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Factorial Experiments with Minimum Changes in Run Sequences*

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SUMMARY

Randomization of run sequences in factorial experiments may result in large number of changes in factor levels which will make the experimentation expensive, time-consuming and difficult. Experiments, in which it is difficult to change the levels of factor(s), use of run sequences with minimum changes may often be preferable to a random run sequence. Here, we have developed a general method to obtain symmetric factorial with minimum changes in run sequence along with general expression of factor-wise number of level changes. Further, minimally changed run sequences to develop half replicate of 2-level factorial experiments and 2-level factorial in which only highest order factorial effect get confounded, have also been obtained. For providing readymade solution to the end users, SAS macro have been developed for generating symmetric factorial and half replicate of 2-level factorial with minimum changes in run sequence along with its parameters.

Keywords: Confounding, Factorial, Fractional factorial, Macro, Minimal change, Randomization, Run sequence.

1. INTRODUCTION

Factorial experiments are experiments wherein two or more factors each at two or more levels are used simultaneously. Such an experiment allows studying the effect of each factor as well as the effects of interactions between factors on the response variable. These experiments may be either complete or incomplete. If the number of combinations in a full factorial design is too high to be logistically feasible, a fractional factorial design may be done, in which some of the possible combinations are omitted. Both full factorial and fractional factorial experiments find profound applications in agricultural and industrial experiments. It is always advisable that the order of execution of factorial and fractional factorial designs should be random as randomization of run sequences can avoid bias in the estimates of the effects of interest which might result from a time trend. However, under factorial experiment setup, randomization can induce a large number of changes in factor levels which will ultimately make the experimentation expensive, time-consuming and difficult. The number of level changes is of serious concern to experimenters in many agricultural, post-harvest and processing, engineering and industrial experiments as in such experiments one may come across some situations where it is physically very difficult to change levels of some factors. The recommended practice under such situations where it is difficult or near impossible to change the level of a particular factor, is to fix the level of that particular factor and randomize the levels (combination of levels) of other factor (s)

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accordingly. Although, well adopted in many experimental situations where it is physically near impossible to change the level of only one of the factor, however, this approach suffers from the potential disadvantages under the circumstances where minimum number of changes in the level is required for more than one factor. Since here randomization is performed on the levels or combination of levels of remaining factors, the probability is very small for getting a run sequence where total number of level changes is minimum. So, in these situations, instead of adopting the above traditional approach, one should construct run sequences where total number of level changes is minimum.

Experimental Situation 1: In an experiment of soil microbial diversity (community level physiological profiling) through BIOLOG ecoplates with the purpose of identifying the best treatment, three factors have been tried viz. (i) CO_2 [Two levels: Elevated CO_2 and Ambient CO_2], (ii) Fertilizer (Two levels: Organic and Inorganic) and (iii) Variety (Four different wheat varieties). Here, the levels of factor fertilizer can easily be changed from organic to inorganic, but changing the levels of other two factors are costly. Therefore, if randomization done on the level combination, it will increase the cost of the experiment to a great extend since the experiment involved two hard-to-change factors. Hence, use of factorial design where the number of factor level changes is kept small, is an alternative.

Experimental Situation 2: [Joiner and Campbell, 1976]: For designing an experiment to evaluate the sensitivity of spectrophotometer using a “ruggedness test” factors to be examined are:

a. Lamp position (one second to change factor levels)
b. Burner position (60 seconds to change)
c. Burner height (one second to change)
d. Type of flame (60 seconds to change)
e. Flow rate (120 seconds to change)

The measurements were believed to drift linearly with time due to carbon build-up. In addition, it is necessary to stop the measurements and remove all of the built up carbon after about every 20 measurements. The design problem then reduced to finding an approximately optimal time order, bearing in mind the approximately linear time drifts in blocks of about 20 measurements, and the differential costs of changing factor levels. It is thus desirable to execute the experiment in such an order that the number of factor level changes is kept small.

The search for experimentation run orders that offer the minimum number of factor level changes and at the same time minimize the possible influence of undesirable factors is not new. Cox (1951) began the study of systematic designs for the replicated varietal trials. A lot of work is available in literature which deals with the criteria of number of factor level changes in factorial experiments [for example Draper and Stoneman (1968), Dickinson (1974), Cheng *et al.* (1998), De León *et al.* (2005), Correa *et al.* (2009), Correa *et al.* (2012) etc.].

