

Calibration Approach based Estimation of Finite Population Total under Two Stage Sampling

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SUMMARY

Auxiliary information is often used to improve the precision of estimators of finite population total. Calibration approach is widely used for making efficient use of auxiliary information in survey estimation. We proposed the regression type estimators of the population total using the calibration approach under the assumption that the population level auxiliary information is available at secondary stage unit level under two stage sampling design. The variance and the estimator of the variance of the proposed estimators were developed. We carried out limited simulation studies to demonstrate the empirical performance of proposed estimators. Our empirical results show that the proposed estimators outperforms the usual regression estimator under two stage sampling design in terms of the criteria of relative bias and relative root mean square error.

Keywords: Auxiliary information, Calibration approach, Regression type estimator, Secondary stage unit, Two stage sampling.

1. INTRODUCTION

Auxiliary information is often used to improve the precision of estimators of finite population total. Calibration approach is widely used for making efficient use of auxiliary information in survey estimation. Estimation in sample surveys is conducted mainly by attaching weights to sample data and then computing weighted averages. When available, auxiliary information may be employed to improve survey estimates. In this context, a set of sample weights which is said to have the calibration property are considered, if it reproduces exactly known population quantities when applied to the sample values of the corresponding auxiliary variables. It is based on the argument that "weights that perform well for the auxiliary variables also should perform well for the study variable" (Deville and Sarndal 1992). Auxiliary information is often used by survey statisticians to increase the precision of estimators of commonly used parameters. Some examples of estimators of population mean or population

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total, which use auxiliary information, are ratio and regression estimators. In the past twenty years or so, calibration itself became an important topic in survey research and a large amount of literature has been devoted to it, so much so that it gained significant attention not only in the field of survey methodology, but also of survey practice, calibrated weights, mainly derived using the techniques in Deville and Sarndal (1992), are currently employed by several national statistical agencies to produce official estimates from large scale surveys. Following Deville and Sarndal (1992) a lot of work has been carried out in the context of calibration estimation i.e. Singh et al. (1998, 1999), Folsom and Singh (2000), Farrell and Singh (2002), Wu and Sitter (2001), Sitter and Wu (2002), Kott (2006), Estevao and Sarndal (2002, 2006), Plikusas and Pumputis (2010), Sud et al. (2014), Aditya et al. (2016) but most of the studies in this context is only restricted to single stage or two phase sampling designs whereas in large scale surveys two stage or multistage sampling designs are generally used. In this study, we have developed calibration estimators under two stage sampling design by modifying the sampling design weight with the help of auxiliary information by minimising the chi-square type distance function between the proposed weight and the sampling design weight with respect to certain calibration constraint equations in the presence of complex auxiliary information. We consider case of availability of complete auxiliary information at the secondary stage unit (ssu) level under two stage sampling design.

In what follows, regression type estimators have been proposed using the calibration approach under two stage sampling design in the presence of complex auxiliary information and the regression line does not pass through the origin. In fact, the generalized regression (GREG) estimator is a special case of the calibration estimator when the chosen distance function is the Chi-square distance (Deville and Sarndal 1992). The main difference between the GREG approach and the calibration approach is that in the GREG approach the predicted values are generated using an assisting model whereas in calibration approach it does not depend on any assumption about the assisting model. Assisting model, an imagined relationship between study variable and auxiliary variable which can have many forms: linear, nonlinear, generalized linear, mixed (model with some fixed, some random effects), and so on. Expressions for variance and variance estimator of the proposed calibration approach based estimators have been developed. The improved performance of the proposed estimator over the usual regression estimator under two stage sampling design was demonstrated through a simulation study and concluding remarks were made.

2. CALIBRATION BASED ESTIMATION UNDER TWO STAGE SAMPLING DESIGN

We consider a simple case where information on only one auxiliary variable is available. Let, the population of elements $U = \{1, ..., k, ..., N\}$ is partitioned into N_1 clusters, $U_1, U_2, ..., U_i, ..., U_{N_1}$. They are also called the primary stage units (psus) when there are two stages of selection. The size of *i*-th cluster or psu (U_i) is denoted as N_i . We have

$$U = \bigcup_{i=1}^{N_{\rm I}} U_i$$
 and $N = \sum_{i=1}^{N_{\rm I}} N_i$.

