

Available online at www.isas.org.in/jisas JOURNAL OF THE INDIAN SOCIETY OF AGRICULTURAL STATISTICS 69(3) 2015 315-325

# On Choice of Explanatory Variables in Linear Model for Estimation of Crop Production at Smaller Geographical Area

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Received 15 April 2014; Revised 03 September 2015; Accepted 31 December 2015

#### SUMMARY

Crop production statistics for a small area like Community Development Block (generally referred to as Block) or Panchayat are now essential in view of decentralized planning process at micro-level in India. Generally, estimates of crop production or yield through crop cutting experiments (CCEs) are being reported at district level and these estimates are aggregated at state and country level. If reasonably precise estimates are required for further smaller geographical levels such as Block or Gram Panchayat the number of CCEs is expected to increase enormously. However, conducting requisite number of area specific CCEs is neither operationally feasible nor is economically viable. Sisodia and Singh (2001) proposed a scale down approach using multiple regression model to obtain the block level estimate from the district level crop-production estimate. Singh *et al.* (2012) proposed a predictive approach of estimation of crop-production. Three options are considered (i) using the auxiliary variables as such in the model, (ii) application of principal component analysis and (iii) step-wise regression analysis. It is assumed that the auxiliary information available at district level is also available at block level. An empirical study with wheat production data of Sultanpur district of the State of Uttar Pradesh, India shows that approach works well and provides reliable estimates of wheat production at block level by applying technique of principal component analysis. The estimator based on least squares adjustment has performed better in most of the cases than other estimators in terms of percent standard error.

Keywords: Block level estimate, Principal component analysis, Regression model.

## **1. INTRODUCTION**

Small area estimation (SAE) techniques have been developed within the frame of design based sample surveys. In such techniques the observations on sampled units are utilized to obtain direct/ indirect (synthetic) estimates of parameters of small area. The classical literature on SAE methodologies is given in Rao (2003). In the context of agriculture surveys for cropstatistics in India, the estimates of crop production or yield are being reported at district level through scientifically designed Crop-Cutting Experiments (CCEs) under the scheme of General Crop Estimation Survey (GCES) and these estimates are aggregated at state and country level. The estimates of crop production at Block or Panchayat level are in great demand in recent times by the planners for policy formulation in the context of de-centralized planning process at micro level in India. If reasonably precise estimates of crop production are required for Block or Panchayat level then the number of CCEs is expected to increase enormously. Thus, conducting requisite number of area specific CCEs is neither operationally feasible is economically viable.

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An attempt, quite earlier, was made by Panse et al. (1966) to develop some methodologies for estimating the crop yield at block level using double sampling approach, but it could not succeed due to certain physical constraints. Stasny et al. (1991) have made use of a regression model to predict wheat production at county level in USA. They, in fact, developed the multiple regression models to exhibit best possible relationship between farm production and some predictor variables related with farm production for predicting wheat production at county level. Sisodia and Singh (2001) advocated a scale down approach which involved scaling down of available district level estimate of crop production to block level. Their approach did not require any additional CCEs. They attempted to establish a best possible functional relationship between time series data on crop production and its related auxiliary variables at district level, which was used to develop block level estimates of crop production assuming that the data on the same auxiliary variables were also available at block level. Sharma et al. (2004) developed estimators of crop yield at panchayat level using eye estimate and farmers' appraisal within the frame of design based sample survey. Sisodia and Chandra (2012) and Sisodia and Singh (2012) have recently made further attempts to improve the estimates for block level crop-production following the approach of Sisodia and Singh (2001). Singh et al. (2012) proposed a predictive approach for block level estimate of crop production by fitting multiple regression model between time-series data on crop yield and related auxiliary variables.

The question in the present paper is that in what way we make best use of the set of auxiliary variables related to dependent variable y in the regression model to obtain best prediction of block level crop-production. In other worlds, it can be treated as a problem of choice of explanatory variables in the model. Three options are plausible. First, one used by Singh *et al.* (2012) where available set of the auxiliary variables is used as explanatory variables. Second, use few best principal components of the

auxiliary variables as explanatory variables in the regression model, since the auxiliary variables are generally correlated. It may be pointed out that the use of principal components in regression analysis was earlier dealt with by Jolliffe (1982). Third, use step-wise regression analysis approach to arrive at the best set of explanatory variables from the complete set of auxiliary variables.

A brief review of the prediction approach of Singh *et al.* (2012) is presented in section 2. Methodology based on principal component analysis has been described in Section 3. Stepwise regression analysis approach is well known and is available in many standard books (see, Montgomery and Peak, 1982). An empirical study has been carried out to illustrate the relative merits of the three aforesaid options in Section 4. Discussion of the results and specific concluding remarks are given in Section 5.

# 2. A BRIEF REVIEW OF PREDICTION APPROACH OF SINGH *ET AL.* (2012)

Singh *et al.* (2012) proposed the following multiple regression linear model between the crop yield and auxiliary variables at district level,

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \dots + \beta_p X_{ip} + \varepsilon_i$$
(2.1)

where  $Y_i$  is the crop yield in the i<sup>th</sup> year (*i*= 1, 2, ..., *n*),  $X_{ij}$ 's are the auxiliary variables, *j*= 1, 2, ...,*p*;  $\beta'=(\beta_0, \beta_1, \ldots, \beta_p)$  is vector of model parameters and  $\varepsilon_i$ 's are error terms assumed to follow independently normal distribution with mean zero and variance  $\sigma^2$ . Let the model (2.1) fitted with the data at district level by least squares technique be denoted by

$$\hat{Y}_{i} = \hat{\beta}_{0} + \hat{\beta}_{1} x_{i1} + \dots + \hat{\beta}_{p} x_{ip}$$
(2.2)

where  $\hat{\beta}$  is the least square estimate of  $\beta$  and  $\hat{Y}_i$  is the estimated value of  $Y_i$  for corresponding values of  $X_{ij}$ 's in the i<sup>th</sup> year. They used the estimate in (2.2) directly at block level to predict the  $Y_q$ , the yield at block q (q=1, 2, ...Q). Let  $\hat{Y}_q$  be the predicted value of the crop yield  $Y_q$  during a particular year of interest, which is given by

$$\hat{Y}_{q} = \hat{\beta}_{0} + \hat{\beta}_{1}x_{q1} + \dots + \hat{\beta}_{p}x_{qp}$$
 (2.3)

where  $X_{qj}$ 's; j= 1, 2, ..., p, are values of auxiliary variables for the q<sup>th</sup> block. Therefore, an unbiased estimator of  $Z_q$ , the crop production at q<sup>th</sup> block is given by

$$\hat{Z}_{q} = \delta_{q} \hat{Y}_{q} \tag{2.4}$$

where  $\delta_q$  is the area under the crop at q<sup>th</sup> block, which is known through complete enumeration by State Governments in India. Note that  $\hat{Z}_q$  is an unbiased estimator of  $Z_q$  as  $E(\hat{Z}_q) = Z_q$ because  $E(\hat{Y}_q) = Y_q$  under the assumption of regression model (2.1). Following Montgomery and Peck (1982), the variance of  $\hat{Z}_q$  is given by

$$V(\hat{Z}_{q}) = \delta_{q}^{2} \sigma^{2} C_{q}' (X'X)^{-1} C_{q}$$

$$(2.5)$$

where  $\sigma^2$  is the residual variance corresponding to the regression model (2.1), **X** is matrix of **X**<sub>ij</sub>'s at district level given by