Majority of the work are based on two level factorial experiments and no methods seems to be available in literature to obtain a general method for the development of run sequences in case of symmetric factorial experiments with minimum changes in factor levels. Further, no methods seems to available in literature which describes the case of confounded factorial with minimum number of level changes in run sequences even with 2-levels. In this article, a general method irrespective of the number of levels for obtaining minimally changed run sequences in symmetric factorial experiments has been discussed. The general expression has been obtained for calculating the factor wise number of level changes in the developed minimally changed run sequences. Further, we have also obtained half replicate of 2-level factorial experiments with minimum level changes in factor levels based on method of generator. Keeping the importance of confounding in mind, a method has been obtained to develop the blocks of 2-level confounded factorial with only one (highest order interaction effect) confounded effect. For providing a readymade solution to the end users, SAS macros for the generation of the minimally changed run sequences in case of symmetric factorial and also for the generation of half replicate of 2-level factorial with minimum changes in run sequences have been developed. The run sequences with minimum changes in factorial and fractional factorial experiments may not be unique and it is possible to obtain a number of run sequences for a particular factorial and fractional factorial experiment where total number of changes in the levels of factors is minimum. Therefore, an exhaustive search has also been implemented through SAS to obtain all such run sequences.

2. SYMMETRIC FACTORIAL WITH MINIMUM CHANGES IN RUN SEQUENCES

2.1 Conditions for a Minimally Changed Run Sequences in Symmetric Factorial

The number of changes that is necessary to perform on the factor levels has significant impact on effort and costs of running an experiment specially when a great amount of effort is required in changing the levels of a factor (e.g. change of mould), or when it is necessary to wait a certain amount of time (e.g. change of temperatures), or for some other reasons (e.g. cleaning the reactor). Therefore, the criteria for the number of level changes for factors are possibly one of the most important elements to consider when taking on an experimentation process.

Run Sequences	**Factor**		
	A	**B**	**C**
1	-1	-1	-1
2	1	-1	-1
3	-1	1	-1
4	1	1	-1
5	-1	-1	1
6	1	-1	1
7	-1	1	1
8	1	1	1
Factor wise number of level changes	7	3	1

Experiments in which it is difficult to change the levels of factors, one possible alternative may be the adaptation of factorial experiments with minimum number of change in run sequences. For example, consider the following 2^3 factorial run sequences in standard order:

Here, the total number of factor level changes is 11. By rearranging the run sequences of standard order, following run sequences for a 2^3 factorial can be obtained where the total number of factor level changes is 7 which is the minimum possible for a 2^3 factorial:

Run sequences	**Factor**		
	A	**B**	**C**
1	-1	-1	-1
2	1	-1	-1
6	1	-1	1
8	1	1	1
4	1	1	-1
3	-1	1	-1
7	-1	1	1
5	-1	-1	1
Factor wise number of level changes	2	2	3

The total number of factor level changes will be minimum if the level of only one factor is changed on two consecutive experimental trials, or equivalently when only one sign is changed by passing from one row to the following in the design matrix. Therefore, in a s^k factorial [where k is the number of factors and s is the level for all the factor] experiment with minimum changes in the run sequences, the total number of level changes will be $s^k - 1$.

2.2 General Method of Construction

Here, a method for constructing symmetric factorial with minimum level changes in run sequences has been discussed. Let, there are k factors such that the levels of all the factor is constant (say s). Further, let the levels of a particular factor are denoted by

$-s/2, -(s-2)/2, -(s-4)/2, \ldots, -[s-(s-2)]/2, [s-(s-2)]/2, \ldots, (s-4)/2, (s-2)/2, s/2$ for even s and

$-(s-1)/2, -(s-3)/2, -(s-5)/2, \ldots, -[s-(s-2)]/2, 0, [s-(s-2)]/2, \ldots, (s-5)/2, (s-3)/2, (s-1)/2$ for odd s.

The method starts with a single factor with s levels denoted as defined above, forming s rows and a single column for a factor. Then in order to add the next factor, repeat each row consecutively (s-1) more number of times resulting in S^2 rows and single column. Now for the additional factor, add a new column starting with entries add a new column starting with entries $- s / 2$, $- (s -2)/2$, $- (s -4)/2,\ldots, - [s - (s -2)]/2$, $[s - (s -2)]/2,\ldots, (s - 4)/2, (s -2)/2, s/2$ for even s and $- (s -1)/2, - (s -3)/2, - (s -5)/2,\ldots, - [s - (s -2)]/2, 0, [s -(s -2)]/2,\ldots, (s -5)/2, (s -3)/2, (s - 1)/2$ for odd s and then taking its fold over. This will give rise to s^2 factorial with minimum changes in run sequences with total number of change as S^2 -1 which is minimum for the s^2 factorial experiments.

