



VAR-MGARCH Models for Volatility Modelling of Pulses Prices: An Application

Achal Lama¹, Girish K. Jha², Bishal Gurung¹, Ranjit Kumar Paul¹ and Kanchan Sinha¹

¹ICAR-Indian Agricultural Statistics Research Institute, New Delhi

²ICAR-Indian Agricultural Research Institute, New Delhi

Received 23 April 2016; Revised 14 June 2016; Accepted 15 June 2016

SUMMARY

In this paper an attempt has been made to model the volatile pulses prices using the VAR-MGARCH approach. The different forms of MGARCH models such as BEKK, CCC and DCC have also been explored. To deal with the presence of excess kurtosis in the series Student-t distribution innovation was also considered. Further, it was empirically found that the MGARCH-DCC model with Student-t distribution was the most suitable among all for modelling the pulses series.

Keywords: Volatility, VAR, MGARCH, BEKK, CCC, DCC.

1. INTRODUCTION

Pulses in India are cultivated mainly on marginal lands under rain fed conditions. Due to the high level of fluctuations in pulse production and prices, farmers are not readily taking up cultivation of pulses despite government ensuring high wholesale pulse prices in recent years. Among the kharif pulses, Arhar and Urud are the major ones which regularly have high demand and also have high wholesale prices. In addition to it, these two pulses face the inherent problem of fluctuation in production and prices. Thus, it becomes necessary to understand the phenomenon that might govern the prices of these crops. Further, to have better insight into the process one needs to deal with both the crops together and come to a conclusion. Modelling of more than one time-series together at present has become the need of the hour. With advent of large data storage and availability, as well as the fast computing systems one can easily deal with multiple series at once. Moreover, it has been noticed that the series tend

to have a higher degree of interaction among them or they move together over time, which compels the researcher to consider all the series to better understand the phenomenon under consideration. Different multivariate time-series models are used for forecasting, but after the pioneering work of Sims (1980), Vector Autoregressive (VAR) model have become the most popular among them for correlated series. This phenomenon of correlation is also observed in agricultural data series. The VAR model is useful only for modelling the mean or the first order moment of the series. Thus to have a better understanding of the series, modelling and forecasting volatility has been a major area of time series research for some years now. Traditional econometric models assume a constant one-period forecast variances. To generalize this implausible assumption, Engle (1982) introduced a new class of stochastic process called Autoregressive Conditional Heteroscedasticity (ARCH) which is very useful where underlying forecast variances may change over time and is predicted by the

past forecast errors. However, ARCH model has some drawbacks such as high number of unknown parameters and rapid decay of unconditional autocorrelation function of squared residuals etc. To overcome these difficulties, Bollerslev (1986) proposed the Generalized ARCH (GARCH) model in which conditional variance is also a linear function of its own lags. This model is also a weighted average of past squared residuals, but it has declining weights that never go completely to zero. It provides much more flexible lag structure and it permits more parsimonious descriptions in most of the situations. The ability of GARCH model to capture volatility has been widely studied in literature (Lama *et al.* 2015).

However, with increased globalization it is now widely accepted that agricultural price volatilities interact over time across commodities and markets. This feature can be better understood through a multivariate GARCH (MGARCH) model framework instead of working with separate univariate model for different commodities and markets. A class of MGARCH models have been developed over time. Engle and Kroner (1995) introduced a multivariate structure of GARCH model known as BEKK (Baba, Engle, Kraft and Kroner) model which is the direct generalization of univariate GARCH model and have huge flexibility. Bollerslev (1990) developed a relatively flexible approach known as Constant Conditional Correlation (CCC) model which allowed for combination of univariate GARCH model, with an assumption of constant correlation among the series over time. Engle (2002) proposed a new class of multivariate GARCH model known as Dynamic Conditional Correlation (DCC) model which has the flexibility of the univariate GARCH models coupled with parsimonious parametric model for the correlations. The use of these models for modelling the degree of interactions among various volatile commodities and markets can be widely seen in literature (Chevallier 2012, Teng and Lean 2013 and Li and Lin 2015). It is also interesting to note that most of the financial series exhibits leptokurtic behaviour which has been documented well in the literature. There are number of ways to deal with this problem and the use of Student-t distribution to model the error term is a promising alternative among them