	[1	X <sub>11</sub>	X <sub>12</sub>				$X_{1p}$	
	1	$\mathbf{X}_{21}$	X <sub>12</sub> X <sub>22</sub>				X <sub>2p</sub>	
X =	.			•	•	•		
	•	•	•	•	•	•	•	
	.	•	•	•	•	•		
	1	$X_{n1}$	$X_{n2}$				X <sub>np</sub>	

and  $\mathbf{X}^{*}$  is the transpose of  $\mathbf{X}$  matrix.  $C_q$  is column vector of  $\mathbf{X}_{qj}$ 's at  $\mathbf{q}^{th}$  block level given by

$$\mathbf{C'}_{q}=[1 \mathbf{X}_{q1} \mathbf{X}_{q2} \dots \mathbf{X}_{qp}]$$

Since in general  $\sum_{q=1}^{Q} \hat{Z}_q \neq Z$ , where **Z** is the crop-

production for the district based on CCEs, an adjusted estimator of  $\mathbf{Z}_{q}$  by scaling  $\hat{\mathbf{Z}}_{q}$  is given by

$$\tilde{Z}_{q} = a_{q}\hat{Z}_{q} \tag{2.6}$$

where  $a_q$  is constant such that  $\sum_{q=1}^{Q} \tilde{Z}_q = Z$  or

 $\sum_{q=1}^{Q} a_q \hat{Z}_q = Z.$  Three alternative choices as

suggested by Sisodia and Chandra (2012) were used by Singh *et al.* (2012) to obtain different scaled estimator  $\tilde{Z}_q$ . These choices along with scaled estimators and their conditional variance/ means square error (MSE) are given below.

#### 2.1 Choice-I: Ratio Adjustment

For 
$$a_q = a$$
 for all q, we have  $a = \frac{Z}{\sum_{q=1}^{Q} \hat{Z}_q}$ , and

a new scaled estimator of  $Z_q$  is

$$\widetilde{Z}_{q}^{(1)} = \widehat{Z}_{q} \left[ Z / \sum_{q=1}^{Q} \widehat{Z}_{q} \right]$$
(2.7)

with conditional MSE

$$MSE[\tilde{Z}_{q}^{(1)}] = \frac{\left(Z - \sum_{q=1}^{Q} \hat{Z}_{q}\right)^{2} Z_{q}^{2}}{\left(\sum_{q=1}^{Q} \hat{Z}_{q}\right)^{2}} + \frac{Z^{2}}{\left(\sum_{q=1}^{Q} \hat{Z}_{q}\right)^{2}} V(\hat{Z}_{q}) \quad (2.8)$$

Note that the estimator  $\widetilde{Z}_{q}^{(1)}$  is not an unbiased estimator of  $Z_{q}$ .

#### 2.2 Choice-II: Least Squares Adjustment

Another choice of  $a_q$  is obtained by minimizing the sum of squared differences between  $\tilde{Z}_q$  and  $\hat{Z}_q$  subject to condition that  $\sum_{q=1}^{Q} a_q \hat{Z}_q = Z$ . This results in  $a_q = \frac{\left(Z - \sum_{q=1}^{Q} \hat{Z}_q\right)}{Q\hat{Z}_q} + 1$  (2.10)

and resultant scaled estimator is

$$\tilde{Z}_{q}^{(2)} = \hat{Z}_{q} + \frac{Z - \sum_{q=1}^{Q} \hat{Z}_{q}}{\Omega}$$
(2.11)

with conditional variance

$$V(\tilde{Z}_{q}^{(2)}) = \frac{Q-2}{Q}V(\hat{Z}_{q}) + \frac{1}{Q^{2}}\sum_{q=1}^{Q}V(\hat{Z}_{q}) \qquad (2.12)$$

Note that  $\tilde{Z}_{q}^{(2)}$  is an unbiased estimator of  $\hat{Z}_{q}$ .

## 2.3 Choice-III: Relative Least Squares Adjustment

The third choice of  $a_q$  is obtained by minimizing the sum of squares of relative differences  $(\tilde{Z}_q - \hat{Z}_q)/\hat{Z}_q$  subject to condition that  $\sum_{q=1}^{Q} a_q \hat{Z}_q = Z$ . This results in

$$a_{q} = 1 + \frac{Z - \sum_{q=1}^{Q} \hat{Z}_{q}}{\sum_{q=1}^{Q} \hat{Z}_{q}^{2}} \hat{Z}_{q}$$
(2.13)

and a new scaled estimator of  $Z_q$  is

$$\tilde{Z}_{q}^{(3)} = \hat{Z}_{q} + \frac{\hat{Z}_{q}^{2}}{\sum_{q=1}^{Q} \hat{Z}_{q}^{2}} \left( Z - \sum_{q=1}^{Q} \hat{Z}_{q} \right)$$
(2.14)

with conditional MSE, for given  $X_{ij}$ ,

$$MSE[\tilde{Z}_{q}^{(3)}] = \left[1 + \frac{Z - \sum_{q=1}^{Q} \hat{Z}_{q}}{\sum_{q=1}^{Q} \hat{Z}_{q}^{2}} \hat{Z}_{q}\right]^{2} V(\hat{Z}_{q})$$
(2.15)

Note that  $\tilde{Z}_{q}^{(3)}$  is not an unbiased estimator of  $\hat{Z}_{q}$ 

Comparison for relative efficiency of  $\tilde{Z}_q^{(1)}, \tilde{Z}_q^{(2)}$  and  $\tilde{Z}_q^{(3)}$  over,  $\hat{Z}_q$ 

Taking difference of  $V(\hat{Z}_q)$  and  $MSE(\tilde{Z}_q^{(1)})$ , i.e.  $V(\hat{Z}_q)$  -  $MSE(\tilde{Z}_q^{(1)})$  and after simplification, we get the inequality

$$V(\hat{Z}_{q}) \ge \frac{\left(z - \sum_{q=1}^{Q} \hat{z}_{q}\right)^{2} z_{q}^{2}}{\left[\left(\sum_{q=1}^{Q} \hat{z}_{q}\right)^{2} - z^{2}\right]}$$
(2.16)

for  $\tilde{Z}_q^{(1)}$  to be efficient than  $\hat{Z}_q$ . However, it seems that inequality (2.16) may not hold true in general.

Taking difference  $\hat{Z}_{q}$  of  $V(\hat{Z}_{q})$  and  $V(\tilde{Z}_{q}^{(2)})$ , i.e.  $V(\hat{Z}_{q}) V(\tilde{Z}_{q}^{(2)})$ , and after simplification, we get the following inequality

$$\mathbf{V}(\hat{\mathbf{Z}}_{q}) \ge \frac{1}{2Q} \sum_{q=1}^{Q} \mathbf{V}(\hat{\mathbf{Z}}_{q})$$
(2.17)

for  $\tilde{Z}_q^{(2)}$  to be efficient than  $\hat{Z}_q$ . It is obvious from the inequality (2.17) that the  $V(\hat{Z}_q)$  must be greater than the half of the average of variances of  $\hat{Z}_q$  (q=1, 2, ..., Q), which is quite possible in general.