Similarly for s^3 factorial, consider the above s^2 factorial with minimum changes in run sequences. By following the same procedure as mentioned above, the last column i.e. third factor is added by taking the fold over till the last entry of the column. This procedure can be extended for s^k factorial [where total number of changes will be s^k-1] by starting with a s^{k-1} factorial with minimum changes in run sequences. For this minimally changed run sequences, the general expression for factor wise number of level changes have also been obtained. For i^{th} factor (for i = 1, 2,..., k), the expression has been obtained as (s-1) s^{i-1}. Following is an example:

Example 1: To construct a 3^3 factorial with minimum changes in run sequences, start with a single factor with three levels denoted by -1, 0 and 1, forming three rows and a single column. Therefore, repeat each row of the initial array consecutively three more number of times. Add another column for second factor with three levels starting with entries -1,0,1 and then taking its fold over. The resultant run sequences will produce a 3^2 run sequences with minimum changes (where total number of changes will be 8) as follows:

3^2 Factorial with Minimally Changed Run Sequences	
-1	-1
-1	0
-1	1
0	1
0	0
0	-1
1	-1
1	0
1	1
Factor wise Number of Level Changes	
2	**6**

Now, in order to obtain a 3^3 factorial with minimum changes in run sequences, we can start with the above run sequences for a 3^2 factorial by repeating each row of the same consecutively two more number of times. The levels of last factor can now be added by a column starting with entries -1,0,1 and then taking its fold over. The process will continue till the last entry of the column as a result we can get a 3^3 factorial with minimum changes in run sequences (where total number of changes will be 26) as follows:

3^3 Factorial with Minimally Changed Run Sequences		
-1	-1	-1
-1	-1	0
-1	-1	1
-1	0	1
-1	0	0
-1	0	-1
-1	1	-1
-1	1	0
-1	1	1
0	1	1
0	1	0
0	1	-1
0	0	-1
0	0	0
0	0	1
0	-1	1
0	-1	0
0	-1	-1
1	-1	-1
1	-1	0
1	-1	1
1	0	1
1	0	0
1	0	-1
1	1	-1
1	1	0
1	1	1
Factor wise Number of Level Changes		
2	**6**	**18**

Example 2: To construct a 4^3 factorial with minimum changes in run sequences, we can start with the minimally changed run sequences for a 4^2 factorial following the method as discussed in Section 2.2. Now, by repeating each row of the same consecutively three more number of times and then adding a new column starting with entries -2,-1,1,2 and then taking its fold over, we can get a 4^3 factorial with minimum changes in run sequences (where total number of changes will be 63). The factor wise number of level changes in case of the developed 4^3 factorial will be as follows:

Factor Wise Number of Level changes for 4^3 Factorial with Minimum Changes in Run Sequences		
3	12	48

It can be verified that, in all the cases, a specific pattern can be observed in factor wise number of changes which can be easily obtained based on the general expression for the same as discussed earlier in Section 2.2.

2.3 Time Count Effect: New *vs.* Original Factorial with Minimally Changed Run Sequences

In literature, in order to measure the influence of factors alien to the experimentation, authors like Draper and Stoneman (1968), Dickinson (1974), Correa *et al.* (2009) and others have repeatedly used one term known as ***time count***. A general relationship between the time count effects of a 2^{k+1} design with minimum number of level changes obtained based on a 2^k design with minimum number of level changes has been established by Correa *et al.* (2009).

Here, the relationship has been generalized between time count effects of a higher order symmetric factorial with minimum changes in run sequences and that of a lower order factorial minimum changes in run sequences based on which, the higher order factorial with minimum changes in run sequences have been obtained.

Let, a s^k factorial with minimum changes in run sequences has been obtained based on the proposed method as describes in Section 2.2. Therefore, the design matrix of the s^k factorial with minimum changes in run sequences (except the last added factor) is nothing but the design matrix of s^{k-1} factorial with minimum changes in run sequences except each row of s^k factorial with minimum changes in run sequences has been repeated s-1 more number of times in the final array. Thus, the relationship between time count value for any effect of a s^{k-1} factorial with minimum changes in run sequences and the same effect in a s^k factorial with minimum changes in run sequences obtained from the first one, by following the described procedure in Section 2.2, can be expressed as

$$\sum_{u=1}^{F} S_u\, u \sum_{u=1}^{F} S_u \left[us^2 - \frac{s(s-1)}{2}\right] \qquad (1)$$

where, $s^{k-1} = F$, S_u represents the value in the u^{th} row corresponding to the column of the calculated effect. As $\sum_u S_u = 0$, the time count value for any effect of a s^k factorial with minimum changes in run sequences is always S^2 times the value of the same effect in a S^{k-1} factorial with minimum changes in run sequences obtained from the first one since

$$\sum_{u=1}^{F} S_u \left[us^2 - \frac{s(s-1)}{2}\right]$$

$$= S^2 \sum_{u=1}^{F} S_u\, u - \frac{s(s-1)}{2} \sum_{u=1}^{F} S_u = S^2 \sum_{u=1}^{F} S_u\, u$$

Example 3: Consider the Example 1 where we have constructed a 3^3 factorial with minimum changes in run sequences using a 3^2 factorial with minimum changes in run sequences. Table 1 shows the time count value for any effect of a 3^2 factorial with minimum changes in run sequences and the same effect in a 3^3 factorial with minimum changes in run sequences obtained from the first one by repeating each row of the 3^2 factorial with minimum changes in run sequences consecutively two more number of times.