(Ku 2008). In this study, we have modelled the volatile agricultural price series using the class of MAGRCH model to understand the transmission of volatility among the series and its movement over time. Further, keeping in mind the inherent kurtosis nature of the series Student-t innovation have been incorporated in the models to better explain the characteristics of the series. The paper is organised as follows: In Section 2 brief details of the VAR, MGARCH models and testing of MARCH effect have been described, followed by empirical results in Section 3. Section 4 of the paper deals with the discussion of the results. Finally, the paper is concluded in Section 5.

2. A BRIEF DESCRIPTION OF MODELS

2.1 VAR Model

Let $Y_t = (y_{1t}, y_{2t}, \dots, y_{nt})$ denote an $(n \times 1)$ vector of time series variables. The basic p -lag vector autoregressive VAR(p) model has the form:

$$Y_t = A + B_1 Y_{t-1} + B_2 Y_{t-2} + B_3 Y_{t-3} + \dots + B_p Y_{t-p} + \varepsilon_t \quad (1)$$

where, A is $n \times 1$ vector of intercepts, B_i ($i = 1, 2, \dots, p$) is $k \times k$ matrices of parameters and $\varepsilon_t \sim iidN(0, \Sigma)$

The number of parameters to be estimated in the VAR model is $k(1+kp)$ which increases with the number of variables (k) and number of lags (p).

2.2 MGARCH Model

For a multivariate time series $y_t = (y_{1t}, \dots, y_{kt})$ the MGARCH model is given by:

$$y_t = H_t^{1/2} \varepsilon_t \quad (2)$$

where, H is $k \times k$ positive-definite matrix and of the conditional variance of $C_t k$ is the number of series and $t = 1, 2, \dots, n$ (number of observations). It is with the specification of conditional variance that the MGARCH model changes.

Engle and Kroner (1995) introduced the BEKK model which is the direct generalization of the univariate GARCH model. The resulting variance is dependent on the amount of currently available information. A general GARCH (p, q)

model (Bollerslev 1986) can be defined as:

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \dots + \alpha_p \varepsilon_{t-p}^2 + \beta_1 h_{t-1} + \dots + \beta_q h_{t-q},$$

$$\alpha_i > 0, \beta_i > 0, \alpha_i + \beta_i < 1 \quad (3)$$

where, h_t is the conditional variances which depends on the previous error terms as well as previous conditional variances of the process.

Equation (2) can be transferred into multivariate GARCH model with a generalization of the resulting variance matrix H_t

$$H_t = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \quad (4)$$

Each element of H_t depends on the p delayed values of the squared ε_t , the cross product of ε_t and on the q delayed values of elements from H_t . In general, multivariate GARCH (1, 1) model can be written as:

$$H_t = C_0 C_0 + \dots + \begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix} \begin{pmatrix} \varepsilon_1^2 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_3 \\ \varepsilon_2 \varepsilon_1 & \varepsilon_2^2 & \varepsilon_2 \varepsilon_3 \\ \varepsilon_3 \varepsilon_1 & \varepsilon_3 \varepsilon_2 & \varepsilon_3^2 \end{pmatrix}$$

$$+ \dots + \begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix} + \dots + \begin{pmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{pmatrix}$$

$$+ \dots + \begin{pmatrix} h_1^2 & h_1 h_2 & h_1 h_3 \\ h_2 h_1 & h_2^2 & h_2 h_3 \\ h_3 h_1 & h_3 h_2 & h_3^2 \end{pmatrix} \begin{pmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{pmatrix} \quad (5)$$

In compact form, the above equation can also be written as:

$$H_t = C_0' + A' \varepsilon_{t-1} \varepsilon_{t-1}' A + B' H_{t-1} B \quad (6)$$