Taking difference of  $V(\hat{Z}_q)$  and  $MSE^{(\widetilde{Z}_q^{(3)})}$ , i.e.  $V^{(\hat{Z}_q)} - MSE^{(\widetilde{Z}_q^{(3)})}$ , we get the following inequality

$$1 - \left\{ 1 + \frac{z - \sum_{q=1}^{Q} \hat{z}_{q}}{\sum_{q=1}^{Q} \hat{z}_{q}^{2}} \hat{z}_{q} \right\}^{2} \ge 0$$
 (2.18)

which is not possible as the value of LHS of the above inequality will always be negative. This shows that  $\hat{Z}_q$  will always be efficient than  $\tilde{Z}_q^{(3)}$  in general.

The above comparisons clearly indicate that the  $\tilde{Z}_q^{(2)}$  is expected to perform better than  $\hat{Z}_q, \tilde{Z}_q^{(1)}$  and  $\tilde{Z}_q^{(3)}$ .

# 3. METHODOLOGY BASED ON PRINCIPAL COMPONENT ANALYSIS

The technique of principal component analysis is available in many standard books of multivariate analysis [(See, for example Anderson (1984)]. Using the time series data on  $X_{ij}$ 's, the principal component analysis can be carried out at both district and block levels. Let  $P_1, P_2, \ldots, P_k$ be the first k (k < p) principal components explaining variability up to 90 percent at the district level. Similarly, let,  $P_1, P_2, \ldots, P_k$  be the first k principal components at the  $q^{\text{th}}$  block level ( $q = 1, 2, \ldots, Q$ ).

Using district level k principal components as the explanatory variables, we fit the following regression model at district level.

$$Y_{i} = \beta_{0} + \beta_{1} P_{1} + \beta_{2} P_{2} + \dots + \beta_{k} P_{k} + \varepsilon_{i},$$
  
*i*=1, 2, ... *n* (3.1)

where  $Y_i$  is the crop-yield based on CCEs during  $i^{th}$  year in a district.  $\beta' = (\beta_0, \beta_1, ..., \beta_k)$  is vector of unknown model parameters and  $\varepsilon_i$ 's are error terms assumed to follow independently normal distribution with mean zero and variance  $\sigma^2$ . We denote the fitted model by least squares technique as

 $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 P_1 + \beta P_2 + \dots + \hat{\beta}_k P_k \quad ; \text{ where}$ hat (^) denote the estimate . (3.2)

Using block level principal components in place of district level principal components in the above fitted model (3.2), the crop- yield  $Y_q$  for  $q^{\text{th}}$  block (q= 1, 2, ..., Q) for a given year of interest can be predicted as follows.

$$\hat{Y}_{q} = \hat{\beta}_{0} + \hat{\beta}_{1} P_{q_{1}}^{'} + \beta P_{q_{2}}^{'} + \dots + \hat{\beta}_{k} P_{q_{k}}^{'}; q = 1, 2, \dots, Q$$

$$(3.3)$$

An estimator for  $Z_q$ , crop- production for  $q^{\text{th}}$  block, can be obtained as

$$\hat{Z}_{q} = \delta_{q} \hat{Y}_{q}$$
,  $q = 1, 2, ..., Q$  (3.4)

where  $\delta_q$  is the area under the crop in  $q^{\text{th}}$  block.  $\hat{Z}_q$ is the unbiased estimator of  $Z_q$  as  $E(\hat{Z}_q) = \delta_q E(\hat{Y}_q) = \delta_q Y_q$ , because  $E(\hat{Y}_q) = Y_q$  under the model (2.3). The variance of  $\hat{Z}_q$  is given by  $V(\hat{Z}_q) = \delta_q^2 \sigma^2 C'_q (X'X)^{-1} C_q$  (3.5) where  $\sigma^2$  is the residual variance corresponding to the fitted regression model (3.2), **X** is matrix of Principal Components at district level given by

	[1	P 11	P 12				P <sub>1k</sub>
	1	P 21	P 22				P <sub>2k</sub>
X =				•	•	•	
	.		•	·	·	·	·
	.						.
	[1	$P_{n1}$	P <sub>12</sub> P <sub>22</sub> P <sub>n 2</sub>			•	P <sub>nk</sub>

and **X**' is the transpose of **X** matrix. Cq is column vector of Principal Components at  $q^{th}$  block level given by

$$\mathbf{C}_{q}^{\prime} = \begin{bmatrix} \mathbf{l} & \mathbf{P}_{q1}^{\prime} & \mathbf{P}_{q2}^{\prime} & \dots & \mathbf{P}_{qk}^{\prime} \end{bmatrix}$$

Again in general,  $\sum_{q=1}^{Q} \hat{Z}_q \neq Z$ . Then, similarly a scaled estimator of  $Z_q$  is given by

$$\widetilde{Z}_q = a_q \hat{Z}_q$$

where  $a_q$  is such that  $\sum_{q=1}^{Q} \widetilde{Z}_q = Z$  or  $\sum_{q=1}^{Q} a_q \hat{Z}_q = Z$ 

The choices of  $a_q$  and resultant three scaled estimators along with their conditional MSE/variance given in Section 2 will be exactly same in this case also. Only the difference will be that  $\hat{Z}_q$  is obtained on the basis of principal component analysis.

**Remark:** Note that difference in the methodologies described in Section 2 and 3 lies in the inclusion of the explanatory variables in the regression model.

#### 4. AN EMPIRICAL ILLUSTRATION

An empirical study has been carried out to illustrate the relative merits of these options under consideration. The time series data on the  $Y_i$  (wheat yield) and related auxiliary variables  $(X_{ij})$ , i.e. percent irrigated area under wheat  $(X_I)$ , relative area under wheat as percentage of the gross-cropped area (GCA)  $(X_2)$  and fertilizer consumption in Kg/ha  $(X_3)$  pertaining to the period 1984-85 to 2008-09 for the Sultanpur district of U.P., India, have been used for the study. The time series data on yield and auxiliary variables have been splitted into three overlapping sets. Set-I: 1992-93 to 2006-07, Set-II: 1994-95 to 2008-09 and Set-III: 1984-85 to 2008-09. The objective is to estimate the wheat production at block level in Sultanpur district. The time series data on the aforesaid auxiliary variables  $X_1$ ,  $X_2$  and  $X_3$  are also available at block level for the same period.

## 4.1 Results based on the Methodology Due to Singh *et al.* (2012)

The model (2.1) has been fitted with three sets of data separately by ordinary least square technique. The results of the fitted model are summarized in Table 4.1.1.

Table 4.1.1. Details of Fitted Model

Set	Fitted Model	R2 (%)	$\mathbf{SE}^{(\hat{Y})}$	$\hat{\sigma}^2$
Set 1	Y= -325.284+3.556X1- 0.432X2+0.129X3*	78.38	1.36	1.834
	(333.58) (3.492) (0.459) (0.045)	66.21	1.43	2.034
Set 2	Y=-382.161+4.142X1- 0.336X2+0.076X3# (357.006) (3.737) (0.464) (0.041)	89.50	1.33	1.788
Set 3	Y=-178.333+1.951X1- 0.061X2+0.117X3** (119.381) (1.301) (0.287) (0.022)			

#P<0.10, \*P<0.05 and \*\*P<0.01.  $\hat{\sigma}^2$  is estimated residual variance The figures in parentheses are standard error.