Table 1. Time Count for an Effect of the Initial 3^2 Factorial with Minimum Changes in Run Sequences and the same Effect of the 3^3 Factorial with Minimum Changes in Run Sequences Design Obtained from it

Time Count Value for 3^2 Factorial with Minimum Changes in Run Sequences	**Time Count Value for 3^2 Factorial with Minimum Changes in Run Sequences (Excluding the Last Factor)**
$S_1.1$	$S_1.(1+2+3) = S_1.6 = S_1(9\times1-3)$
$S_2.2$	$S_2.(4+5+6) = S_2.15 = S_1(9\times2-3)$
$S_3.3$	$S_3.(7+8+9) = S_3.24 = S_1(9\times3-3)$
$S_4.4$	$S_4.(10+11+12) = S_4.33 = S_1(9\times4-3)$
$S_5.5$	$S_5.(13+14+15) = S_5.42 = S_1(9\times5-3)$
$S_6.6$	$S_6.(16+17+18) = S_6.51 = S_1(9\times6-3)$
$S_7.7$	$S_7.(19+20+21) = S_6.60 = S_1(9\times7-3)$
$S_8.8$	$S_8.(22+23+24) = S_6.69 = S_1(9\times8-3)$
$S_9.9$	$S_9.(25+26+27) = S_6.78 = S_1(9\times9-3)$
$\sum_{u}^{9} S_u\ u$	$\sum_{u}^{9} S_u\ [3^2 u - 3]$

2.4 Time Count Value for the Added Factor

Time count effect for the added factor i.e. for the k^{th} factor in a 3^3 factorial with minimum changes in run sequences obtained from 3^2 factorial with minimum changes in run sequences, can be calculated based on whether the levels are even or odd. In case of even number of levels, the time count for the added factor will always be zero whereas for odd number of levels, the time count value for the added factor will follow a complex pattern. The final expression is as follows:

Time count for the added factor = $\frac{s(s-3)+(s+1)}{2}$ + (time count value of the factor with levels as odd number which just precede s) (2)

Remark 1: It is to be noted here that, for 3^k factorial with minimum changes in run sequences obtained from 3^{k-1} factorial with minimum changes in run sequences, time count value for the last factor is always 2. This time count value can be obtained by putting, s = 3 in the first part of Equation 2 i.e. in $\frac{s(s-3)+(s+1)}{2}$ [as, 3 is the first odd number in the set of odd numbers and no other odd number precede 3].

Example 4: Let us consider the Example 1 where a 3^3 factorial with minimum changes in run sequences was obtained from a 3^2 factorial with minimum changes in run sequences. Here the time count for the added factor can be calculated as

Column Representing the Last Factor in the Developed 3^3 Factorial with Minimum Changes in Run Sequences	**Row Number**	**Time Count Value for the Last Factor in the Developed 3^3 Factorial with Minimum Changes in Run Sequences (Coulum^1 × Column^2)**
-1	1	-1
0	2	0
1	3	3
1	4	4
0	5	0
-1	6	-6
-1	7	-7
0	8	0
1	9	9
1	10	10
0	11	0
-1	12	-12
-1	13	-13
0	14	0
1	15	15
1	16	16
0	17	0
-1	18	-18
-1	19	-19
0	20	0
1	21	21
1	22	22
0	23	0
-1	24	-24
-1	25	-25
0	26	0
1	27	27
Total		**2**

The result can be verified with Remark 1.

Example 5: Let a 5^3 factorial with minimum changes in run sequences is obtained based on 5^2 factorial with minimum changes in run sequences following the methodology as described in Section 2.2. Therefore, in this case, from Equation (2), the time count value for the last factor will be 8+2 [where 2 is the time count value of last factor if we consider a 3^k factorial with minimum changes in run sequences as 3 is the odd number which just precede 5].

3. HALF REPLICATE OF 2-LEVEL FACTORIAL WITH MINIMUM CHANGES IN RUN SEQUENCES

In fractional factorial designs, factors that appear in the design matrix can be viewed as divided into two groups:

a. Factors that accommodate to a full matrix (e.g., in 2^{4-1}, any three factors can be accommodated to a full 2^3 matrix), and

b. Factors that are generated on the basis of interactions with the former.

