parameter estimates of best fitted ARIMA and GARCH model are furnished in Table 4, Table 5 and Table 6 along with their significance level. Though some of the parameters are non-significant here indicating possible improvement in their hybrid counterpart which is described in the next section.

Wavelet-GARCH model is fitted to seasonally adjusted series for the markets under study. At first, decomposition at multi-resolution level is done using MODWT implementing Haar filter. Maximum level J_0 is taken as 4 here for all the cases. Here, W_1 to W_4 denote the wavelet details components, and V_4 denotes the smoothed component of MODWT. Table 7 shows that for Haldwani market, best suited GARCH model were fitted to W_1 and W_3; for Agra market GARCH model were implemented at detail level W_3 and W_4;

Table 3. Tests for ARCH disturbances based on OLS residuals

Order	Haldwani Market				Agra Market				Lucknow Market			
	Q	Pr>Q	LM	Pr>LM	Q	Pr>Q	LM	Pr>LM	Q	Pr>Q	LM	Pr>LM
1	51.1	<.0001	49.8	<.0001	22.5	<.0001	29.71	<.0001	93.70	<.0001	91.80	<.0001
2	67.0	<.0001	51.9	<.0001	23.0	<.0001	29.90	<.0001	151.15	<.0001	92.88	<.0001
3	73.9	<.0001	52.0	<.0001	23.3	<.0001	30.10	<.0001	188.90	<.0001	93.10	<.0001
4	77.7	<.0001	52.0	<.0001	23.8	<.0001	30.40	<.0001	217.60	<.0001	93.45	<.0001
5	81.0	<.0001	53.0	<.0001	25.3	0.0001	31.70	<.0001	236. 40	<.0001	94.00	<.0001
6	84.1	<.0001	53.4	<.0001	26.9	0.0001	32.00	<.0001	245.90	<.0001	94.30	<.0001
7	86.2	<.0001	53.6	<.0001	27.3	0.0003	32.10	<.0001	250.10	<.0001	94.30	<.0001
8	86.6	<.0001	54.9	<.0001	27.7	0.0005	33.70	<.0001	252.70	<.0001	94.50	<.0001
9	86.7	<.0001	55.0	<.0001	28.2	0.0009	33. 70	0.0001	255.57	<.0001	95.40	<.0001
10	87.3	<.0001	56.0	<.0001	29.9	0.0009	39.00	<.0001	258.50	<.0001	95.60	<.0001
11	87.6	<.0001	56.1	<.0001	33.6	0.0004	41.10	<.0001	260.30	<.0001	95.80	<.0001
12	87.6	<.0001	56.7	<.0001	39.4	<.0001	45.00	<.0001	261.00	<.0001	95.80	<.0001

Table 4. Parameter estimates of the ARIMA (1,1,0) and ARIMA (1,1,0)-GARCH (1,0) model for Haldwani Market

Model	Parameters	Estimate	Probability
ARIMA(1,1,0)	C	2.276	0.7863
	AR 1	-0.231	0.0100
ARIMA(1,1,0)-GARCH(1,0)	Mean equation		
	C	2.1260	0.8769
	AR 1	0.1378	0.2163
	Variance equation		
	C	13270	<.0001
	ARCH 1	0.298	0.0871

Table 5. Parameter estimates of the ARIMA (1,1,0) and ARIMA (1,1,0)-GARCH (1,0) model for Agra Market

Model	Parameters	Estimate	Probability
ARIMA(1,1,0)	C	1.412	0.9075
	AR 1	0.155	0.0851
ARIMA(1,1,1)-GARCH(1,0)	Mean equation		
	C	4.804	0.7875
	AR 1	-0.058	0.6573
	Variance equation		
	C	13160	<.0001
	ARCH 1	0.352	0.1141

for Lucknow market W_2, W_3 and smooth level V_4were considered for fitting GARCH. The specific model fitted to individual wavelet components are listed in Table 8.Original series is reconstructed through inverse wavelet transform from detail and smooth level.

Table 6. Parameter estimates of the ARIMA (1,1,0) and ARIMA (1,1,0)-GARCH (1,0) model for Lucknow Market

Model	Parameters	Estimate	Probability
ARIMA(1,1,0)	C	2.548	0.8455
	AR 1	0.170	0.0891
ARIMA(1,1,0)-GARCH(1,0)	Mean equation		
	C	2.728	0.8704
	AR 1	-0.104	0.3863
	Variance equation		
	C	13976	<.0001
	ARCH 1	0.349	0.1150

Table 7. Perusal of Wavelet- GARCH model

Market	W_1	W_2	W_3	W_4	V_4
Haldwani	GARCH	ARIMA	GARCH	ARIMA	ARIMA
Agra	ARIMA	ARIMA	GARCH	GARCH	ARIMA
Lucknow	ARIMA	GARCH	GARCH	ARIMA	GARCH

4.5 Evaluation of Forecasting Performances

The forecasting performance of all the three models has been computed for an out of-sample cross-validation period of 6 observations (i.e., 6months). Predictive abilities of different models have been compared using Relative mean absolute prediction error (RMAPE)and Root mean square prediction error (RMSPE).The corresponding results of forecast comparison have been reported in Table 8.