The fitted model (2.1) for the three sets of data at district level has been directly used to predict the block estimates of wheat yield using block wise data on  $X_1$ ,  $X_2$  and  $X_3$  for the year 2006-07. In fact, the block level wheat yield based on CCEs for the year 2006-07 was available from a Pilot-Survey conducted by Directorate of Agricultural Statistics and Crop Insurance, Govt. of U.P., India. So, the data on wheat yield of block level of the year 2006-07 were used to validate the block level estimates of wheat production obtained from the proposed estimators. The block estimates of wheat production based on four estimators are given in Appendix-I. The percent standard errors of the estimates have been computed and are presented in Table 4.1.2. To measure how far the four estimates based on four estimators are from the  $Z_q^*$  (crop production based on CCEs), an average distance between  $Z_{q}^{*}$  and estimates based on four estimators has been work out by following formula.

**Table 4.1.2.** The Percent Standard Errors of the Estimates of wheat Production based on Different Estimators and Different Sets of Data for the year 2006-07 using Methodology of Singh *et al* (2012)

S. No.	Block	% Standard Error											
			Se	t I			Se	t II			Set	III	
		$\hat{Z}_{q}$	$\widetilde{Z}_{q}^{\left(l\right)}$	$\widetilde{Z}_{q}^{\left(2\right)}$	$\widetilde{Z}_{q}^{\left( 3\right) }$	$\hat{Z}_q$	$\widetilde{Z}_{q}^{\left(l\right)}$	$\widetilde{Z}_{q}^{(2)}$	$\widetilde{Z}_{q}^{\left( 3\right) }$	$\hat{Z}_q$	$\widetilde{Z}_{q}^{(1)}$	$\widetilde{Z}_{q}^{(2)}$	$\widetilde{Z}_{q}^{\left( 3\right) }$
1	Shukul bazaar	2.61	4.43	3.30	2.68	3.80	5.19	4.21	3.84	1.76	2.93	2.13	1.79
2	Jagdishpur	6.05	6.91	6.24	6.24	6.14	6.83	6.18	6.20	3.58	4.25	3.69	3.65
3	Musafhirkhana	9.16	9.78	9.39	9.40	8.76	9.19	8.74	8.83	6.01	6.48	6.09	6.10
4	Waldi Rai	22.57	22.79	22.69	23.12	23.07	22.75	22.47	23.25	16.93	17.11	16.85	17.14
5	Jamo	4.94	6.25	5.19	5.11	4.64	5.80	4.78	4.69	2.38	3.45	2.56	2.43
6	Shahgarh	28.91	29.06	28.92	29.66	31.75	31.14	30.81	32.01	15.37	15.45	15.06	15.68
7	Gauriganj	6.75	7.78	6.87	6.98	8.57	9.19	8.43	8.66	4.56	5.15	4.55	4.66
8	Amethi	8.11	8.65	8.25	8.36	11.22	11.39	11.02	11.32	5.00	5.35	5.01	5.11
9	Bhetua	4.20	5.14	4.71	4.31	5.52	6.19	5.79	5.56	2.41	3.11	2.73	2.46
10	Bhadar	9.97	10.36	10.16	10.25	14.11	14.09	13.85	14.22	5.77	6.04	5.80	5.87
11	Sangrampur	10.56	10.93	10.96	10.79	15.38	15.33	15.24	15.47	5.73	6.01	5.89	5.81
12	Dhanpatganj	2.15	4.53	2.80	2.21	2.20	4.47	2.78	2.22	1.49	3.00	1.84	1.53
13	Kurebhar	2.21	4.14	2.78	2.29	2.94	4.53	3.32	2.98	1.51	2.75	1.82	1.54
14	Jai Singh Pur	3.28	4.85	3.62	3.40	4.49	5.65	4.63	4.54	2.46	3.36	2.59	2.51
15	Kurwar	2.38	4.84	3.04	2.46	3.03	5.09	3.47	3.05	1.45	3.13	1.85	1.47
16	Dube Pur	3.86	5.33	4.23	3.98	5.64	6.63	5.74	5.69	2.80	3.66	2.95	2.86
17	Bhadaiyeea	2.27	3.75	2.89	2.35	2.59	3.89	3.08	2.62	1.45	2.44	1.40	1.48
18	Dostpur	3.91	5.44	4.25	4.04	4.39	5.74	4.60	4.44	2.32	3.44	2.52	2.37
19	Akhand Nagar	5.76	7.27	5.85	5.99	3.87	5.61	3.98	3.92	2.92	4.07	2.98	3.00
20	Lambhua	3.70	5.20	4.00	3.83	3.90	5.23	4.09	3.94	2.75	3.60	2.85	2.81
21	Pratap Pur Kamaicha	8.93	9.63	9.16	9.18	12.94	13.18	12.73	13.05	5.45	5.90	5.49	5.55
22	Kadipur	5.08	6.16	5.01	5.43	6.35	7.19	6.16	6.47	2.86	3.67	2.81	2.98

$$D_{i} = \sqrt{\frac{\sum_{q=1}^{Q} \left(Z_{q}^{*} - \hat{\theta}_{qi}\right)^{2}}{Q}} \quad (i = 1, 2, 3, 4)$$
(4.1.1)

where  $\hat{\theta}_{qi}$  is the estimated crop production of

 $Z_q^*$  from the i<sup>th</sup> estimator. The percent standard error of the estimates has been computed as follows

Percent standard error (PSE)=  

$$\sqrt{\frac{MSE / Variance of the estimator}{Estimate}} \times 100$$
 (4.1.2)

## 4.2 Results based on Principal Component analysis

Using three sets of time series data on three auxiliary variables  $X_1$ ,  $X_2$ , and  $X_3$ , the principal component analysis has been carried out at district and block level both.

Since the first principal component has been able to explain the variation from 74 to 90 percent of the total variability in  $X_{ij}$ 's, it has been included in the linear regression model as explanatory variable at district level given as follows

$$Y_i = \beta_0 + \beta_1 P_{1i} + \varepsilon_i; i = 1, 2, ..., n.$$
 (4.2.1)

where  $Y_i$  is wheat yield,  $P_{1i}$  is first principal component during i<sup>th</sup> year at district level.  $\beta_0$  and  $\beta_1$  are model parameters and  $\epsilon_i$  is error term assumed to follow NID (0,  $\sigma^2$ ). The results of fitted model (4.2.1) with three sets of principal components by least square technique are presented in the Table 4.2.1.

**Table 4.2.1.** Fitted Models based on Principal Component

 Analysis

Set	Fitted Model	R2 (%)	$\mathbf{SE}(\hat{\mathbf{Y}})$	$\hat{\sigma}^2$
Set 1	$Y = 24.74 + 2.03P_1 *$ (0.43) (0.44)	61.96	1.65	2.73
Set 2	$Y = 25.84 + 1.51P_1 *$ (0.42) (0.44)	47.85	1.62	2.66
Set 3	$Y = 23.15 + 3.53P_1 ** (0.31) (0.32)$	83.77	1.453	2.52

\*P<0.05, \*\*P<0.01 and  $\hat{\sigma}^2$  is the residual variance. The figures in parentheses are standard errors.

