Available online at www.isas.org.in/jisas
JOURNAL OF THE INDIAN SOCIETY OF AGRICULTURAL STATISTICS 70(2) 2016 159–166

General Efficiency Balanced Designs in Circular Blocks with Correlated Observations*

S.G. Patil, A. Majumder and Manjunatha G.R.
Bidhan Chandra Krishi Viswavidyalaya, Nadia

Received 30 September 2014; Revised 25 July 2016; Accepted 26 July 2016

SUMMARY

Neighbor balance provides protection against the effects of correlated observations or potentially unknown trends highly correlated with plot positions within a linear block (Keifer and Wynn 1981, Cheng 1983, Stroup and Mulitze 1991, Jacroux 1998, Ahmed *et al*. 2011). Concept of circular blocks was introduced by Rees (1967). In 1983, Morgan developed the optimality criteria of Cyclic Neighbor balanced designs with correlated observations. These circular neighbor balanced designs require a large number of blocks, specially, for even number of treatments. Construction of circular block designs for even number of treatments with correlated observations with lesser number of blocks is a challenge to researchers. In the present paper, an attempt has been made to construct optimal First order circular block designs with lesser number of blocks considering the correlated observations for even number of treatments. The developed designs are general efficiency balanced (GEB) circular block designs (Das and Ghosh 1985 and Kageyama and Mukerjee 1986) with correlated observations for even (v + 1) number of treatments.The structure of the C- matrix of the design has been derived and A and D-efficiencies of the resultant GEB designs are also obtained for different values of the correlation coefficient ρ ($0 \le \rho \le 1$).

Keywords: GEB designs, Correlated observations, First order circular neighbor balanced designs.

1. INTRODUCTION

Neighbor balanced designs are important if it is known or thought that the effect of a particular plot is influenced by its neighboring plots, in such cases nearest neighbor analysis is considered to be more efficient than classical analysis methods (Wilkinson *et al.* 1983). The construction of nearest neighbor balanced designs in one-dimension has received much attention from several authors e.g., Kiefer and Wynn (1981), Cheng (1983), Shukla and Gill (1986), Morgan and Chakraborti (1988), Ahmed and Akhtar (2008) and others.

Circular neighbor balanced designs were initially used in serology. Rees (1967) presented a technique used in virus research, which requires the arrangement in circles of samples from a number of virus preparations in such a way that over the whole set, a sample from each virus preparation appears next to a sample from every other virus preparation. Since then, the concept of construction of circular neighbor balanced design has become an important topic in statistics and its optimality criteria been studied in a much broader way.

1.1 Definition

A block of treatments of a design is called *circular* if the treatments allotted to its first and last plots are considered as neighbors to each other. If all the blocks of a design are circular then the design is called a *circular block design*. In a circular block each treatment has one left and one right neighbor. If any two treatments appear as immediate neighbor (at distance 1) equally often

*The paper was presented on 30 January 2015 in Dr. G.R. Seth Memorial Young Scientist Award Session during the 68th Annual Conference of the Indian Society of Agricultural Statistics held at ICAR-IASRI, New Delhi during 29-31 January, 2015.
Corresponding author: S.G. Patil
E-mail address: san.santoshpatil@gmail.com

then such design is called as *First Order Circular Neighbor Balanced* (NN1) BIB design.

1.2 One Dimensional Neighbor Models

Let d be a block design with v treatments in distinct b blocks of size k. Then the additive model considered for analysis will be

$$Y = 1\,\mu + X_1\,\tau + X_2\,\beta + \varepsilon.$$

where $Y_{n\times 1}$ is response vector, $1_{n\times 1}$ is vector with all elements as 1, μ is the overall mean, τ_{vx1} is vector of treatment effects, β_{bx1} is vector of block effects, X_1 and X_2 are incidence matrices for treatments and blocks of size $n \times v$ and $n \times b$, respectively. The elements of X_1 and X_2 are either 0 or 1 for a binary block design. Here, $\varepsilon_{n\times 1}$ is a vector of random errors with the assumptions $E(\varepsilon) = 0$ and $Var(\varepsilon) = \sigma^2\,\mathbf{V}$, where $\mathbf{V}$ is some symmetric positive definite matrix. When the observations are uncorrelated, then $\sigma^2\mathbf{V} = \sigma^2\,\mathbf{I}_n$.