Let NC(X) and NC(Y) be the number of changes corresponding to factors X and Y respectively such that the design matrix represents a minimally changed run sequence. If the new factor is generated based on the interaction of XY, then, the number of changes in this factor will be the sum of the number of changes of individual factor used in generation of the concerned factor i.e. NC(XY) = NC(X) + NC(Y). This property is valid for any complete matrix with the minimum number of changes. This is so because if the sign of column X changes from one row to the next, then the same will happen to column XY, since the complete matrix is restricted to only one change when moving from one row to the next. This property is valid for any complete matrix with the minimum number of changes. For the same reason, if a factor is generated by the interaction of three or more factors with a minimum number of changes in the design matrix, the number of changes in the new factor will be equal to the sum of the number of changes of the factors that intervene in the interaction with which it is aliased. Here, we have discussed the construction of half replicate of 2-level factorial with minimum level changes. Let, there are k factors each at 2 level. In order to construct half replicate of 2^k factorial with minimum number of changes, where the identity group of contrast will be the highest order interaction effect, the following steps are required:

Step I: Construct a $2^{k\text{-}1}$ factorial with minimum changes in run sequences as describe in Section 2.2.

Step II: Generate a new factor by taking the product of all the k-1 factor in the developed $2^{k\text{-}1}$ factorial with minimum changes in run sequences.

The resultant run sequences will be a half replicate of 2^k factorial with minimum number of changes, where the identity group of contrast will be the highest order interaction effect. The number of changes for the last factor will be the sum of number of changes of all the individual factor. Following is a minimally changes run sequences for $\frac{1}{2}\left(2^4\right)$ with identity group of contrast as I = ABC and total number of changes as 14. It is a resolution III plan.

A	B	C	D=ABC
-1	-1	-1	-1
-1	-1	1	1
-1	1	1	-1
-1	1	-1	1
1	1	-1	-1
1	1	1	1
1	-1	1	-1
1	-1	-1	1
Factor wise Number of Level Changes			
1	**2**	**4**	**7**

Here, first three columns represents the 2^3 design with minimum changes in run sequences obtained based on the method as discussed in Section 2.2. The remaining column has been generated by method of generator. It can be observed that the number of changes in the new columns is the sum of the number of changes of the factors that intervene in the interaction with which it is aliased.

4. CONFOUNDING IN 2-LEVEL FACTORIAL EXPERIMENTS WITH MINIMUM LEVEL CHANGES IN RUN SEQUENCES

When number of factors and levels of each factors involved in the experiments, became large, it became difficult to maintain the homogeneity within each block. In such situations, it is advisable to use the technique of confounding in factorial experiments to maintain the homogeneity within each block. Confounding in factorial experiments is a technique of reducing block size by making one or more interaction contrasts (preferably higher order interaction) identical with block contrast. No literature is available which deals with the technique of confounding in factorial design with minimum level changes. Here, we have obtain a general method to construct a (2^k, 2^{k-1}) confounded factorial with minimum level changes in two blocks. The method will give rise a confounded factorial with highest order interaction effect as the confounded effect. The steps involved are as follows:

Step I: Construct a 2^{k-1} factorial with minimum changes in run sequences as described in Section 2.2.

Step II: The levels of last factor will be added alternatively starting from -1 till the last entry of the column.

The result will give rise a key block of a 2^k confounded factorial with minimum level changes in 2^{k-1} number of plots per block where the confounded effect will be the highest order interaction. The number of change of the last factor will be 2^{k-1} -1. The remaining block can then be easily generated by multiplying the last column of the key block with -1. Here, the number of factor wise level changes and total number of changes will be same in both the block.

Example 6: In order to construct a (2^3, 2^2) confounded factorial with minimum level changes, first one need to construct a 2^2 factorial with minimum level changes. Therefore, the levels of last factor will be added alternatively staring from -1 in order to get the key block. The remaining block can be generated by multiplying the last column with -1. Following are the two blocks of the developed (2^3, 2^2) confounded factorial with minimum level changes [where total number of changes will be 6 for both the blocks]:

Block I (Key Block)		
-1	-1	-1
-1	1	1
1	1	-1
1	-1	1
Factor wise Number of Level Changes		
1	2	3

Block II		
-1	-1	1
-1	1	-1
1	1	1
1	-1	-1
Factor wise Umber of Level Changes		
1	2	3

Example 7: Following are the two blocks of a (2^4, 2^3) confounded factorial with minimum level changes constructed based on the above method [where total number of changes will be 14 for both the blocks]:

Block I (Key Block)			
-1	-1	-1	-1
-1	-1	1	1
-1	1	1	-1
-1	1	-1	1
1	1	-1	-1
1	1	1	1
1	-1	1	-1
1	-1	-1	1
Factor wise Number of Level Changes			
1	2	4	7

Block II			
-1	-1	-1	1
-1	-1	1	-1
-1	1	1	1
-1	1	-1	-1
1	1	-1	1
1	1	1	-1
1	-1	1	1
1	-1	-1	-1
Factor wise Number of Level Changes			
1	2	4	7

5. GENERATION OF MINIMALLY CHANGED RUN SEQUENCES

The method of construction of run sequences where total number of change in factors level is minimum will involve theoretical understanding and it may not be very easy for the end users to understand the theoretical logic behind the construction methods. So, considering this, user friendly SAS macros have been developed for generation of symmetric factorial and half replicate of 2^k factorial with minimum level changes in run sequences respective.