For 2 variable case the model can be represented as:

$$H_t = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1} \varepsilon_{2,t-1} \\ \varepsilon_{2,t-1} \varepsilon_{1,t-1} & \varepsilon_{2,t-1}^2 \end{bmatrix}$$

$$+ \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} H_{t-1} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$

$$h_{11,t} = c_{11} + a_{11}^2 \varepsilon_{1,t-1}^2 + 2a_{11} a_{21} \varepsilon_{1,t-1} \varepsilon_{2,t-1}$$

$$+ a_{21}^2 \varepsilon_{2,t-1}^2 + g_{11}^2 h_{11,t-1} + 2g_{11} g_{21} h_{12,t-1}$$

$$+ g_{21}^2 h_{22,t-1}$$

$$h_{12,t} = c_{12} + a_{11} a_{21} \varepsilon_{1,t-1}^2$$

$$+ (a_{21} a_{12} + a_{11} a_{22}) \varepsilon_{1,t-1} \varepsilon_{2,t-1}$$

$$+ a_{21} a_{22} \varepsilon_{2,t-1}^2 + g_{11} g_{12} h_{11,t-1}$$

$$+ (g_{21} g_{12} + g_{11} g_{22}) h_{12,t-1} + g_{21} g_{22} h_{22,t-1}$$

$$h_{22,t} = c_{22} + a_{12}^2 \varepsilon_{1,t-1}^2 + 2a_{12} a_{22} \varepsilon_{1,t-1} \varepsilon_{2,t-1} + a_{22}^2 \varepsilon_{2,t-1}^2$$

$$+ g_{12}^2 h_{11,t-1} + 2g_{12} g_{22} h_{12,t-1} + g_{22}^2 h_{22,t-1}$$

$$H_t = D_t R D_t$$

The core issues in MGARCH model is to construct the conditional variance-covariance matrix H_t . A relatively flexible approach is the CCC model introduced by Bollerslev (1990). This model assumes the conditional correlations to be constant. This restriction strongly reduces the number of unknown parameter and thus simplified the estimation. In case of CCC model the H_t represented as follows:

$$H_t = D_t R D_t$$

where, $D_t = \text{diag}(h_{11,t}^{1/2}, \dots, h_{kk,t}^{1/2})$ and R is a symmetric positive-definite matrix whose elements are (constant) conditional correlations $\rho_{ij}, i, j = 1, 2, \dots, k (\rho_{ij} = 1, i = j)$. Thus each conditional covariance is given by:

$$h_{ij,t} = \rho_{ij} \sqrt{h_{ii,t} h_{jj,t}} \quad (7)$$

In case of DCC the R matrix is also time varying thus making it dynamic. The representation of the model is as follows:

$$H_t = D_t R_t D_t$$

$$R_t = \text{diag}(Q_t)^{-1/2} Q_t \text{diag}(Q_t)^{-1/2}$$

where $Q_t = (1 - \alpha - \beta)R + \alpha u_{t-1} - u_{t-1} u_{t-1}' + \beta Q_{t-1}$ and $U_t = D_t^{-1} y_t$

R is the unconditional covariance matrix and the conditional covariances are given by :

$$h_{ij,t} = q_{ij,t} \sqrt{h_{ii,t} h_{jj,t}} / \sqrt{q_{ii,t} q_{jj,t}} \quad (8)$$

Q_t is written as GARCH (1,1) type equation and then transformed to get R_t .

The parameters of the models are estimated using the maximum likelihood estimation procedure.

2.3 Testing of MARCH Effect

The appropriateness of VAR model depends largely on the behaviour of the residuals remaining constant over time. Thus, the MARCH – Lagrange multiplier (LM) test was carried out on the square of the residuals obtained after fitting the VAR model on two series to test whether the residuals do in fact remain constant. The results of the test revealed the presence of ARCH effect for both the series (Table 3).