The value of first principal component of block q (q=1, 2, ..., Q) for the year 2006-07, say,  $P_{1q}$  has been used to predict the wheat yield for  $q^{th}$  block by using the fitted models at district level as given in Table 4.2.1.

**Table 4.2.2.** The Percent Standard Errors of the Estimates of wheat Production based on Different Estimators and Different Sets of Data for

 the year 2006-07 using Principal Component Analysis

S. No.	Block					(	% Standa	ard Erro	r				
			Se	t I			Se	t II				III	
		$\hat{Z}_q$	$\widetilde{Z}_{q}^{(1)}$	$\widetilde{Z}_{q}^{(2)}$	$\widetilde{Z}_{q}^{(3)}$	$\hat{Z}_{q}$	$\widetilde{Z}_{q}^{(1)}$	$\widetilde{Z}_{q}^{(2)}$	$\widetilde{Z}_{q}^{(3)}$	$\hat{Z}_q$	$\widetilde{Z}_{q}^{(1)}$	$\widetilde{Z}_{q}^{(2)}$	$\widetilde{Z}_{q}^{(3)}$
1	Shukul bazaar	2.71	2.72	2.67	2.72	2.80	2.86	2.79	2.85	1.76	1.82	1.72	1.95
2	Jagdishpur	2.78	2.82	2.75	2.81	2.91	2.96	2.88	2.96	1.68	2.15	1.67	1.86
3	Musafhirkhana	2.40	2.47	2.49	2.46	2.50	2.52	2.53	2.51	1.76	2.21	1.75	2.01
4	Waldi Rai	2.33	2.35	2.39	2.34	2.41	2.44	2.47	2.43	1.53	2.03	1.45	1.77
5	Jamo	2.87	2.91	2.82	2.91	2.72	2.79	2.70	2.79	1.81	2.25	1.28	1.95
6	Shahgarh	2.23	2.23	2.21	2.25	2.95	2.99	2.92	2.99	1.78	2.23	1.63	1.97
7	Gauriganj	2.75	2.76	2.67	2.76	2.72	2.77	2.68	2.77	1.83	2.27	1.30	1.98
8	Amethi	2.60	2.65	2.59	2.64	3.00	3.01	2.94	3.01	1.65	2.13	1.71	1.83
9	Bhetua	2.40	2.45	2.40	2.45	2.73	2.75	2.72	2.75	1.73	2.19	1.85	1.95
10	Bhadar	2.35	2.36	2.38	2.36	2.52	2.56	2.50	2.56	1.77	2.22	1.70	2.01
11	Sangrampur	2.50	2.54	2.63	2.53	2.50	2.53	2.62	2.53	1.84	2.27	1.69	2.17
12	Dhanpatganj	2.52	2.53	2.47	2.53	2.78	2.81	2.73	2.81	1.72	2.18	1.47	1.88
13	Kurebhar	2.57	2.58	2.51	2.58	2.90	2.98	2.89	2.98	1.83	2.26	1.41	1.99
14	Jai Singh Pur	2.67	2.70	2.62	2.69	2.60	2.64	2.56	2.63	1.83	2.27	1.34	1.98
15	Kurwar	2.69	2.70	2.63	2.70	2.69	2.72	2.64	2.71	1.70	2.16	1.53	1.87
16	Dube Pur	2.43	2.44	2.39	2.44	2.50	2.55	2.49	2.54	1.79	2.23	1.51	1.96
17	Bhadaiyeea	2.58	2.61	2.55	2.61	2.59	2.62	2.56	2.62	1.79	2.23	1.47	1.96
18	Dostpur	2.91	2.92	2.83	2.92	2.86	2.89	2.81	2.88	1.55	2.05	1.59	1.71
19	Akhand Nagar	1.78	1.80	1.75	1.79	2.98	2.99	2.87	2.99	1.72	2.18	1.07	1.83
20	Lambhua	2.43	2.44	2.37	2.44	3.90	4.00	3.85	4.00	1.69	2.15	1.31	1.83
21	Pratap Pur Kamaicha	2.62	2.64	2.61	2.63	2.67	2.68	2.65	2.68	1.84	2.28	1.75	2.07
22	Kadipur	2.43	2.45	2.34	2.44	2.68	2.71	2.60	2.71	1.58	2.07	0.87	1.66

Let  $\hat{\mathbf{Y}}_{q}$  be the predicted wheat yield for  $q^{\text{th}}$  block given by

$$\hat{\mathbf{Y}}_{q} = \hat{\beta}_{0} + \hat{\beta}_{1} \mathbf{P}_{1q}^{'}$$
 (4.2.2)

The estimates of wheat production  $(Z_q)$  for each block have been computed from four estimators described in Section-2 using  $\hat{Y}_q$  given in (4.2.2) and are presented in Appendix-II.

The percent standard errors of the estimates based on four estimators have been computed and are presented in Table 4.2.2.

## 4.3 Results based on Step-wise Regression Analysis

The step-wise regression analysis of three sets of data at district level has finally provided the following fitted regression models, which are presented in the Table 4.3.1.

Table 4.3.1. Fitted Models by Step-wise Regression Analysis

Set	Models	R2 (%)	$\mathbf{SE}^{(\hat{\mathbf{Y}})}$	$\hat{\sigma}^2$
Set 1	Y= 5.54+0.21X3** (1.82) (0.02)	84.80	1.24	1.54
Set 2	Y= 14.50+0.11X3** (2.47) (0.02)	62.40	1.38	1.92
Set 3	Y= -156.72+1.71X1*+0.12X3** (59.92) (0.62) (0.02)	89.48	1.31	1.71

\*P<0.05, \*\*P<0.01 and  $\hat{\sigma}^2$  is the estimated residual variance. The figures in parentheses are standard error.

Using the fitted models given in the Table 4.3.1, the block estimates of wheat production based on four estimators have been computed and are presented in Appendix-III. The percent standard errors of the estimates based on four estimates have been computed and are presented in the Table 4.3.2.

**Table 4.3.2.** The Percent Standard Errors of the Estimates of Wheat Production based on Different Estimators and Different Sets of Data for the Year 2006–07 using Step-wise Regression Analysis

