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Fitting Exponential Smooth Transition Autoregressive Nonlinear Time-Series Model using Particle Swarm Optimization Technique

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SUMMARY

Exponential Smooth Transition Autoregressive (ESTAR) family of parametric nonlinear time-series models is considered. The methodology for estimation of its parameters using a powerful Particle Swarm Optimization (PSO) technique is discussed. Further, simulation study is also carried out to test the validity of the proposed methodology. A heartening feature of ESTAR model is that, as opposed to some other nonlinear models involving regimes switching, the change between the extreme regimes is smooth and is assumed to be defined by a bounded continuous function of a transition variable. Further, it is capable of describing those datasets that depict cyclicity. As an illustration, it is employed for modelling and forecasting of Oil sardine, Mackerel and Bombay duck time-series landings data in India. Finally, the performance of fitted ESTAR model is also compared by computing goodness-of-fit statistic and various measures of forecast performance. It is concluded that fitted ESTAR model perform better than ARIMA methodology for the datasets under consideration.

Keywords: Exponential Smooth Transition Autoregressive model, Particle Swarm Optimization, Cyclicity, Goodness-of-fit, Forecast performance, Fish landings data, ARIMA.

1. INTRODUCTION

Time-series is a sequence of observations taken sequentially in time. Understanding, description of the generating mechanism and then forecasting of future values based on past values are prime objectives of time-series.Statisticians possess a well-established methodology based on linear time-series models, called the ARIMA methodology, for modelling and forecasting of such data. These models are quite popular due to their relative simplicity and also due to the fact that there exists numerous computer software incorporating the same. The ARIMA model, however,is insufficient as it is not able to capture many important features. This methodology was employed by Noble and Sathianandan (1991) for investigating the trend analysis in all-India Mackerel catches. The three statistical methodologies, viz. regression, univariate and multivariate time-series methods for modelling and forecasting fish catches were compared by Venugopalan and Srinath (1998). An elaborate study of the all-India landings of Oil sardine, Mackerel and Bombay duck fish species using the more advanced technique of Spectral decomposition was done by Sathianandan and Alagaraja (1998). Nampoothiri and Balakrishna (2000) applied the Threshold autoregressive model for a time-series data. For over three decades, time-series analysis has moved towards the nonlinear domain, which is generally more appropriate for accurately describing dynamics of a time-series, for making better multi-step-ahead forecasts and also in terms of fitting as well as forecasting when the data are nonlinearly related with their past.

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Kanai *et al.* (2010) have developed the estimation function approach for fitting conditional heteroscedastic autoregressive family. Exponential autoregressive model, which is capable of describing cyclical data, belongs to this family. One important family of parametric nonlinear time-series model is the Smooth Transition Autoregressive (STAR) family of parametric nonlinear time-series models, which was introduced for modelling and forecasting of "cyclical" data. One approach for parameter estimation of this model is through nonlinear least squares given by Terasvirta (1994). But, it is only an approximate method and moreover it has problems with convergence of the estimates of parameters. A new method proposed is through the minimization of objective function based on Particle Swarm Optimization (PSO) technique.This approach gives a simple and efficient algorithm to handle a nonlinear model.

In this paper, after a brief introduction in Section 1, the next section gives a description of ESTAR models. In Section 3, PSO along with its working principle for parameter estimation is described. Section 4 illustrates the fitting of ESTAR models to Oil sardine, Mackerel and Bombay duck landings data ('000 tonnes) and simulation is carried out to study performance of estimation procedure. Finally, Section 5 is concerned with some concluding remarks on the merits of PSO and delineation of some problems for future research work.

2. DESCRIPTION OF ESTAR MODELS

The main idea of Threshold autoregressive (TAR) model is to describe a given stochastic process by a piecewise linear autoregressive model, where the determination as to whether each of the sub-models is active or not depends on the value of a known variable. The STAR model proposed by Terasvirta (1994) is a generalization of the threshold models, avoiding discontinuities in the autoregressive parameters since the transition from one regime to the next is driven by a continuous function. Other important feature of the STAR model is that it is also capable of capturing the non-Gaussian characteristics of a time-series.