2.4 Akaike Information Criterion and Bayesian Information Criterion

Standard model evaluation criteria, such as Akaike information Criterion (AIC) and Bayesian Information Criterion (BIC), are used to compare the performance of different models. These criteria penalize the decrease in the degrees of freedom when more variables are added. The AIC and BIC values for GARCH model with Gaussian distributed errors are computed by:

$$AIC = -2 \log(\text{likelihood}) + 2T$$

$$BIC = -2 \log(\text{likelihood}) + \log(Tk)$$

where, k is model degrees of freedom.

3. EMPIRICAL RESULTS

3.1 Data and Implementation

In this study, we have used two pulses series namely price index of Arhar and Urud. The data was collected from the Office of the Economic Adviser, Ministry of Commerce and Industry, Government of India. These series illustrate the complexity and variation of typical agricultural price data (Figs. 1-2). Each series contained 261 data points (April, 1994 to January, 2016) and the entire series was used for modelling. The characteristics of the data sets used are presented in the Table 1. The visual inspection of these series (Figs. 1-2) clearly indicated the presence of volatility at several time-epochs. In addition, the skewness and kurtosis coefficients suggested the asymmetry and fat-tailed distribution of

the series and the correlation coefficient (0.92) suggested the linear inter-dependence of the two series (Table 1). The presence of cointegration among the two series was also tested and the results (Table 2) clearly indicted the absence of it, with the alternate hypothesis being the presence of cointegration. This result was needed to have an understanding of the fact that the series have a long run dependency or not. This motivated us to use the VAR-MGARCH model with multivariate Student-t distribution instead of the usual multivariate normal distribution. The MARCH – Lagrange multiplier (LM) test was carried out on the square of the residuals obtained after fitting the VAR model on the two series to test whether the residuals exhibit heteroscedasticity (Table 3). The heteroscedasticity or volatility clustering phenomenon is observed in the price series and justifies the implementation of MGARCH models. We also explore the Stationarity of the series by implementing the augmented Dickey–Fuller (ADF) test and we find that the series are stationary after first order differencing (Table 4).

3.2 Fitting of GARCH and MGARCH Models

Both the price series at first were modelled individually by univariate GARCH model and then by the MGARCH models. The results of the GARCH models are reported in Table 5. The results clearly indicate the persistence of volatility in both the series by the high value of the coefficient b_j . For modelling the two series together we first modelled them with VAR(2) model to understand the causality relationship of the series among them, in terms of their mean response. After fitting of the VAR, residuals obtained from them were used as an input for MGARCH models. The estimates of VAR model is reported in Table 6. In case of MAGRCH we have first used the BEKK model and the results indicated the transmission of volatility among the series. The conditional volatilities obtained from this model are depicted in Fig. 3. The transmission of volatility was more from series b (Urud) to series a (Arhar), as evident from the values of the coefficients A21 and B21 (Table 7). The problem of this model lies in the fact that all the coefficients in the model were not statistically significant at 5% or 10% level of significance. Thus, the

accuracy of results obtained is questionable. In light of this finding we then focused on the CCC model. The estimates of the model were found to be significant and the series transmitted volatility among them with a correlation of 0.52 (Table 8). After achieving a positive insight of the conditional correlation, we tested for the presence of dynamic conditional correlation using Tse's (2000) Lagrange Multiplier (LM) test. The χ^2 statistic (27.52) was rejected at 5% level of significance, confirming the presence of dynamic properties of the series. Further, we modelled the series using DCC model with multivariate normal distribution. The results of the DCC model were encouraging as all the parameters were found to be significant at 5% level of significance (Table 9). Further, to incorporate the leptokurtic effect DCC model with multivariate Student-t distribution was fitted to the series. The results obtained here were also encouraging and in line with that of DCC model with multivariate normal distribution. Along with it the shape parameter (γ) of the Student-t distribution was also found to be significant at 1% level of significance (Table 10). The superiority of DCC-MARCH model with Student-t distribution to model leptokurtic pulses price series goes with the findings in literature (Ku 2008).