At stage one, a sample of psus, s_i , is selected from U_i according to the design $p_i(.)$ with the inclusion probabilities π_{li} and π_{lij} at the psu level. The size of s_i is n_i psus. The sampling units at the second stage (ssu) are population elements, labeled k = 1, ..., N. Given that the psu U_i selected at the first stage, a sample s_i of size n_i units is drawn from U_i according to some specified design $p_i(.)$ with inclusion probabilities $\pi_{k/i}$ and $\pi_{kl/i}$. For the second stage sampling we are assuming the invariance and independence property. The whole sample of elements and its size is defined as,

$$s = \bigcup_{i=1}^{S_{\rm I}} s_i \text{ and } n_s = \sum_{i=1}^{n_{\rm I}} n_i.$$

The inclusion probabilities at the first stage is given as,

$$\pi_{Ii} = \Pr(i \in s_{I}),$$

$$\pi_{Iij} = \begin{cases} \Pr(i \& j \in s_{I}), i \text{ and } j \text{ belongs to different psus} \\ \pi_{Ii}, i \text{ and } j \text{ belongs to same psus} \end{cases}$$

The inclusion probabilities for the second stage is given as,

$$\pi_{k/i} = \Pr\left(k \in s_i \mid i \in s_1\right) \text{ and}$$
$$\pi_{kl/i} = \begin{cases} \Pr(k \& l \in s_i \mid i \in s_1), k \text{ and } l \text{ are different} \\ \pi_{k/i}, k \text{ and } l \text{ are same} \end{cases}$$

Let the study variable be y, where y_k is observed for all $k \in s$. The parameter to estimate

is the population total
$$t_y = \sum_{k=1}^{N} y_k = \sum_{i=1}^{N_1} t_{yi}$$
 where
 $t_{yi} = \sum_{k=1}^{N_i} y_k = i$ -th psu total.

Let us consider the case where population level complete auxiliary information (x_k) is available at the ssu level. As an example, consider a survey conducted in a large city and let the city blocks are the primary stage units (psus) and the buildings within as the ssus. The study variable is measurement of some aspects of the *k*-th building such as habitable carpet area or the quantity of certain equipments in the building. Suppose we have from the city register, an up to date list of all the buildings in the entire city with some useful information x_k attached to the k-th building, k = 1, 2, ..., N. Further, in agricultural surveys, while estimating the crop area at sub-district level, we can consider villages as the psus and households within each village as ssus. For crop area enumeration, we do complete enumeration of all the households. In that situation, we have complete information at the ssu level for both crop area along with information on active family members, number of cattle the household owns, their family status etc. which can be used as the frame of auxiliary information. This kind of situation leads to the scenario of complete auxiliary information.

As population level complete auxiliary information is available at the unit (ssu) level i.e. the auxiliary information x_k was known for all elements $k \in U$. U was the population of size N. The simple estimator of the population total in this case will be,

$$\hat{t}_{HT} = \sum_{i=1}^{n_I} a_{Ii} \sum_{k=1}^{n_i} a_{k/i} y_k = \sum_{k=1}^{n_s} a_k y_k \tag{1}$$

where,

$$a_{Ii} = \frac{1}{\pi_{Ii}}, \ a_{k/i} = \frac{1}{\pi_{k/i}} \text{ and } a_k = a_{Ii} \times a_{k/i}.$$

The proposed calibration estimator of the population total in this case was given by

$$\hat{t}_{y\pi u}^{s} = \sum_{k=1}^{n_{s}} w_{k} y_{k}$$
(2)

 w_k was the calibrated weight. In this situation we minimize the chi-square type distance function using lagrangian multiplier technique to obtain the calibrated weight. In this case we have minimized

the chi-square type distance given as $\sum_{k=1}^{n_s} \frac{\left(w_k - a_k\right)^2}{a_k q_k}$

subject to
$$\sum_{k=1}^{n_s} w_k x_k = \sum_{k=1}^{N} x_k$$
 and $\sum_{i=1}^{n_s} w_k = \sum_{i=1}^{n_s} a_k$.