Table 8. Predictive Abilities for 3 different models

Market	Validation Criterion	ARIMA	ARIMA-GARCH	WAVELET-GARCH
Haldwani	RMAPE (%)	14.36	5.71	4.96
	RMSPE	102.02	43.10	43.02
Agra	RMAPE (%)	19.68	13.63	8.08
	RMSPE	179.72	110.30	66.61
Lucknow	RMAPE (%)	8.51	5.44	5.00
	RMSPE	71.74	47.97	43.13

Comparing the validation results of individuals as well as hybrid forecasting models, it is observed that Wavelet-GARCH produces the best result over the other methods in terms of RMAPE (%) and RMSPE. Modeling with Wavelet-GARCH hybrid yielded RMAPE (%) value as 4.96, 8.08 and 5.00 respectively which are lower than traditional parametric models. Hence, individual ARIMA or GARCH models cannot be considered at all for forecasting purpose. Prediction results clearly reflect that wavelet approach is outperforming the other models. General GARCH models can only describe the overall volatility features of the series, but cannot describe partial volatility features of the series and multi-scale information. Further, if we ignore partial volatility features and multi-scale information of the time-series that will hamper the precision of the model. To take advantage of partial volatility features and multi-scale information of variables when forecasting, MODWT-GARCH model is implemented by combining wavelet analysis theory with the GARCH. Marked accuracy in out of sample forecasting results also we can expect, if we rely on Wavelet-GARCH combinatory. Table 9 presents the out-of-sample forecast for the potato price series in the three selected markets for the year 2016. The actual vs. predicted plot for the three markets are reported in Figs. 2-4. A visual inspection of these figures indicates that the model performance is quite satisfactory in order to capturing the volatility in the price series. To this end, the residuals coming out from fitted hybrid models have been investigated in order to check for any autocorrelation and non-normality in it. It is found that the residuals are independent and normally distributed confirming the adequacy of the fitted models.

Table 9. Out of sample forecast for potato prices series (Rs/Q)

Months	Haldwani Market	Agra Market	Lucknow Market
January, 2016	445	514	548
February, 2016	479	426	586
March, 2016	558	586	616
April, 2016	581	764	760
May, 2016	682	870	949
June, 2016	835	833	981
July, 2016	916	848	1028
August, 2016	903	849	959
September 2016	768	895	923
October, 2016	760	990	902
November, 2016	644	868	1074
December, 2016	458	537	684

5. CONCLUSION

Price volatility is a fundamental feature of agricultural markets and probably one of the main sources of risk in international agricultural trade. In this light, implementing wavelet approach for modeling and forecasting volatile price series has a great and varied importance. The utility of nonparametric Wavelet methodology in frequency domain for modeling and forecasting purposes employing Haar wavelet is highlighted. Superiority of this approach over traditional ARIMA model and GARCH model is demonstrated for potato price series. The underlying assumptions of linearity and homoscedastic error variance in ARIMA methodology make it quite impossible to deal with series exhibiting high volatility or periods of instability such as agricultural commodity price series. Whereas, potato price exhibits high instability or volatility in all the selected markets. Therefore GARCH model is useful to forecast in the non-linear dynamics and also to take care of conditional heteroscedasticity. But both the models assume to have stationarity and parametric structure of the time series which is not sufficiently extensive for the monthly potato price series of the selected markets under study. In this aspect Wavelet-GARCH combinatory semi parametric modeling strategy out-yielded both the traditional, parametric models. Wavelet approach renders a check in loss of information through capturing both the partial and global volatility features and provides a better explanation of the spectral pattern.