S. No.	Block	% Standard Error											
			Se	t I			Set				Set	III	
		$\hat{Z}_q$	$\widetilde{Z}_{q}^{(1)}$	$\widetilde{Z}_{q}^{(2)}$	$\widetilde{Z}_{q}^{(3)}$	$\hat{Z}_q$	$\widetilde{Z}_{q}^{(1)}$	$\widetilde{Z}_{q}^{(2)}$	$\widetilde{Z}_{q}^{(3)}$	$\hat{Z}_q$	$\widetilde{Z}_{q}^{(1)}$	$\widetilde{Z}_{q}^{(2)}$	$\widetilde{Z}_{q}^{(3)}$
1	Shukul bazaar	2.12	9.67	2.37	2.29	3.30	10.00	3.32	3.36	1.67	3.08	1.74	1.70
2	Jagdishpur	2.58	9.29	2.79	2.81	3.42	8.91	3.40	3.50	2.33	3.44	2.34	2.38
3	Musafhirkhana	3.01	9.93	3.36	3.22	4.12	9.62	4.13	4.20	2.62	3.77	2.64	2.66
4	Waldi Rai	1.31	11.33	1.92	1.37	1.67	10.61	1.97	1.69	1.48	3.25	1.71	1.50
5	Jamo	1.65	11.41	1.89	1.78	1.58	10.21	1.66	1.61	2.26	3.57	2.27	2.31
6	Shahgarh	2.95	6.51	3.14	3.22	4.44	7.39	4.39	4.53	2.46	2.96	2.46	2.51
7	Gauriganj	1.77	9.96	1.93	1.95	2.73	9.99	2.72	2.79	1.42	2.98	1.46	1.46
8	Amethi	3.08	7.85	3.27	3.36	4.72	8.85	4.67	4.82	2.61	3.33	2.60	2.66
9	Bhetua	2.77	8.07	3.05	2.98	4.33	8.88	4.33	4.41	2.35	3.19	2.38	2.39
10	Bhadar	3.87	7.52	4.14	4.19	6.07	9.12	6.01	6.18	3.61	4.09	3.59	3.68
11	Sangrampur	4.66	7.90	5.10	4.98	7.61	10.23	7.60	7.72	4.74	5.14	4.75	4.82
12	Dhanpatganj	1.48	11.00	1.70	1.61	2.05	10.69	2.11	2.09	1.37	3.17	1.44	1.40
13	Kurebhar	1.87	9.60	2.03	2.04	2.82	9.71	2.83	2.88	1.47	2.93	1.51	1.50
14	Jai Singh Pur	2.05	9.52	2.20	2.26	2.98	9.73	2.96	3.05	1.57	2.97	1.60	1.61
15	Kurwar	1.41	11.79	1.65	1.52	1.98	11.36	2.05	2.02	1.34	3.34	1.42	1.37
16	Dube Pur	2.17	9.70	2.34	2.36	3.33	10.04	3.32	3.41	1.65	3.07	1.64	1.69
17	Bhadaiyeea	1.67	8.30	1.80	1.82	2.51	8.30	2.54	2.56	1.40	2.57	1.47	1.44
18	Dostpur	1.94	10.58	2.04	2.10	3.34	10.82	3.33	3.41	1.50	3.19	1.56	1.54
19	Akhand Nagar	1.83	13.05	1.99	2.01	1.72	11.56	1.75	1.76	2.58	4.05	2.55	2.65
20	Lambhua	1.58	9.73	1.70	1.73	2.67	9.66	2.67	2.74	1.34	2.88	1.37	1.38
21	Pratap Pur Kamaicha	3.38	9.11	3.64	3.66	5.34	10.40	5.30	5.44	2.93	3.83	2.94	2.99
22	Kadipur	3.13	9.31	3.14	3.78	4.84	10.38	4.69	5.05	2.76	3.75	2.68	2.89

# 5. DISCUSSION AND CONCLUDING REMARKS

It can be observed from the results presented in the Tables 4.1.2, 4.2.2 and 4.3.2 that the percent standard error (PSE) of the block estimates of wheat production based on  $\hat{Z}_q$  is uniformly smaller than that based on  $\tilde{Z}_q^{(3)}$ , which also supports the finding based on theoretical comparison of  $\hat{Z}_q \, \mbox{and} \, \, \widetilde{Z}_q^{(3)}$  in section -2 . The superiority of  $\widetilde{Z}_{q}^{(1)}$  and  $\widetilde{Z}_{q}^{(2)}$  over  $\hat{Z}_{{}_{q}}$  is theoretically conditional and it is also very evident from the results of the aforesaid Tables. If we compare the percent standard error of  $\hat{Z}_{q}$  ,  $\widetilde{Z}_{q}^{(l)}$  and  $\widetilde{Z}_{q}^{(2)}$  from the aforesaid tables, we find that PSEs of block estimates of wheat production based on  $\widetilde{Z}_{q}^{(2)}$  are less than those based on  $\hat{Z}_{q}$  and  $\widetilde{Z}_{q}^{(l)}$  in case of 10, 11 and 9 blocks in the Table 4.1.2 (Singh et al methodology) for Sets I, II and III, respectively. PSEs of  $\widetilde{Z}_q^{(2)}$  are less than PSEs of  $\hat{Z}_{q}$  and  $\widetilde{Z}_{q}^{(1)}$  in case of 18, 19 and 21 blocks in the Table 4.2.2 for respective sets (Principal component methodology). Similarly, the PSEs of  $\widetilde{Z}_{q}^{(2)}$  are less than PSEs of  $\hat{Z}_{q}$  and  $\widetilde{Z}_{q}^{(l)}$  in case of 14, 18 and 16 blocks in the Table 4.2.3 for respective sets (step-wise regression methodology). Therefore, it is very obvious that the PSEs of  $\tilde{Z}_{q}^{(2)}$  are smaller than PSEs of  $\hat{Z}_{q}$ and  $\widetilde{Z}_{q}^{(1)}$  in most of the 22 blocks when methods of principal component analysis and step- wise regression analysis are applied for estimation of wheat production at block level. It can also be observed from the results of the Tables 4.1.2, 4.2.2 and 4.3.2 that the PSEs of  $\widetilde{Z}_q^{(2)}$  based on principal component analysis are smaller in most of the blocks except in one or two blocks than those based on Singh et al methodology and method of step- wise regression analysis. The results in terms of PSEs of the estimators based

on three sets of data have been found to be consistent for all three methodologies. It is, however, obvious from the results of the Tables that PSEs of the estimators are smaller for the larger set of the data.

It can also be observed from the results presented in Appendix-I, II, and III that the block estimates of wheat production based on  $\hat{Z}_{q}$ ,  $\tilde{Z}_{q}^{(1)}$ ,  $\tilde{Z}_{q}^{(2)}$  and  $\tilde{Z}_{q}^{(3)}$  are close to  $Z_{q}^{*}$  (based on CCEs) in most of the blocks except in few blocks. The block estimates which are not found close to  $Z_{q}^{*}$  might be probability because of local effects of blocks such as soil types, land types etc. which may not be uniformly similar across the blocks. The average distance (D) has been found to be uniformly less for  $\tilde{Z}_{q}^{(2)}$  in comparison with that of  $\hat{Z}_{q}$ ,  $\tilde{Z}_{q}^{(1)}$  and  $\tilde{Z}_{q}^{(3)}$ .

In view of the above discussion of the results of the empirical study the specific conclusions are as follows:

- 1. The  $\hat{Z}_q$  always performs better than  $\tilde{Z}_q^{(3)}$
- 2. The performance of  $\tilde{Z}_q^{(1)}$ , and  $\tilde{Z}_q^{(2)}$  over  $\hat{Z}_q$  are conditional as it is evident from the inequalities (2.16) and (2.17).
- 3. In the light of the empirical results,  $\tilde{Z}_q^{(2)}$  has generally performed better than  $\hat{Z}_{q'}$ ,  $\tilde{Z}_q^{(2)}$ and  $\tilde{Z}_q^{(3)}$ . Therefore, the estimator  $\tilde{Z}_q^{(2)}$  can be recommended for estimation of crop production at block level.
- 4. Larger sets of data should be used for better precision of the estimates.
- 5. Method of principal component analysis is recommended as it has provided more precise estimates in comparison to other methods.
- 6. If more number of auxiliary variables are available at district and block levels, the use of larger set of the auxiliary variables may

further improve the estimates of crop production at block level.