The objective function in this case is given as,

$$\phi = \sum_{k=1}^{n_s} \frac{\left(w_k - a_k\right)^2}{a_k q_k} - \lambda_1 \left(\sum_{k=1}^{n_s} w_k x_k - \sum_{k=1}^{N} x_k\right)$$
$$-\lambda_2 \left(\sum_{i=1}^{n_s} w_k - \sum_{i=1}^{n_s} a_k\right)$$

Now using the lagrangian multiplier technique minimizing ϕ with respect to w_{μ} , we get,

$$w_k = a_k + \frac{\lambda_1}{2} a_k q_k x_k + \frac{\lambda_2}{2} a_k q_k$$

Now putting the value of w_k in the constraint equations we get,



Putting the values of λ_1 and λ_2 in w_k we obtain the calibrated weight as,

$$w_{k} = a_{k} + \left[\frac{\left(\sum_{k=1}^{N} x_{k} - \sum_{k=1}^{n_{k}} a_{k} x_{k}\right)}{\sum_{k=1}^{n_{k}} a_{k} q_{k} x^{2}_{k} - \frac{\left(\sum_{k=1}^{n_{k}} a_{k} q_{k} x_{k}\right)^{2}}{\sum_{k=1}^{n_{k}} a_{k} q_{k}} \right] a_{k} q_{k} x_{k}$$



where, $a_k = a_{Ii}a_{ki}$ and $n_s = \sum_{i=1}^{n_i} n_i$ = total sample size at the unit level. Here we had considered

 $q_k = 1$ and the estimator in the Eq (2) takes the form of a regression estimator which is given by,







$$=\sum_{k=1}^{n_{s}}a_{k}y_{k}+\hat{\beta}^{*}\left(\sum_{k=1}^{N}x_{k}-\sum_{k=1}^{n_{s}}a_{k}x_{k}\right),$$

which takes the form of the regression estimator. Now the estimator can also be expressed as,

$$=\sum_{i=1}^{N_I}\sum_{k=1}^{N_i}\hat{y}_k + \sum_{k=1}^{n_s}w_k e_{ks},$$





Through, the model assisted survey sampling approach by Sarndal *et al.* (1992), the estimator can also be expressed as,

$$\hat{t}_{y\pi u}^{s} = \sum_{k=1}^{n_{s}} a_{k} g_{ks} y_{k}$$
.

The Approximate variance of the proposed estimator was obtained by first order Taylor series linearization technique and was given by

$$V(\hat{t}_{y\pi u}^{s}) = \sum_{i=1}^{N_{I}} \sum_{j=1}^{N_{I}} \Delta_{lij} \frac{t_{E_{i}}}{\pi_{li}} \frac{t_{E_{j}}}{\pi_{lj}} + \sum_{i=1}^{N_{I}} \frac{1}{\pi_{li}} \sum_{k=1}^{N_{i}} \sum_{l=1}^{N_{i}} \Delta_{kl/i} \frac{E_{k}}{\pi_{k/i}} \frac{E_{l}}{\pi_{l/i}}$$

where,

$$E_{k} = y_{k} - \beta x_{k}, \quad t_{E_{i}} = \sum_{k=1}^{N_{i}} E_{k},$$

$$\Delta_{lij} = (\pi_{lij} - \pi_{li}\pi_{lj}), \quad \Delta_{kl/i} = \pi_{kl/i} - \pi_{k/i}\pi_{l/i} \text{ and }$$

$$\beta = E(\hat{\beta}).$$

Following the model assisted survey sampling approach by Sarndal *et al.* (1992), the Yates– Grundy form of estimator of variance of the calibration estimator) was given by,