In case of SAS macro for generation of symmetric factorial with minimum level changes, user only need to enter the levels of factors each separated by comma. If user run the macro after entering the levels separated by commas, the SAS Macro will generate the layout under the heading Minimally changed run sequences for factorial experiment. Along with the design, the macro will also generate Factor-wise number of changes in the run sequence and Total number of changes in the run sequence under two different arrays. If user run the macro after entering the levels of factors without separation by comma, the macro will display a message as Enter the number of levels each factors separated by comma. Further, in case of any level less than two, the macro will display the message as The level of each factor should be >=2. Once user run the macro, every time the SAS macro would also generate a word file containing the output. User can then save the word file. The macro is already available in public domain at http://www.iasri.res.in/sscnars/sftsmcrs aspx.]. The output of SAS macro for generation of 4^2 factorial with minimum level changes has been given in **ANNEXURE-IA**.

In case of SAS macro for generation of half replicate of 2^k factorial with minimum level changes, user only need to enter the value of *k*. Once user run the macro after entering the value of *k*, the SAS Macro will generate the layout under the heading Minimally changed run sequences for half replicate of 2-level factorial experiment along with Factor-wise number of changes in the run sequence and Total number of changes in the run sequence respectively under two different arrays. The output of SAS macro for generation of half replicate of 2^5 factorial with minimum level changes has been given in **ANNEXURE-IB**.

6. EXHAUSTIVE SEARCH TO GENERATE ALL POSSIBLE MINIMALLY CHANGED RUN SEQUENCES

Factorial experiments in which number of factors level changes are required to be minimum, it may be the case that there may be more than one run sequence with minimum number of change i.e., Minimally changed run sequences in factorial experiments is not unique. Considering this, here, we have developed an exhaustive search procedure to generate all possible minimally changed run sequences in factorial experiments. This algorithm would be helpful to the experimenter for randomly identifying any minimally changed run sequences out of all possible run sequences.

The procedure starts with entering the levels of all the factors each separated by commas in case of symmetric factorial. Once the levels are entered, the algorithm generates a factorial design in lexicographic order. Based on the initial design, the algorithm generates minimally changed run sequences for the corresponding factorial by performing all possible row permutation of the initial array keeping in mind that only one factor is allowed to change in level between any pair of adjacent runs. Along with the design, the algorithm also generates factor wise number of level changes separately for all the minimally changed run sequences obtained based on the algorithm. A flow diagram highlighting all the steps involved in the algorithm to systematically generate all possible run sequences in factorial experiments with minimum changes is illustrated in **ANNEXURE-II A**.

The algorithm has been implemented by writing programme in SAS 9.3. The programme has been executed for three different cases of symmetric factorial viz. 2^2, 2^3, and 3^2 and two different cases of fractional factorial viz. $\frac{1}{2}\left(2^3\right)$ and $\frac{1}{2}\left(2^4\right)$ respectively to obtain all possible run sequences with minimum changes. Table 2 shows the total number of possible minimally changed run sequences for all the above cases:

Table 2. All Possible Minimally Changed Run Sequences

Factorial Experiment	**Total Possible Number of Minimally Changed Run Sequences based on Exhaustive Search Algorithm**
22	8
23	144
32	1512
$\frac{1}{2}\left(2^3\right)$	24
$\frac{1}{2}\left(2^4\right)$	13824

A complete list of all the minimally changed run sequences for 2^2 and $\frac{1}{2}$ (2^3) factorial with minimum level changes are given in **ANNEXURE-IIB** and **ANNEXURE-IIA.**

Remark 2: It should be noted that, the approach where minimum number of changes in the level is required for more than one factor, may be viewed as a split-plot design taking the hard-to-change factor in the main plot. In case of split plot designs the main interest of the experimenter lies in the more precise estimation of sub plot treatment effects along with the main plot × sup plot treatment effects. However, in agricultural and allied experiments, many situations may arise where experimenter wants to estimate all the effects with equal precision even though there may be some factors for which it is difficult to change the levels. For example, in an agronomical trial involving three parameters viz. temperature, relative humidity and fertilizer. Here, the levels of fertilizer can be easily changed whereas temperature and relative humidity remains hard to change factors. The experimenter wants to study the effects of all the factors with equal precision. These makes the present research practically distinguishable from split plot designs as here the interest lies in the estimation of all the effects with equal precision. So, the design obtained under the present investigation can serve the dual purpose of accommodating hard-to-change factors and equal precision for all the effects in the same experiments which cannot be possible for split plot designs.