According to Keifer and Wynn (1981), here we consider the covariance structure in a circular block design with correlated structure as

$$Cov\,(y_{jr}, y_{j'r'}) = \begin{cases} \sigma^2, & \text{if } j = j' \text{ and } r = r' \\ \rho\sigma^2, & \text{if } j = j' \text{ and } |r - r'| = 1 \\ 0, & \text{otherwise,} \end{cases} \tag{1.1}$$

where, y_{jr} is the observation from the r^{th} plot in j^{th} block, *i.e.* all observations have the same variance, no correlation exists between blocks, and neighboring plots in the same block have the same amount of correlation, $r = 1, 2, \ldots, k$; $j = 1, 2, \ldots, b$, where k is the size of each block.

Considering the above model (1.1) for a BIB design (v, b, r, k, and λ) in circular blocks, we have the following relationships. We know that in a circular block, there is no end plot and all plots will be considered as inner plots.

Let $g\,(j, r)$ = the treatment number of the r^{th} plot in the j^{th} block.

A_i = the set of blocks in which treatment i occurs.

Let, $N_{ii'} = \#\{\,j\!: g(j,r) = i \text{ or } g\,(j,s) = i', |r - s| = 1\}$, i.e. $N_{ii'}$ is the number of times i and i' are adjacent in a circular block. Thus for the above BIB design, $\sum_{i'(\neq i)} N_{ii'} = 2r$.

When the circular neighbor effect exists among the adjacent neighbors only, the covariance structure can be also written as $\mathbf{V} = \mathbf{I}_b \otimes \mathbf{W}_k$ ($\mathbf{I}_b$ is an identity matrix of order b, $\otimes$ denotes the Kronecker product and $\mathbf{W}_k$ is the correlation matrix of k observations within a block).

Where,

$$\mathbf{W}_k = \begin{bmatrix} 1 & \rho & 0 & \ldots & 0 & \rho \\ \rho & 1 & \rho & \ldots & 0 & 0 \\ 0 & \rho & 1 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & 0 \\ \vdots & \vdots & \vdots & & \vdots & 0 \\ 0 & 0 & 0 & \ldots & 0 & 0 \\ \rho & 0 & 0 & & \rho & 1 \end{bmatrix}$$

Let Q_i, denote the vector of adjusted i-th treatment total

$$kQ_i = kT_i - \Sigma_j\, n_{ij} B_j$$

where T_i denotes the total yield of the i-th treatment and B_j denotes the total yield for the j-th block and n_{ij} is the (i, j)-th element of the incidence matrix $\mathbf{N}$ of the design d.

For nearest neighbor structure in linear blocks it is proved that (*cf.* Shah and Sinha (1989)

$$k^2\sigma^{-2}\,\text{var}(Q_i) = r[k(k-1) - 2(k+1)\rho] + 2k\rho e_i$$

$$k^2\,\sigma^{-2}\,\text{cov}(Q_i, Q_j) = -\lambda[k + 2(k+1)\rho] + k\rho\,(kN_{ij} + e_{ij});\ (\,i \neq j = 1, 2, \ldots, v) \tag{1.2}$$

where, e_i = Number of blocks with treatment i at an end; N_{ij} = Number of blocks with treatments i and j in adjacent positions and e_{ij} = Number of blocks with treatments i and j such that either one of them occurs at an end position. In case of a circular block design, the e_i and e_{ij} terms vanishes as there is no end plots in a circular block.