4. DISCUSSIONS

The two series under consideration were modelled using the VAR-MGARCH approach. Order of the VAR model was determined with the help of AIC and SIC criterion in which VAR (2) model was found to have the lowest values 16.16 and 16.29 respectively. The diagnostic checking after fitting of VAR model clearly indicated the need for further use of MGARCH model to have a clear understanding of the series behaviour. We started by fitting the MGARCH-BEKK model to the residuals series to understand the pattern of volatility transmission among them. The results obtained clearly indicated the transmission of volatility from series *a* to series *b* (-0.40) and from *b* to *a* (0.50). The value of -0.40 can be interpreted as the transmission of persistent negative impact on series *b* due to the presence of volatility in series *a*. In a similar manner the value 0.50 is the positive impact that series *a* has on series *b* in terms of transmission of volatility between them.

But, these effects are lower than their individual effects on their past shocks. The conditional variance graph obtained after fitting of the MGARCH-BEKK model shows a large amount of variability toward the end. This goes with the original series which too is more volatile at the latter half of the series, indicating its adequacy to capture volatility. Although, the BEKK model is adequate to study the effect of volatility transmission, it becomes necessary to explore the possibility of conditional correlation that might exist in the structure of conditional variance. In order to have an insight in this area, we have further used the MGARCH-CCC and MGARCH-DCC models in the residual series. Results of the CCC model clearly suggested the presence of conditional correlation in the conditional variance exhibited by the series. The magnitude of constant conditional correlation being 0.52, whereas the dynamic conditional correlation obtained from DCC multivariate normal model is 0.24 for both ARCH and GARCH effects and 0.23 and 0.35 respectively for ARCH and GARCH effects from DCC multivariate student-t model.

5. CONCLUSION

In this paper the performance of MGARCH models namely BEKK and DCC have been studied using monthly agricultural commodity price indices. The DCC model was further studied with multivariate normal and student-t distribution of error to incorporate the leptokurtic behaviour of the series. For both the series, the VAR (2)-MGARCH (1,1) was found to be suitable for modelling due to its low AIC and SIC values. The superiority of the DCC-MGARCH model for modelling the series is highlighted by the low AIC and SIC values and by the presence of dynamic conditional correlation in the series (Fig. 4) than by the corresponding BEKK-MAGARCH model. The DCC-MGARCH model with Student-t distribution was found to be superior for modelling than the DCC-MGARCH model due to the leptokurtic behaviour of the series. The methodology employed in this paper can also be used for modelling other multivariate agricultural time-series data exhibiting transmission of volatility among them.