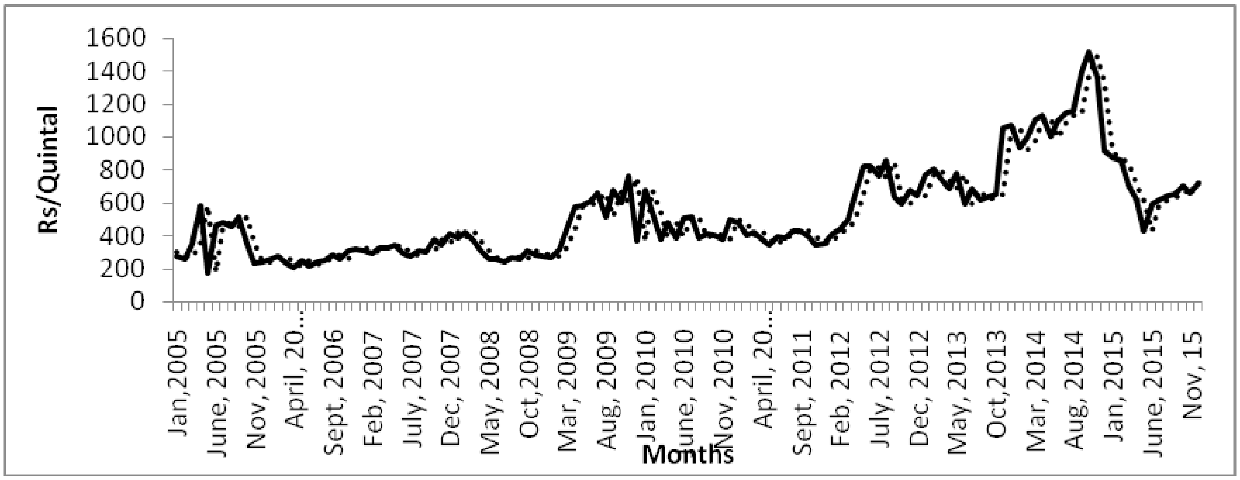

Fig 2: Actual price series (solid line) and seasonally adjusted series (dashed line) for Haldwani market

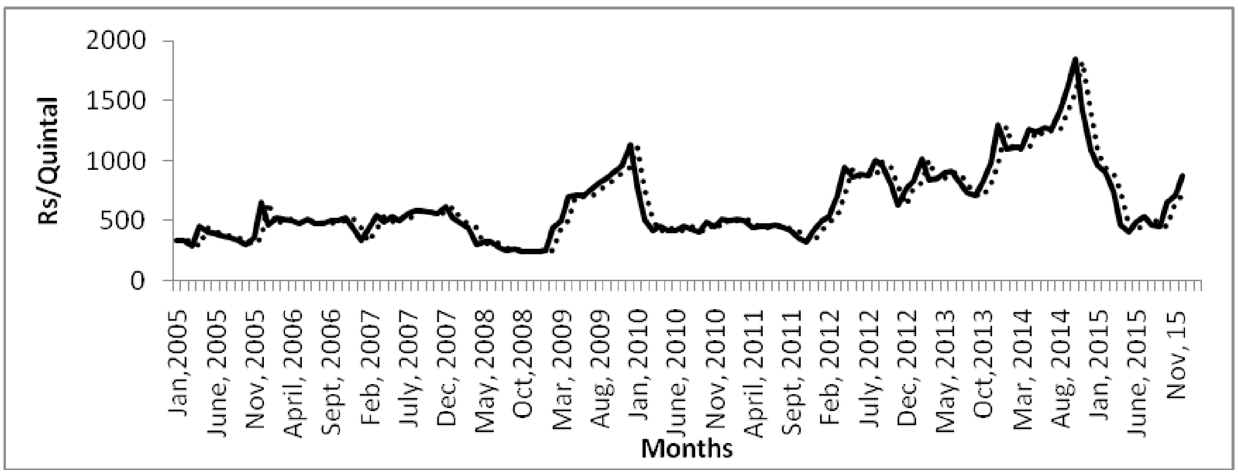

Fig. 3: Actual price series (solid line) and seasonally adjusted series (dashed line) for Agra market

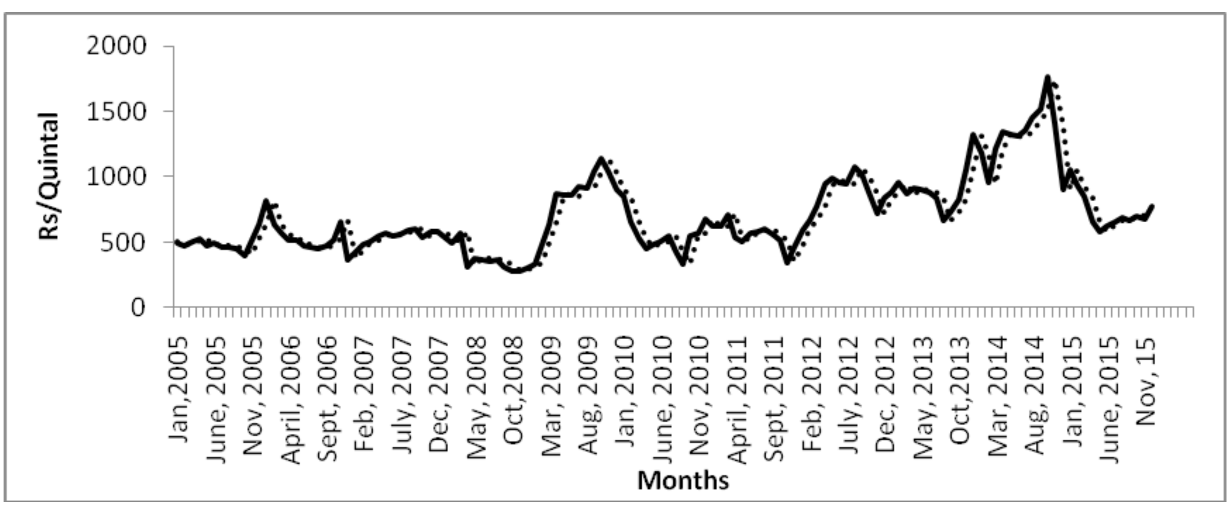

Fig. 4: Actual price series (solid line) and seasonally adjusted series (dashed line) for Lucknow market

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