Kiefer and Wynn (1981) also showed that under the model (1.1), a BIB design with linear blocks in which all the quantities of $(e_{ii'} + kN_{ii'})$, $(i \neq i')$ are equal, possess strong optimality properties in the set of BIB designs and the design will be consider as first order Neighbor Balanced, where $e_{ii'}$ is the number of blocks in which i occurs at an end plus the number where i' occurs at an end. It is obvious that BIB designs in circular blocks in which all the quantities of $N_{ii'}$ ($i \neq i'$) are equal (here $e_{ii'}$ is zero), possess strong optimality properties in the set of BIB designs and the design will be considered as first order Circular Neighbor Balanced.

Several authors have developed BIB designs with equal ($e_{ii'} + kN_{ii'}$) (e.g. Kiefer and Wynn 1981, Cheng 1983 etc.) in linear blocks. Normally, for any values of $k \geq 3$, minimum requirement of number of blocks is $mv(v$-1)/2 where m is an integer. If v is odd, minimum value of b (number of blocks) will be $v(v - 1)/2$ and if v is an even number b will be $v(v - 1)$ (Kiefer and Wynn 1981, Cheng 1983, Morgan and Chakraborti 1988 etc.). However, Kiefer and Wynn (1981) presented one NN balanced BIB design with parameters (7, 14, 8, 4, 4).

Actually, the construction of such designs either in circular blocks or in linear blocks, when v is a prime product or even number, is very difficult as it requires a large number of blocks. Thus construction of designs when v is even with correlated observations as suggested in model (1.1) is still a challenge to present day statisticians. Apart from balanced block designs, other types of balanced designs with correlated observations may reduce the number of blocks and thereby serve the purpose. Das and Ghosh (1985) introduced the concept of 'General efficiency balanced' (GEB) designs to unify the definitions of variance balanced (VB) and efficiency balanced (EB) designs. Several series of (GEB) designs were constructed by many authors (Kageyama and Mukerjee 1986, Kageyama *et al*. 1997 etc.).

Constantine (1983) showed by adding a control treatment once in every block of a BIB design is A-optimal in the restricted class of block designs having a single replication of the control treatment in each block.

Kageyama and Mukerjee (1986) had constructed GEB designs through method of reinforcement of a BIB design with parameters (v, b, r, k and λ). They found that if one new treatment is added to each block of the BIB design then the resultant design will be a GEB design with v+1 treatments. This type of designs can also be considered as Balanced Treatment Incomplete Block (BTIB) designs with one control treatment as proposed by Bechhofer and Tamhane (1981).

Jaggi *et al*. (1997) defined and constructed General Efficiency Balanced block designs with unequal block sizes (GEBUB) for comparing treatments belonging to two disjoint sets, each set consisting of two or more treatments.

In the present study, we have tried to construct a series of GEB designs for v+1 number of treatments (v is a prime or prime power) in circular blocks by following the method described by Kageyama and Mukerjee (1986) from a series of First Order Neighbor Balanced (NN1) BIB designs. Considering the correlated observations as in (1.1), C-matrix of the newly constructed GEB designs has been obtained. The A and D-efficiencies values of the resultant circular GEB designs are calculated for different ρ values ($0 \leq \rho \leq 1$).

2. METHOD OF CONSTRUCTION

Sahu and Majumder (2012) developed a general series of all order neighbor balanced BIB designs. Here we consider the above series of neighbor balanced BIB designs (d) with parameters v, $b = pv$, $r = pk$, k, $\lambda = r(k$-1)/(v-1), with $\lambda_1 = 2p(k$-1)/(v-1), where v is a prime power, p is number of initial blocks, λ_1 is the occurrence of any pair of treatments as First order neighbors.