#### ACKNOWLEDGEMENT

The authors are thankful to the referee for his valuable suggestions and comments which have improved significantly the earlier version of the paper.

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# APPENDIX

Appendix I. Estimates of Wheat Production based on Different Estimators using Singh et al. Methodology

S. No.	Block	Zq*					Block Wi	ise Estim	ate (Lak	h Tonne	2)			
				Se	t I			Set				Set	III	
			$\hat{Z}_q$	$\widetilde{Z}_{q}^{\left(l\right)}$	$\widetilde{Z}_{q}^{\left(2\right)}$	$\widetilde{Z}_{q}^{\left( 3\right) }$	$\hat{Z}_{q}$	$\widetilde{Z}_{q}^{\left(l\right)}$	$\widetilde{Z}_{q}^{\left(2\right)}$	$\widetilde{Z}_{q}^{\left( 3\right) }$	$\hat{Z}_{q}$	$\widetilde{Z}_{q}^{\left(l\right)}$	$\widetilde{Z}_{q}^{\left(2\right)}$	$\widetilde{Z}_{q}^{\left( 3\right) }$
1	Shukul bazaar	1.883	1.916	1.849	1.841	1.859	1.853	1.831	1.828	1.835	1.876	1.833	1.827	1.840
2	Jagdishpur	1.981	2.162	2.086	2.086	2.090	2.162	2.136	2.137	2.137	2.031	1.985	1.983	1.989
3	Musafhirkhana	1.663	1.762	1.700	1.686	1.714	1.762	1.741	1.737	1.745	1.598	1.561	1.549	1.572
4	Waldi Rai	1.426	1.630	1.572	1.554	1.589	1.630	1.610	1.605	1.616	1.319	1.289	1.270	1.301
5	Jamo	2.304	2.197	2.120	2.122	2.123	2.268	2.241	2.243	2.240	2.157	2.107	2.108	2.109
6	Shahgarh	1.361	1.721	1.661	1.646	1.676	1.696	1.676	1.672	1.681	2.054	2.007	2.005	2.011
7	Gauriganj	2.428	2.292	2.212	2.217	2.211	2.279	2.252	2.255	2.252	2.367	2.313	2.319	2.310
8	Amethi	1.690	2.041	1.969	1.965	1.976	1.933	1.910	1.908	1.913	2.083	2.035	2.034	2.039
9	Bhetua	1.466	1.807	1.744	1.732	1.757	1.707	1.687	1.682	1.692	1.757	1.717	1.709	1.726
10	Bhadar	1.381	1.800	1.736	1.724	1.750	1.655	1.635	1.630	1.640	1.830	1.788	1.782	1.796
11	Sangrampur	1.121	1.458	1.407	1.383	1.426	1.295	1.279	1.270	1.286	1.447	1.414	1.399	1.426
12	Dhanpatganj	2.331	2.131	2.056	2.055	2.061	2.119	2.094	2.094	2.095	2.100	2.052	2.052	2.056
13	Kurebhar	2.180	2.271	2.191	2.196	2.192	2.223	2.196	2.198	2.196	2.221	2.170	2.173	2.171
14	Jai Singh Pur	2.320	2.371	2.287	2.295	2.284	2.318	2.290	2.293	2.289	2.365	2.311	2.316	2.308
15	Kurwar	2.360	2.044	1.972	1.969	1.980	2.031	2.006	2.006	2.009	1.992	1.946	1.944	1.952
16	Dube Pur	2.147	2.132	2.057	2.057	2.062	2.078	2.053	2.053	2.055	2.135	2.086	2.087	2.089
17	Bhadaiyeea	1.762	2.152	2.076	2.077	2.081	2.123	2.098	2.098	2.099	2.108	2.060	2.060	2.063
18	Dostpur	2.295	2.214	2.136	2.139	2.139	2.147	2.122	2.122	2.123	2.113	2.065	2.065	2.068
19	Akhand Nagar	3.147	2.586	2.495	2.510	2.483	2.708	2.676	2.683	2.669	2.600	2.540	2.551	2.531
20	Lambhua	2.350	2.350	2.267	2.275	2.265	2.337	2.309	2.312	2.308	2.371	2.316	2.322	2.314
21	Pratap Pur Kamaicha	1.772	1.799	1.735	1.723	1.749	1.688	1.668	1.663	1.673	1.839	1.797	1.790	1.804
22	Kadipur	4.130	4.320	4.168	4.245	4.032	4.034	3.986	4.009	3.947	4.199	4.103	4.151	4.020
	Total	45.496	47.157	45.496	45.496	45.496	46.045	45.496	45.496	45.496	46.562	45.496	45.496	45.496
	Di (Average Distance)		0.255	0.249	0.244	0.255	0.209	0.211	0.208	0.214	0.280	0.278	0.276	0.281