$$\hat{V}_{YG}(\hat{t}_{y\pi u}) = \frac{1}{2} \sum_{i=1}^{n_I} \sum_{j=1}^{n_I} d_{Iij} \left(\frac{\hat{t}_{E_i}}{\pi_{Ii}} - \frac{\hat{t}_{E_j}}{\pi_{Ij}} \right)^2 + \frac{1}{2} \sum_{j=1}^{n_I} \frac{1}{\pi^2} \sum_{i=1}^{n_i} \sum_{l=1}^{n_i} d_{kl/i} (w_k e_{ks} - w_l e_{ls})^2 ,$$

where, $d_{Iij} = \frac{(\pi_{Ii} \pi_{Ij} - \pi_{Iij})}{\pi_{Iij}}, d_{kl/i} = \frac{(\pi_{k/i} \pi_{l/i} - \pi_{kl/i})}{\pi_{kl/i}}$
and $\hat{t}_{E_i} = \sum_{k=1}^{n_i} \frac{g_{ks} e_{ks}}{\pi_{k/i}} .$

3. EMPIRICAL EVALUATION

In this Section, we report the results from simulation studies that aim at assessing the performance of the developed calibration estimator under two stage sampling design with respect to simple regression estimator when complete auxiliary information is available at ssu level as given in Sarndal (1992, p.322). These proposed estimators were described as in Table 1.

 Table 1. Definition of various estimators considered in simulation studies.

Estimators	Description	
$\hat{t}_{y\pi u}$	Calibration estimator of the population total under two stage sampling design when population level complete auxiliary information (x_k) was available at the ssu level	
\hat{t}_{yBr}	simple regression estimator of the population total when complete auxiliary information is available at ssu level	

In this study we have considered the case of two stage sampling where sample selection at each stage is governed by equal probability without replacement sampling design (SRSWOR). Here, we also have considered the situation that the size of the ssus is fixed. For empirical evaluation, a Bi-variate normal population is generated and used for the study where BVN (22, 25, 2, 5, r). For the case of simplicity we have assumed that, $N_i = 50$ and $N_i = 100$ whereas the selected samples are of size $n_1 = 15$, $n_i = 30$ and $n_1 = 20$, $n_i = 40$ and there is availability of auxiliary information for ssu level. For the study we have selected a total of 1000 samples from the population using two stage SRSWOR and also considered different levels of correlation between the study variable and the auxiliary variable. For the empirical evaluation a SAS macro was developed for selection of the samples under Two Stage SRSWOR sampling design. The developed SAS Macro is as follows,

%macro two_stage;

proc surveyselect data=pop_psu out=sample_ psu sampsize=&sample_n seed= -111 noprint

run;

data temp_1;

set sample_psu;

sample_no=_n_;

run;

proc plan seed=-27371;

factors sample_no=&sample_n;

output out=Randomized;*domain no and sample size should be given equal otherwise the program for domain will not work;

run;

proc sort data=Randomized;

by sample_no;

run;

data temp1;

merge Randomized temp_1;

by sample_no;

run;

%do j=1 %to &sample_n;

proc surveyselect data=pop_ssu out=sample_ ssu_&j sampsize=&sample_m seed=-111 noprint;

run;

data sample_ssu_&j; set sample_ssu_&j; sample_no=&j;

run;

%end;

data temp2;

set %do k=1 %to &sample_n;

sample_ssu_&k; %end;;

by sample_no;run;

proc sql; create table temp3 as select * from temp1, temp2 where temp1.sample no=temp2.sample no; quit; data temp3; set temp3; drop sample_no; run: proc sql; create table final sample as select * from temp3, pop where temp3.i=pop.i and temp3.j=pop.j; quit; proc sql; %do k=1 %to &sample_n; drop table sample_ssu_&k; %end; drop table Sample_psu, Temp1, Temp2,

quit;

% mend;

Temp3, Temp_1, Randomised;