Sahu and Majumder (2012) developed First Order Neighbor Balanced (NN1) BIB designs in linear blocks, with $e_{Li} = e_{Ri}$ and $e_i = 2e_{Li} = 2e_{Ri} = a$ constant, $\forall\, i$ (= 1, 2, ..., v) where e_i be the number of blocks for which treatment i occurs at end plot, e_{Li} be the number of blocks for which treatment i occurs at left end plot and e_{Ri} be the number of blocks for which treatment i occurs at right end plot. Obviously, $e_i = e_{Li} + e_{Ri}$, as e_i is a constant, $\forall$ i (= 1, 2, .., v) and e_{ij} is also a constant , $\forall\, i, j$ ($i \neq j$) = 1, 2, .., v.

Illustration 2.1: Let D be a BIB design with parameters v =7, b = 21, r =12, k = 4, λ = 6, p = 3, whose solution of D is given by:

(1 3 2 6), (2 4 3 0), (3 5 4 1), (4 6 5 2), (5 0 6 3), (6 1 0 4), (0 2 1 5)

(3 2 6 4), (4 3 0 5), (5 4 1 6), (6 5 2 0), (0 6 3 1), (1 0 4 2), (2 1 5 3)

(2 6 4 5), (3 0 5 6), (4 1 6 0), (5 2 0 1), (6 3 1 2), (0 4 2 3), (1 5 3 4)

Here, elements in the parenthesis denote the contents of b = 21 blocks. The illustration 2.1 shows a First Order Neighbor Balanced BIB design. Thus each pair of treatments in the design which are immediately neighbor to each other is occurring 3 times *i.e.*, λ_1 =3. It is also seen that in the above design with correlated error has $N_{ii'}$ = 3.

2.1 Method of Construction of GEB Designs in Circular Blocks with Correlated Observations

Theorem 2.1: If there exists a First Order Neighbor Balanced (NN1) BIB design in linear blocks (D: v, $b = pv$, $r = pk$, k, $\lambda = r(k-1)/(v-1)$) then there will be a GEB design in circular blocks (D*) with parameters $v^* = v + 1$, $b^* = b$, $r^* = [r\mathbf{1}'_v, b]$, $k^* = k + 1$ with correlated observations as given in model (1.1) and any pair of treatments (i, i' ($i \neq i'$) = 1, 2, …, v) will occur λ number of times but any pair of treatments i, j (i (= 1, 2, …, v) and $j = v$ + 1) occurs λ^* (= r) number of times.

According to theorem 2.1, the elements of C-Matrix of D* will be

$$C_{ii} = \frac{r\left[(k+1)^2 - (k+1)(1+2\rho)\right]}{(k+1)^2}; \forall (i = 1, 2, ..., v)$$

$$C_{ii'} = \frac{-\lambda(k+1)(1+2\rho) + (k+1)^2 \rho N_{ii'}}{(k+1)^2};$$

$$\forall (i, i'(i \neq i') = 1, 2, ..., v)$$

$$C_{ij} = \frac{-\lambda(k+1)(1+2\rho) + (k+1)^2 \rho N_{ii'}}{(k+1)^2};$$

$$\forall (i = 1, 2, ..., v \text{ and } j = v+1) \text{ and}$$

$$C_{jj} = \frac{b\left[(k+1)^2 - (k+1)(1+2\rho)\right]}{(k+1)^2}; (j = v+1).$$

***Proof*:** Let us introduce one extra treatment to each end plot of each block of above First Order Neighbor Balanced BIB design (D) then the resultant design will be treated as a GEB design (D*) with $v^* = v + 1$, $b^* = b$, $\mathbf{r}^{*\prime} = [r\mathbf{1}_v', b]$, $k^* = k + 1$. As blocks are circular there is no question of e_i and $e_{ii'}$.