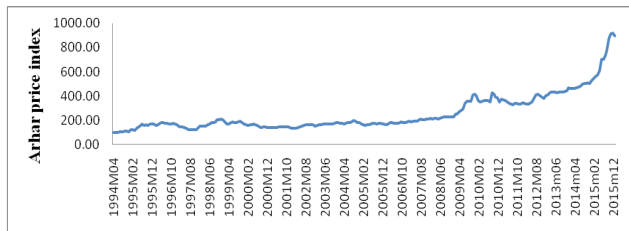


Fig. 1. Time plot of arhar price index

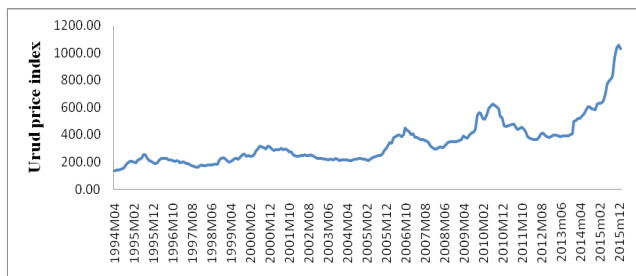


Fig. 2. Time plot of urud price index

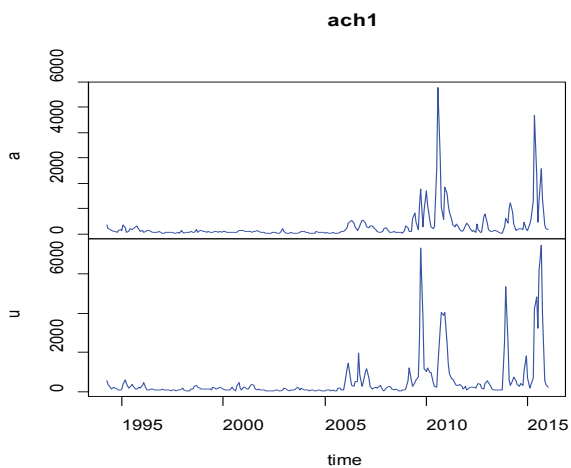


Fig. 3. Conditional variance of arhar (a) and urud(u) after fitting MGARCH-BEKK model

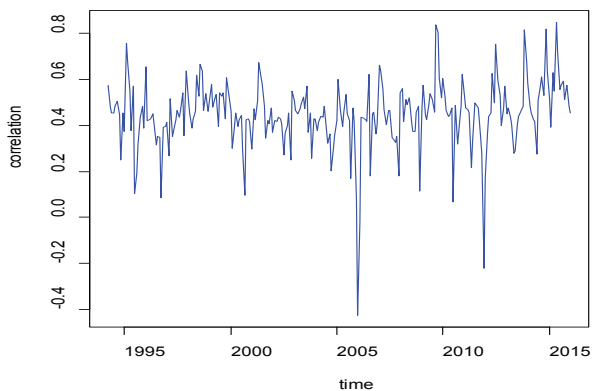


Fig. 4. Dynamic correlation between arhar and urud

Table 1. Descriptive statistics

| | Arhar | Urud |
|-------------|--------|---------|
| Mean | 255.03 | 336.96 |
| Median | 181.40 | 293.80 |
| Maximum | 916.77 | 1059.01 |
| Minimum | 100.30 | 134.10 |
| Std. Dev. | 152.36 | 163.24 |
| Skewness | 1.91 | 1.72 |
| Kurtosis | 7.19 | 6.87 |
| Jarque-Bera | 351.79 | 292.82 |
| Probability | <0.001 | <0.001 |
| C.V (%) | 59.74 | 48.44 |
| Correlation | 0.92 | |

Table 2. Likelihood ratio tests for cointegration

| Test | Null Hypothesis | Eigen Value | Test Statistic | P value |
|---------------------|-----------------|-------------|----------------|---------|
| Trace | None | 0.03 | 15.42 | 0.53 |
| | At most 1 | 0.02 | 5.61 | 0.51 |
| Maximum Eigen Value | None | 0.03 | 9.81 | 0.63 |
| | At most 1 | 0.02 | 5.61 | 0.51 |

Table 3. MARCH test for the series

| Lags | Q value | P value |
|------|---------|---------|
| 1 | 0.01 | <0.0001 |
| 2 | 0.68 | <0.0001 |
| 3 | 10.27 | <0.0001 |
| 4 | 14.92 | <0.0001 |
| 5 | 28.90 | <0.0001 |
| 6 | 34.75 | <0.0001 |

Table 4. Test for stationarity

| Series | ADF test (lag 12) | P value |
|-------------------|-------------------|---------|
| Arhar price index | Level | 0.20 |
| | Differenced | 12.69 |
| Urud price index | Level | 1.48 |
| | Differenced | 12.09 |

Table 5. GARCH model estimates of individual series

| Series | Model | a_0 | a_i | b_j | $a_i + b_j$ |
|--------|-------------|----------------|----------------|----------------|-------------|
| Arhar | GARCH (1,1) | 1.75 (0.61) | 0.14 (0.03) | 0.81 (0.01) | 0.95 |
| Urud | GARCH (1,1) | 8.59 (7.48) | 0.15 (0.07) | 0.84 (0.07) | 0.99 |