7. POSSIBLE ANALYSIS

Use of minimally changed run sequences may be useful for factorial experiments where it is difficult to change the levels of factors. However, the analysis remains a concern due to lack of randomization of run sequences. One possibility in this direction is to use randomization tests to detect significance of factor levels, but the total number of allowable randomizations is likely to be very small. However, another alternative may be the use of an Analysis of Covariance (ANCOVA) type of models with influence of unknown factor like time trend as a covariate. A restricted randomization approach may be used to select one run sequences with minimum changes by generating all possible run sequences of a given factorial using the exhaustive search. It should be noted that the split plot model for analysis should not be used for this situation as here the experimenter require equal precision for all the effects.

8. DISCUSSION

Factorial experiments where it is difficult to change the levels of factors, run sequences with minimum changes may be useful as compare to a random run sequences since randomization of run sequences will increase the number of level changes among the factors and hence increase the cost and time of the experiment. The run sequences with minimum changes obtained in the present article is useful in many agricultural, post-harvest and processing, engineering and industrial experiments whenever the process of making factor level changes is time consuming and expensive or the system required more time to return to steady state following a change in factor level. However, there is a need to obtain the efficiency of the developed minimally changed run sequences as compare to a random run sequences. The user friendly SAS macro developed for the generation of such run sequences would help the experimenter to get a readymade layout plans.

The exhaustive search to generate all possible run sequences should be effective in selecting any run sequences randomly out of the all possible run sequences where number of change is minimum. Therefore, any systematic bias which may be introduced due to the absence of randomization of run sequences, may be reduced to some extent by randomly choosing any minimally changed run sequences generated based on the algorithm if computational facilities are available. An alternative to this exhaustive search algorithm, is the use of random sample of the run sequences with minimum number of level change. However, it suffers from the potential disadvantage that it will generate only a part of the minimally changed run sequences not all the run sequences.

In the current work, run sequences with minimum changes have been developed under the assumption that the cost of changing a factor level is same for all the factors which itself is a stringent assumption. However, the above problem can be tackled by taking research in future direction under the more general assumption that the cost for changing a factor level varies factor wise. Further, factorial design with minimally changed run sequences in the presence of time trend can also be considered as future research problems.

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ANNEXURE–IA

Output of SAS macro for generation of a 4^2 factorial with minimum changes in run sequences

The SAS System

Minimally Changed Run Sequences for Factorial Experiment

,**Run_Sequence**	
-2	-2
-2	-1
-2	1
-2	2
-1	2
-1	1
-1	-1
-1	-2
1	-2
1	-1
1	1
1	2
2	2
2	1
2	-1
2	-2

Factor-wise number of changes in the run sequence

Factor_Change	
3	12

Total number of changes in the run sequence

Total_Change
15

ANNEXURE–IB

Output of SAS macro for generation of half replicate of a 2^5 factorial with minimum changes in run sequences

The SAS System

Minimally Changed Run Sequences for Factorial Experiment

Run_Sequence				
-1	-1	-1	-1	1
-1	-1	-1	1	-1
-1	-1	1	1	1
-1	-1	1	-1	-1
-1	1	1	-1	1
-1	1	1	1	-1
-1	1	-1	1	1
-1	1	-1	-1	-1
1	1	-1	-1	1
1	1	-1	1	-1
1	1	1	1	1
1	1	1	-1	-1
1	-1	1	-1	1
1	-1	1	1	-1
1	-1	-1	1	1
1	-1	-1	-1	-1