From the above correlated model (1.1 and 1.2) the circular NN1 structure can be written as,

$$(k+1)^2\sigma^{-2} \operatorname{var}(Q_i) = r[(k+1)^2 - (k+1)(1+2\rho)]\},$$
$$(i = 1, 2, \ldots, v)$$

$$(k+1)^2 \sigma^{-2} \operatorname{cov}(Q_i, Q_{i'}) = -\lambda (k+1)(1+2\rho) + (k+1)^2 \rho N_{ii'}; \quad (i \neq i').$$

$$(k+1)^2 \sigma^{-2} \operatorname{cov}(Q_i, Q_j) = -\lambda^* (k+1)(1+2\rho) + (k+1)^2 \rho N_{ij};$$
$$(i = 1, 2, \ldots, v\, ; j = v + 1)$$

$$(k+1)^2 \sigma^{-2} \operatorname{var}(Q_j) = b\,[(k+1)^2 - (k+1)(1+2\rho)]\},$$
$$(j = v + 1).$$

As we know, well known reduced normal equation of block design $d \in \Delta$ (Δ class of block design) as $Q = \mathbf{C}\hat{t}$ & $V(Q) = \sigma^2\mathbf{C}$. To emphasize the dependence of information matrix on design, we write **C** matrix for design D^*,with row sum and column sum of **C** are zero. The elements of **C**–Matrix of D^* will be

$$C_{ii} = \frac{r\left[(k+1)^2 - (k+1)(1+2\rho)\right]}{(k+1)^2} = a;$$
$$\forall (i = 1, 2, ..., v),$$

$$C_{ii'} = \frac{-\lambda(k+1)(1+2\rho) + (k+1)^2 \rho N_{ii'}}{(k+1)^2} = b;$$
$$\forall (i, i(i \neq i') = 1, 2, ..., v),$$

$$C_{ij} = \frac{-\lambda^*(k+1)(1+2\rho) + (k+1)^2 \rho N_{ii'}}{(k+1)^2} = c;$$
$$\forall (i, i'(i \neq i') = 1, 2, ..., v),$$

$$C_{jj} = \frac{b\left[(k+1)^2 - (k+1)(1+2\rho)\right]}{(k+1)^2} = d; (j = v+1).$$

We know that the **C**- matrix of any GEB design for (v + 1) treatments will be:

$$C^* = \boldsymbol{\theta}\left[\begin{bmatrix} s\mathbf{I}_v & 0 \\ 0 & z \end{bmatrix} - g^{-1}\begin{bmatrix} s\mathbf{I}_v \\ z \end{bmatrix}\begin{bmatrix} s\mathbf{I}_v & z \end{bmatrix}\right]$$

Where $g = vs + z$ for any values of 'θ'.

The **C**–matrix of D* can be written in the above form and simply, it will be

$$\mathbf{C} = \begin{bmatrix} a & b & b & \dots & b & c \\ b & a & b & \dots & b & c \\ b & b & a & \dots & b & c \\ \vdots & \vdots & \vdots & & \vdots & c \\ \vdots & \vdots & \vdots & & \vdots & c \\ b & b & b & \dots & a & c \\ c & c & c & & c & d \end{bmatrix} \quad (2.1)$$

Here, the design D* is a GEB design with correlated error structures for first order neighbors for v^* (= v + 1) treatments with b^* (= b) blocks of size k^* (= k + 1). It is obvious that BIB designs in circular blocks in which all the quantities of $N_{ii'}$, ($i \neq$ i') are equal (here $e_{ii'}$ is zero), possess strong optimality properties in the set of BIB designs and the design will be consider as First Order Circular Neighbor Balanced. $N_{ii'}$ is constant = $2\lambda/k$ and $N_{ij} = 4\lambda/k$, $\forall$ i(= 1, 2, ..., v) and j = (v + 1).

Illustration 2.2: Let us add one extra treatment to each block of the design of illustration 2.1. Let D* be a GEB design with parameters v^* = 8, b^* = 21, $r^{*\prime}$ = [12 ,21], k^* = 5, λ^* = [6,12] , p = 3 the solution of D* is given by

(1 3 2 6 7), (2 4 3 0 7), (3 5 4 1 7), (4 6 5 2 7), (5 0 6 3 7), (6 1 0 4 7), (0 2 1 5 7)

(3 2 6 4 7), (4 3 0 5 7), (5 4 1 6 7), (6 5 2 0 7), (0 6 3 1 7), (1 0 4 2 7), (2 1 5 3 7),

(2 6 4 5 7), (3 0 5 6 7), (4 1 6 0 7), (5 2 0 1 7), (6 3 1 2 7), (0 4 2 3 7) & (1 5 3 4 7).