*Values in the parenthesis are Standard Errors

Table 6. Estimates of VAR(2) model

| Series | Arhar | Urud |
|------------|------------------|-----------------|
| Arhar (-1) | 1.23 (0.07) | 0.20 (0.10) |
| Arhar (-2) | -0.21 (0.079) | -0.15 (0.10) |
| Urud (-1) | 0.001 (0.05) | 1.25 (0.07) |
| Urud (-2) | -0.004 (0.05) | -0.29 (0.07) |
| C | -2.58 (2.05) | 0.75 (2.68) |

*Values in the parenthesis are Standard Errors

Table 7. MGARCH-BEKK model estimates

| Coefficients | Estimate | Std. Error | t value | P value |
|--------------|----------|------------|---------|---------|
| C11 | 4.03 | 1.33 | 3.01 | <0.001 |
| C21 | 6.05 | 2.20 | 2.74 | <0.001 |
| C22 | 3.89 | 2.36 | 1.64 | <0.001 |
| A11 | 0.54 | 0.08 | 6.69 | <0.001 |
| A21 | 0.10 | 0.07 | 1.31 | 0.190 |
| A12 | 0.05 | 0.05 | 1.13 | 0.254 |
| A22 | 0.58 | 0.07 | 7.65 | <0.001 |
| B11 | 0.89 | 0.10 | 8.70 | <0.001 |
| B21 | 0.50 | 0.09 | 5.25 | <0.001 |
| B12 | -0.40 | 0.04 | 10.22 | <0.001 |
| B22 | 0.48 | 0.05 | 9.41 | <0.001 |

Table 8. MGARCH-CCC model estimates

| Coefficients | Estimate | Std. Error | t value | P value |
|--------------|----------|------------|---------|---------|
| C11 | 14.16 | 4.85 | 2.92 | 0.003 |
| A11 | 0.72 | 0.18 | 3.90 | <0.001 |
| B11 | 0.40 | 0.10 | 3.79 | <0.001 |
| C22 | 60.24 | 13.64 | 4.42 | <0.001 |
| A22 | 1.47 | 0.26 | 5.56 | <0.001 |
| B22 | 0.12 | 0.05 | 2.22 | 0.027 |
| CCC | 0.52 | 0.04 | 10.65 | <0.001 |

Table 9. Estimates of MGARCH-DCC model with multivariate normal distribution

| Coefficients | Estimate | Std. Error | t value | P value |
|---------------|----------|------------|---------|---------|
| C1 | 0.15 | 0.47 | 0.32 | 0.743 |
| ω_1 | 3.88 | 3.73 | 1.03 | 0.299 |
| A11 | 0.22 | 0.06 | 3.68 | <0.001 |
| B11 | 0.77 | 0.08 | 9.72 | <0.001 |
| C2 | 1.01 | 0.80 | 1.24 | 0.212 |
| ω_2 | 81.08 | 49.33 | 1.64 | 0.100 |
| A22 | 0.66 | 0.18 | 3.66 | <0.001 |
| B22 | 0.32 | 0.14 | 2.27 | 0.022 |
| δ_{12} | 0.24 | 0.08 | 2.72 | 0.006 |
| δ_{21} | 0.24 | 0.11 | 2.12 | 0.033 |

Table 10. Estimates of MGARCH-DCC model with student-t distribution

| Coefficients | Estimate | Std. Error | t value | P value |
|---------------|----------|------------|---------|---------|
| C1 | 0.15 | 0.46 | 0.32 | 0.743 |
| ω_1 | 3.88 | 3.75 | 1.03 | 0.301 |
| A11 | 0.22 | 0.06 | 3.66 | <0.001 |
| B11 | 0.77 | 0.07 | 9.73 | <0.001 |
| C2 | 1.01 | 0.81 | 1.24 | 0.214 |
| ω_2 | 81.08 | 48.55 | 1.66 | 0.094 |
| A22 | 0.66 | 0.18 | 3.65 | <0.001 |
| B22 | 0.32 | 0.13 | 2.38 | 0.017 |
| δ_{12} | 0.23 | 0.08 | 2.73 | 0.006 |
| δ_{21} | 0.35 | 0.13 | 2.59 | 0.009 |
| γ | 4.32 | 0.47 | 9.07 | <0.001 |

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