Factor-wise Number of Changes in the Run Sequence

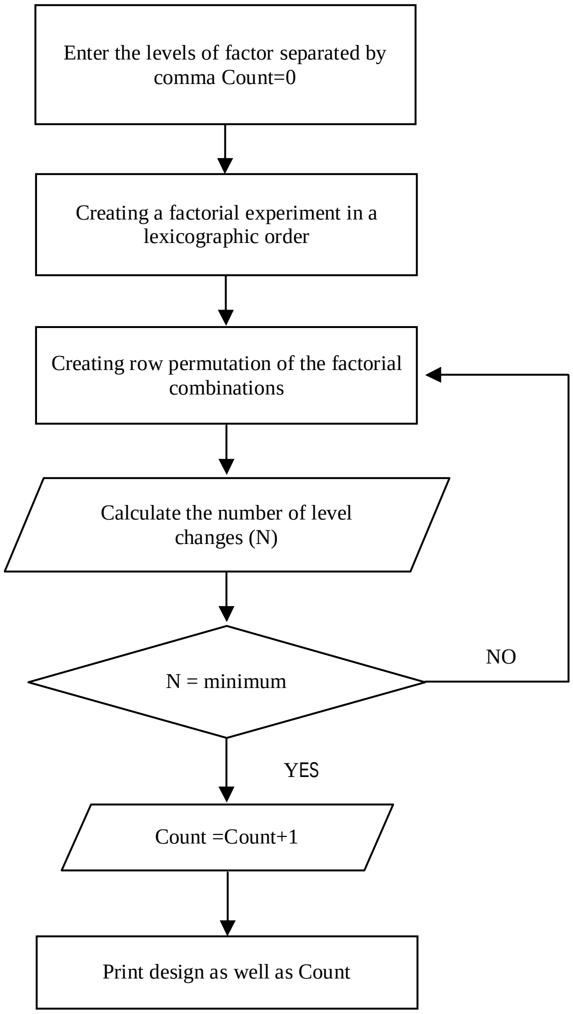
Factor_Change				
1	2	4	8	15

Total Number of Changes in the Run Sequence

Total_change
30

ANNEXURE–IIA

Simplified flow diagram for the exhaustive search algorithm to generate all possible minimally changed run sequences in symmetric factorial experiments



ANNEXURE-IIB

All possible minimally changed run sequences for 2^2 factorial generated through exhaustive search procedure

The SAS System

Design	
-1	-1
-1	1
1	1
1	-1
Count	
1	2

Design	
-1	-1
1	-1
1	1
-1	1
Count	
2	1

Design	
1	-1
-1	-1
-1	1
1	1
Count	
2	1

Design	
1	1
1	-1
-1	-1
-1	1
Count	
1	2

Design	
1	-1
1	1
-1	1
-1	-1
Count	
1	2

Design	
-1	1
1	1
1	-1
-1	-1
Count	
2	1

Design	
1	1
-1	1
-1	-1
1	-1
Count	
2	1

Design	
-1	1
-1	-1
1	-1
1	1
Count	
1	2

Total_Design
8

ANNEXURE-IIC

All possible minimally changed run sequences for $\frac{1}{2}\left(2^3\right)$ factorial generated through exhaustive search procedure

The SAS System

design		
-1	-1	1
-1	1	-1
1	1	1
1	-1	-1
count		
1	2	3

design		
1	-1	-1
-1	-1	1
-1	1	-1
1	1	1
count		
2	1	3

design		
-1	-1	1
1	1	1
1	-1	-1
-1	1	-1
count		
2	3	1

design		
1	1	1
-1	-1	1
1	-1	-1
-1	1	-1
count		
3	2	1

design		
1	-1	-1
1	1	1
-1	1	-1
-1	-1	1
count		
1	2	3

design		
1	1	1
-1	1	-1
-1	-1	1
1	-1	-1
count		
2	1	3

design		
-1	1	-1
1	-1	-1
1	1	1
-1	-1	1
count		
2	3	1

design		
-1	1	-1
1	-1	-1
-1	-1	1
1	1	1
count		
3	2	1

design		
-1	-1	1
-1	1	-1
1	-1	-1
1	1	1
count		
1	3	2

design		
1	-1	-1
-1	-1	1
1	1	1
-1	1	-1
count		
3	1	2

design		
-1	-1	1
1	1	1
-1	1	-1
1	-1	-1
count		
3	2	1

design		
1	1	1
1	-1	-1
-1	-1	1
-1	1	-1
count		
1	2	3

design		
1	1	1
1	-1	-1
-1	1	-1
-1	-1	1
count		
1	3	2

design		
-1	1	-1
1	1	1
-1	-1	1
1	-1	-1
count		
3	1	2

design		
1	-1	-1
-1	1	-1
1	1	1
-1	-1	1
count		
3	2	1

design		
-1	1	-1
-1	-1	1
1	-1	-1
1	1	1
count		
1	2	3

design		
-1	-1	1
1	-1	-1
-1	1	-1
1	1	1
count		
3	1	2

design		
-1	-1	1
1	-1	-1
1	1	1
-1	1	-1
count		
2	1	3

design		
1	1	1
-1	-1	1
-1	1	-1
1	-1	-1
count		
2	3	1

design		
1	-1	-1
1	1	1
-1	-1	1
-1	1	-1
count		
1	3	2

design		
1	1	1
-1	1	-1
1	-1	-1
-1	-1	1
count		
3	1	2

design		
-1	1	-1
1	1	1
1	-1	-1
-1	-1	1
count		
2	1	3

design		
1	-1	-1
-1	1	-1
-1	-1	1
1	1	1
count		
2	3	1

design		
-1	1	-1
-1	-1	1
1	1	1
1	-1	-1
count		
1	3	2

Total_design
24