From above example we can see that $N_{ii'} = 2\lambda/k$ = 3; $N_{ij} = 4\lambda/k = 6$ where i, i' ($i \neq i'$) = 1, 2, …, 7, j = 8 (= v + 1).

3. EFFICIENCY OF GEB DESIGN WITH CORRELATED OBSERVATATIONS

For a connected block design d, let θ_1, θ_2, …., θ_{v-1}, be the non-zero (v – 1) eigenvalues of **C** matrix of the design. The design d will be universal optimal if all θ_i (i = 1, 2, 3, ……, v – 1) are equal with maximum trace of **C** matrix. Now define

$$\varnothing_A(d) = \left(\frac{1}{v-1}\sum_{i=1}^{v-1}\theta_i^{-1}\right)^{-1} \text{and } \varnothing_D(d) = \prod_{i=1}^{v-1}\theta_i .$$

Then, a design is A– [D–] optimal if it maximizes the $\varnothing_A(d)$, $\varnothing_D(d)$ over D (v, b, k). The A and D efficiencies of a design d over D (v, b, k):

$$e_A(d) = \frac{\varnothing_A(d^*_A)}{\varnothing_A(d)} \text{ and } e_D(d) = \left[\frac{\varnothing_D(d^*_D)}{\varnothing_D(d_D)}\right]^{1/(v-1)}$$

where, d_A^* and d_D^* are the hypothetical A-optimal and D-optimal design over $D_\rho(v, b, k)$, respectively. The $\phi_A(d_A^*)$ and $\phi_D(d_D^*)$ values are calculated in Table 3.1 by following the lines of Ponnuswamy and Santharam (1997). A Design is A (or D) optimal if all θ_i's are equal. However, such a design for a given set of parameters may not exist. The information matrix of the hypothetical A (or D) optimal design would have positive eigenvalue $\theta = (v - 1)^{-1}$ ($\theta_1 + \theta_2 + \ldots + \theta_{v-1}$) with multiplicity ($v$ – 1). Then $\phi_A(d_A^*)$ of a hypothetical design d_A^* and $\phi_D(d_D^*)$ of a hypothetical design d_D^* has been computed as mentioned above with unique eigenvalue θ.

Efficiency values (A and D with respect to ρ = 0) for different values of ρ of different GEB designs in circular blocks with correlated observations is presented in Table 3.1. It is observed that the designs are A-efficient for correlated observations ($0 \leq \rho \leq 0.4$).

3.1 Some Results on GEB Designs in Circular Blocks with Correlated Observations

Result 3.1: The first v – 1 non- zero eigen roots of the GEB design D*, with $v^* = v + 1$, $b^* = b$, $r^{*\prime}$ = [$r\mathbf{1}'$, r'], $k^* = k + 1$ will be linearly dependent on ρ and the relation is $A - B\rho$, where $A = pk(vk - 1)/\{(k + 1)(v - 1)\}$ and $B = 2p(vk - 1)/\{(k + 1)(v - 1)\}$. The remaining non-zero eigen root of D* is also linearly dependent on ρ and the relation is $A' - B'\rho$, where $A' = pk(v + 1)/k + 1$, and $B' = 2p(v + 1)/(k + 1)$.

Result 3.2: The trace of the **C**-matrix of the above GEB design D^*, $v^* = v + 1$, $b^* = b$, $r^{*\prime}$ = [$r.\mathbf{1}'$, r'], $k^* = k + 1$ will be linearly dependent on ρ and the relation is $A - B\rho$, where $A = pvk$ and $B = 2pv$.

The proof of the results (3.1 and 3.2) can be done by simplification of the elements of the **C**- matrix of the design D^*.