 $Z_a^{T}$  Crop production (Lakh tonne) based on CCEs

S. No.	Block	Zq*				Bl	ock Wise	Estimat	e (Lakh ]	Fonne)				
		_		Se	t I			Set	t II			Set	III	
			$\hat{Z}_q$	$\widetilde{Z}_{q}^{\left(l\right)}$	${\widetilde Z}_q^{(2)}$	$\widetilde{Z}_{q}^{\left( 3\right) }$	$\hat{Z}_q$	$\widetilde{Z}_{q}^{\left(l\right)}$	$\widetilde{Z}_{q}^{\left(2\right)}$	$\widetilde{Z}_{q}^{\left( 3\right) }$	$\hat{Z}_q$	$\widetilde{Z}_{q}^{\left(l\right)}$	${\rm \widetilde{Z}}_q^{(2)}$	$\widetilde{Z}_{q}^{\left( 3\right) }$
1	Shukul bazaar	1.883	1.869	1.865	1.864	1.865	1.841	1.841	1.840	1.841	1.843	1.851	1.852	1.850
2	Jagdishpur	1.981	1.966	1.961	1.961	1.962	1.930	1.930	1.930	1.930	1.896	1.905	1.905	1.903
3	Musafhirkhana	1.663	1.433	1.430	1.429	1.431	1.435	1.434	1.434	1.434	1.438	1.445	1.448	1.443
4	Waldi Rai	1.426	1.344	1.341	1.339	1.342	1.356	1.355	1.355	1.355	1.294	1.300	1.304	1.297
5	Jamo	2.304	2.519	2.513	2.514	2.512	2.475	2.474	2.474	2.474	2.476	2.488	2.486	2.489
6	Shahgarh	1.361	1.895	1.891	1.890	1.891	1.948	1.947	1.947	1.947	1.941	1.951	1.951	1.949
7	Gauriganj	2.428	2.448	2.443	2.444	2.442	2.421	2.420	2.420	2.420	2.441	2.452	2.450	2.454
8	Amethi	1.690	1.914	1.910	1.909	1.910	1.886	1.885	1.885	1.885	1.855	1.864	1.865	1.863
9	Bhetua	1.466	1.630	1.626	1.625	1.627	1.619	1.618	1.618	1.619	1.627	1.635	1.637	1.633
10	Bhadar	1.381	1.533	1.530	1.529	1.531	1.548	1.548	1.548	1.548	1.553	1.560	1.563	1.558
11	Sangrampur	1.121	1.126	1.123	1.121	1.125	1.131	1.131	1.130	1.131	1.141	1.146	1.151	1.144
12	Dhanpatganj	2.331	2.185	2.180	2.180	2.180	2.182	2.181	2.181	2.181	2.164	2.174	2.173	2.174
13	Kurebhar	2.180	2.227	2.222	2.223	2.222	2.242	2.242	2.242	2.242	2.246	2.257	2.256	2.257
14	Jai Singh Pur	2.320	2.359	2.353	2.354	2.353	2.345	2.344	2.345	2.344	2.361	2.372	2.371	2.373
15	Kurwar	2.360	2.120	2.115	2.115	2.115	2.096	2.095	2.095	2.095	2.068	2.078	2.078	2.077
16	Dube Pur	2.147	2.075	2.070	2.070	2.071	2.084	2.083	2.083	2.083	2.097	2.107	2.107	2.107
17	Bhadaiyeea	1.762	2.155	2.150	2.150	2.150	2.140	2.139	2.139	2.139	2.151	2.162	2.161	2.161
18	Dostpur	2.295	2.149	2.145	2.145	2.145	2.082	2.081	2.081	2.081	2.001	2.010	2.010	2.009
19	Akhand Nagar	3.147	2.799	2.793	2.794	2.791	2.992	2.991	2.991	2.990	2.967	2.982	2.977	2.986
20	Lambhua	2.350	2.443	2.438	2.439	2.437	2.390	2.389	2.389	2.389	2.420	2.431	2.429	2.432
21	Pratap Pur Kamaicha	1.772	1.641	1.637	1.636	1.638	1.627	1.627	1.627	1.627	1.653	1.661	1.663	1.659
22	Kadipur	4.130	3.769	3.761	3.764	3.755	3.742	3.740	3.741	3.739	3.649	3.666	3.659	3.677
	Total	45.496	45.598	45.496	45.496	45.496	45.512	45.496	45.496	45.496	45.281	45.496	45.496	45.496
	Di (Average Distance)		0.217	0.217	0.217	0.217	0.214	0.214	0.214	0.215	0.230	0.230	0.230	0.229

Appendix II. Estimates of Wheat Production based on Different Estimators using Principal Component Analysis

 $Z_q^*$ Crop production (Lakh tonne) based on CCEs

Appendix III. Estimates of Wheat Production based on Different Estimators using Step Wise Regression Analysis

S. No.	Block			Block Wise Estimate (Lakh Tonne)										
				Se	t I			Se	t II			Set	III	
		Zq*	$\hat{Z}_{q}$	$\widetilde{Z}_{q}^{\left(l\right)}$	$\widetilde{Z}_{q}^{\left(2\right)}$	$\widetilde{Z}_{q}^{\left( 3\right) }$	$\hat{Z}_{q}$	$\widetilde{Z}_{q}^{\left(l\right)}$	$\widetilde{Z}_{q}^{\left(2\right)}$	$\widetilde{Z}_{q}^{\left( 3\right) }$	$\hat{Z}_q$	$\widetilde{Z}_{q}^{\left(l\right)}$	$\widetilde{Z}_{q}^{\left( 2\right) }$	$\widetilde{Z}_{q}^{\left( 3\right) }$
1	Shukul bazaar	1.883	2.016	1.831	1.807	1.863	1.866	1.817	1.810	1.826	1.877	1.830	1.824	1.838
2	Jagdishpur	1.981	2.244	2.038	2.035	2.055	2.244	2.185	2.189	2.186	2.020	1.969	1.966	1.974
3	Musafhirkhana	1.663	1.777	1.614	1.568	1.658	1.777	1.731	1.722	1.740	1.579	1.540	1.526	1.552
4	Waldi Rai	1.426	1.281	1.163	1.072	1.219	1.281	1.248	1.226	1.262	1.272	1.240	1.219	1.254
5	Jamo	2.304	2.063	1.873	1.854	1.903	2.149	2.093	2.094	2.096	2.154	2.100	2.101	2.102
6	Shahgarh	1.361	2.368	2.150	2.159	2.157	2.094	2.039	2.038	2.043	2.120	2.066	2.066	2.070
7	Gauriganj	2.428	2.504	2.274	2.295	2.268	2.368	2.306	2.312	2.303	2.389	2.329	2.335	2.325
8	Amethi	1.690	2.363	2.146	2.154	2.153	2.075	2.021	2.020	2.026	2.101	2.048	2.048	2.052
9	Bhetua	1.466	1.955	1.776	1.746	1.811	1.746	1.700	1.690	1.711	1.758	1.714	1.705	1.724
10	Bhadar	1.381	2.163	1.964	1.954	1.987	1.821	1.773	1.765	1.782	1.847	1.801	1.794	1.809
11	Sangrampur	1.121	1.777	1.614	1.568	1.658	1.433	1.395	1.377	1.409	1.456	1.420	1.403	1.433
12	Dhanpatganj	2.331	2.161	1.963	1.952	1.985	2.088	2.033	2.032	2.037	2.103	2.050	2.049	2.054
_	Kurebhar	2.180	2.339	2.124	2.130	2.133	2.198	2.141	2.143	2.142	2.222	2.166	2.169	2.167
14	Jai Singh Pur	2.320	2.523	2.292	2.314	2.284	2.345	2.284	2.290	2.282	2.374	2.314	2.321	2.311
15	Kurwar	2.360	2.038	1.851	1.829	1.882	1.984	1.932	1.928	1.938	1.990	1.940	1.937	1.946
16	Dube Pur	2.147	2.296	2.085	2.087	2.097	2.119	2.064	2.064	2.067	2.145	2.091	2.092	2.094
17	Bhadaiyeea	1.762	2.191	1.990	1.982	2.011	2.086	2.032	2.031	2.036	2.109	2.056	2.056	2.060
18	Dostpur	2.295	2.229	2.025	2.020	2.042	2.086	2.032	2.031	2.036	2.106	2.053	2.052	2.056
19	Akhand Nagar	3.147	2.460	2.235	2.251	2.233	2.591	2.523	2.536	2.513	2.607	2.541	2.553	2.531
20	Lambhua	2.350	2.474	2.247	2.265	2.243	2.372	2.310	2.316	2.307	2.383	2.323	2.329	2.320
21	Pratap Pur Kamaicha	1.772	2.116	1.922	1.907	1.947	1.828	1.781	1.773	1.790	1.856	1.810	1.803	1.818
22	Kadipur	4.130	4.757	4.321	4.549	3.906	4.166	4.057	4.110	3.965	4.203	4.097	4.150	4.007
	Total	45.496	50.093	45.496	45.496	45.496	46.715	45.496	45.496	45.496	46.671	45.496	45.496	45.496
	Di (Average Distance)		0.444	0.383	0.392	0.385	0.290	0.287	0.285	0.291	0.292	0.288	0.287	0.292

 $Z_q^*$ Crop production (Lakh tonne) based on CCEs