The performance measures used for empirical evaluation were percentage Relative Bias (%RB) and percentage Relative Root Mean Squared Error (%RRMSE). The formula of Relative Bias and Relative Root Mean Squared Error of any estimator of the population parameter θ are given by

$$RB(\hat{\theta}) = \frac{1}{S} \sum_{i=1}^{S} \left(\frac{\hat{\theta}_{i} - \theta}{\theta} \right) \times 100 \text{ and}$$
$$RRMSE(\hat{\theta}) = \frac{1}{\theta} \sqrt{\frac{1}{S} \sum_{i=1}^{S} (\hat{\theta}_{i} - \theta)} \times 100$$

where, $\hat{\theta}_i$ are the value of the estimator generated through simulation study and θ is the overall

population total for the character under study. The results corresponding to % RB for all the calibration estimators developed for the situation when there was availability of population level complete auxiliary information at ssu level were reported at Table 1 whereas the results corresponding to %RRMSE these proposed calibration estimators were reported in Table 2. From Table 1 it can be seen that, with respect to %RB the calibration regression type estimator was performing better than the usual regression estimator under two stage sampling design when population level complete auxiliary information was available at ssu level for all the situations of different sample sizes drawn from the population and different level of correlations between the study variable and the auxiliary variable. Table 2 reveals that, the proposed calibration estimator for the situation of availability of complex auxiliary information at ssu level was performing better than the usual regression estimator under two stage sampling design for all the sample sizes drawn from the population and for all the levels of correlation between the study variable and the auxiliary variable with respect to %RRMSE except for the case $n_i=15, n_i=30, r=0.9$ when both the estimators have almost the same %RRMSE.

Table 1. % RB for the proposed	l estimator $(\hat{t}_{y\pi u})$	with respect to
usual regression estimator (\hat{t}_{vBr})	under two stage	sampling design
when auxiliary information	is available at th	e SSU level

Sample Size and Correlation (r)	$\hat{t}_{y\pi u}$	\hat{t}_{yBr}
$n_1 = 15, n_i = 30, r = 0.5$	0.116	0.137
$n_1 = 15, n_i = 30, r = 0.7$	0.019	0.029
$n_1 = 15, n_i = 30, r = 0.9$	0.154	0.156
$n_i = 20, n_i = 40, r = 0.5$	0.036	0.047
$n_1 = 20, n_i = 40, r = 0.7$	0.085	0.088
$n_1 = 20, n_i = 40, r = 0.9$	0.015	0.023

Table 2. %RRMSE for the proposed estimator $(\hat{t}_{y\pi u})$ with respect to usual regression estimator (\hat{t}_{yBr}) under two stage sampling design when auxiliary information is available at the SSU Level

Sample Size and Correlation(r)	$\hat{t}_{y\pi u}$	\hat{t}_{yBr}
$n_i = 15, n_i = 30, r = 0.5$	0.201	0.209
$n_i = 15, n_i = 30, r = 0.7$	0.011	0.013
$n_1 = 15, n_i = 30, r = 0.9$	0.109	0.110
$n_i = 20, n_i = 40, r = 0.5$	0.014	0.027
$n_i = 20, n_i = 40, r = 0.7$	0.025	0.031
$n_1 = 20, n_i = 40, r = 0.9$	0.052	0.067

4. CONCLUDING REMARKS

Using the calibration approach proposed by Deville and Sarndal (1992) we have been able to develop a regression type estimator of population total when the study and the complex auxiliary variables are linearly related. The proposed calibration type regression estimators of population total performs better than the simple regression estimator as given in Sarndal et al. (1992, p.322) under two stage sampling design when there was availability of population level complete auxiliary information at ssu level with respect to % relative bias for most of the cases of selection of sample out of the population using equal probability without replacement sampling design and three different levels of correlation between the study variable and the auxiliary variable. Further, it can also be seen that the proposed calibration based regression estimators of population total outperforms the simple regression estimator under two stage sampling design with respect to % relative root mean square error for all of the cases of selection of sample. Hence, based on the simulation study, it can be concluded that the developed estimator is better than simple regression estimator as given in Sarndal et al. (1992) under two stage sampling design when there was availability of population level complete auxiliary information at ssu level.

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