Table 3.1: Efficiency values for different values of ρ of different GEB designs in circular blocks with correlated observations

BIBD					GEB Design						Efficiency for different values of ρ									
v	b	r	k	λ	v^*	b^*	r^*	k^*	λ^*		ρ = 0.1	ρ = 0.2	ρ = 0.3	ρ = 0.4	ρ = 0.5	ρ = 0.6	ρ = 0.7	ρ = 0.8	ρ = 0.9	ρ = 1.0
7	21	9	3	3	8	21	9, 21	4	3, 9	A	0.9333	0.8667	0.8000	0.7333	0.6667	0.6000	0.5333	0.4667	0.4000	0.3333
										D	0.6170	0.3673	0.2097	0.1141	0.0585	0.0280	0.0123	0.0048	0.0016	0.0005
7	21	12	4	6	8	21	12, 21	5	6, 12	A	0.9500	0.9000	0.8500	0.8000	0.7500	0.7000	0.6500	0.6000	0.5500	0.5000
										D	0.6983	0.4783	0.3206	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152	0.0078
9	36	12	3	3	10	36	12, 36	4	3, 12	A	0.9333	0.8667	0.8000	0.7333	0.6667	0.6000	0.5333	0.4667	0.4000	0.3333
										D	0.4682	0.2072	0.0859	0.0330	0.0116	0.0036	0.0010	0.0002	0.0000	0.0000
9	36	16	4	6	10	36	16, 36	5	6, 16	A	0.9500	0.9000	0.8500	0.8000	0.7500	0.7000	0.6500	0.6000	0.5500	0.5000
										D	0.6302	0.3874	0.2316	0.1342	0.0751	0.0404	0.0207	0.0101	0.0046	0.0020
11	55	15	3	3	12	55	15, 55	4	3, 15	A	0.9333	0.8667	0.8000	0.7333	0.6667	0.6000	0.5333	0.4667	0.4000	0.3333
										D	0.4682	0.2072	0.0859	0.0330	0.0116	0.0036	0.0010	0.0002	0.0000	0.0000
11	55	20	4	6	12	55	20, 55	5	6, 20	A	0.9500	0.9000	0.8500	0.8000	0.7500	0.7000	0.6500	0.6000	0.5500	0.5000
										D	0.5688	0.3138	0.1673	0.0859	0.0422	0.0198	0.0088	0.0036	0.0014	0.0005
13	78	18	3	3	14	78	18, 78	4	3, 18	A	0.9333	0.8667	0.8000	0.7333	0.6667	0.6000	0.5333	0.4667	0.4000	0.3333
										D	0.4078	0.1556	0.0550	0.0177	0.0051	0.0013	0.0003	0.0000	0.0000	0.0000
13	78	24	4	6	14	78	24, 78	5	6, 24	A	0.9500	0.9000	0.8500	0.8000	0.7500	0.7000	0.6500	0.6000	0.5500	0.5000
										D	0.5133	0.2542	0.1209	0.0550	0.0238	0.0097	0.0037	0.0013	0.0004	0.0001

Table 3.1(contd.): Efficiency values for different values of ρ of different GEB designs in circular blocks with correlated observations

BIBD					GEB Design						Efficiency for different values of ρ									
v	*b*	*r*	*k*	*λ*	*v**	*b**	*r**	*k**	*λ**		ρ = 0.1	ρ = 0.2	ρ = 0.3	ρ = 0.4	ρ = 0.5	ρ = 0.6	ρ = 0.7	ρ = 0.8	ρ = 0.9	ρ = 1.0
17	136	24	3	3	18	136	24, 136	4	3, 24	A	0.9333	0.8667	0.8000	0.7333	0.6667	0.6000	0.5333	0.4667	0.4000	0.3333
										D	0.3095	0.0878	0.0225	0.0051	0.0010	0.0002	0.0000	0.0000	0.0000	0.0000
17	136	32	4	6	18	136	32, 136	5	6, 32	A	0.9500	0.9000	0.8500	0.8000	0.7500	0.7000	0.6500	0.6000	0.5500	0.5000
										D	0.4181	0.1668	0.0631	0.0225	0.0075	0.0023	0.0007	0.0002	0.0000	0.0000

ACKNOWLEDGEMENT

The authors are grateful to the referee for their valuable comments that have helped in improving the paper.

REFERCNCES

Ahmed, R. and Akhtar, M. (2008). Construction of neighbor balanced block designs. *J. Statist. Theory Pract.*, **2**, 551-558.

Ahmed, R., Akhtar, M. and Yasmin, F. (2011). Brief review of one-dimensional neighbor balanced designs since 1967. *Pak. J. Commer. Soc. Sci.,* **5(1)***,* 100-116.

Bechhofer, R.E. and Tamhane, A.C. (1981). Incomplete block designs for comparing treatments with a control, General theory. *Technometrics,* **23**, 45-57.

Cheng, C.S. (1983). Construction of optimal balanced incomplete block designs for corresponding observations. *Ann. Statist.*, **11**, 240-246.

Constantine, G.M. (1983). On the efficiency for control of reinforced BIB designs. *J. Roy. Statist. Soc.,* B, **45**, 31-36.

Das, M.N. and Ghosh. D.K. (1985). Balancing incomplete block designs. *Sankhya*, **B 47***,* 67-77.

Jacroux, M. (1998). On the construction of efficient equineighbored incomplete block designs having block size 3. *Sankhya,* **B(3)60**, 488-495.

Jaggi, S., Parsad R. and Gupta, V.K. (1997). General efficiency balanced block designs with unequal block sizes for comparing two sets of treatments. *J. lnd. Soc. Agril. Statist.,* **50(1)**, 37-46.

Kageyama, S. and Mukerjee, R. (1986). General balanced designs through reinforcement. *Sankhya,* **B 48**, 380-387.

Kageyama, S., Pal, S. and Majumder, A. (1997). General methods of construction of balanced designs. Bull. of the Faculty of School Education, Hiroshima Univ. **19**(II), 57- 64.

Kiefer, J. and Wynn, H.P. (1981). Optimum balanced block design and Latin square designs for correlated observations. *Ann. Statist.*, **9**, 737- 757.

Morgan, J.P. (1983). Optimum Block Designs for Neighbor Type Covariance Structures. Ph.D. Thesis of University of North Carolina at Chapel Hill, USA.

Morgan, J.P. and Chakraborti, I.M. (1988). Block designs for first and second order neighbour correlations. *Ann. Statist.*, **16(3)**, 1206-1224.

Ponnuswamy, K.N. and Santharam, C. (1997**).** Optimality and efficiency of neighbouring design. *J. lnd. Soc. Agril. Statist.,* **50(1)**, 1-10.

Rees, D.H. (1967). Some designs of use in serology. *Biometrics*, **23**, 779-791.

Sahu, A.K. and Majumder, A. (2012). Construction of μ-resolvable all ordered neighbor balanced BIB designs. *Res. Rev.: J. Statist.*, **1(3)**, 7-16.

Shah, K.R. and Sinha, B.K. (1989). *Theory of Optimal Designs.* Springer-Verlag, Lecture Notes in Statistics, **53**, 121-123.

Shukla, G.K. and Gill, P.S. (1986): Nearest neighbour designs for comparative experiments: Optimality and analysis. *Proceedings of the symposium on Optimization, Design of Experiments and Graph Theory.* IIT Bombay. pp 90-102.

Stroup, W.W. and Mulitze D.K. (1991). Nearest neighbour adjusted best linear unbiased prediction. *The Amer. Statist*., **45**, 194-200.

Wilkinson, G.M., Eckert, S.R., Hancock, T.W. and Mayo, O. (1983). Nearest neighbourhood (NN) analysis of field experiments. *J. Roy. Statist. Soc.*, **B 45**, 151- 211.