The STAR equation may be written as:

$$y_t = G(z_t, y_t; \psi) = \boldsymbol{\varphi' w_t} + (\boldsymbol{\theta' w_t}) G_k^L(\gamma, c; y_{t-d}) + \in_t \tag{2.1}$$

where $\in_t$ is a sequence of normal $(0, \sigma^2)$ independent errors, $\varphi = (\phi_0, \phi_1, ..., \phi)'$ and $\theta = (\theta_0, \theta_1, ..., \theta)'$ are $(p + 1)$ parameter vector, $\boldsymbol{w_t} = (1, y_{t-1}, ..., y_{t-p})$ is the vector consisting of an intercept and the first p lags of y_t and $G_k^L(\gamma, c; y_{t-d})$ is known as transition function.The behaviour of the autoregressive coefficients is essentially constant in each regime with a continuous smooth change between regimes, whose speed is controlled by the constant $\gamma_L (> 0)$. The Self-Exciting Threshold Autoregressive (SETAR) model may be interpreted as a special case of the STAR model, when γ tends to infinity. Depending upon the forms of transition function, different forms of STAR models are defined. If the function is written as

$$G_k^L(\gamma, c; y_{t-d}) = 1 - \exp\{-\gamma(y_{t-d} - c)^2\}, \gamma > 0 \tag{2.2}$$

Terasvirta (1994) called this model as Exponential Smooth Transition Autoregressive (ESTAR) model. It has the property that the minimum value of the transition function equals zero.

It may also be kept in mind that other nonlinear as well as linear models appear as special cases of the STAR specifications. The main role of the transition function $G_k^L(\gamma, c; y_{t-d})$ is that it allows the coefficients for lagged values of y_t to change smoothly with y_t. This allows local dynamics of the model to change with, y_t. The ESTAR model allows local dynamics to be different for high and low values of the transition variable, y. To model nonlinear effects of a shock,local dynamics as a function of a lagged value of is modelled.For instance, if a shock of a negative kind pushes a characterisation away from a locally stable regime, the change in the value of G subsequently changes the local dynamics. If this regime contains a pair of explosive complex roots, may be returned to the previous level a lot more quickly than would be the case, if it followed a linear AR process. This means the local dynamics are the same for high as well as for low values of y_{t-d}, whereas behaviour of the variable is different for the mid-range. Further the transition function for ESTAR allows y_t to move smoothly between very small and very large values for which local dynamics are stable.

3. ESTIMATION OF PARAMETERS USING PARTICLE SWARM OPTIMIZATION (PSO) TECHNIQUE

STAR model is generally fitted by using either the nonlinear least squares algorithm or the maximum likelihood method under Gaussian assumption. The function to be optimized is the Normalized AIC (NAIC) criterion. Fitting of nonlinear time-series models is also carried out through "search algorithm" (Tong 1995). However, main drawback of this algorithm is that the number of possible models to be searched is extremely large. Further, it requires a large computation time. When the number of maximum order of lag is small, all possible models could be tried; otherwise, complete enumeration and evaluation is practically not feasible. Genetic algorithm (GA) has also been employed for estimation of parameters in SETAR models, a form of STAR model. But, there are several limitations of the use of GA such as no absolute assurance that it will find a global optimum and also it cannot assure constant optimisation response times. Moreover, GA is capable of finding the solution through evolution operators, such as crossover and mutation. Evolution is inductive. In reality, life does not always evolve towards an optimal solution.

To this end, a very efficient stochastic optimization technique, viz. Particle Swarm Optimization (PSO) technique, can be employed (Parsopoulos and Vrahatis 2010). PSO methodology is capable of rectifying above limitations. In contrast to GA, which exploits the competitive characteristics of biological evolution, PSO simulates cooperative and social behaviour, such as fish schooling, birds flocking, or insects swarming. The PSO algorithm optimizes a problem by iteratively trying to improve a candidate solution with regard to an objective function. PSO optimizes a problem by having a population of candidate solutions, usually referred to as "particles" and moving these particles around in the search-space according to simple mathematical formulae over the particle's position and velocity.

The basic PSO algorithm can be described in vector notation as follows:

$$\vec{\theta}_{t+1} = \vec{\alpha} \otimes \vec{\theta}_t + \vec{b}_1 \otimes \vec{r} \otimes (\vec{p}_1 - \vec{x}_t) + \vec{b}_2 \otimes \vec{r}_2 \otimes (\vec{p}_2 - \vec{x}_t) \qquad (3.1)$$

$$\vec{x}_{t+1} = \vec{c} \otimes \vec{x}_t + \vec{d} \otimes \vec{\theta} \qquad (3.2)$$

The symbol $\otimes$ denotes element-by-element vector multiplication. At iteration t, the velocity is updated depending on its current value which is affected by a momentum factor $(\vec{d})$ and also on a term which attracts the particle towards previously found best positions: its own previous best position (P_1) and globally best position in the whole swarm (P_2). The strength of attraction is given by the two coefficients b_1 and b_2. The particle position is updated iteratively using its current value and the newly computed velocity $\vec{x}$, affected by coefficients c and d respectively and can be set to unity without loss of generality. Randomness which is useful for good state space exploration is introduced by making use of the vectors of random numbers c and d. They are generally selected to be a uniform random numbers in the range [0, 1].

Parsopoulos and Vrahatis (2010) have given a good description on PSO.

4. AN ILLUSTRATION

4.1 Preliminary Data Analysis

Three time-series datasets, viz. Oil sardine, Mackerel and Bombay duck landings data ('000 tonnes)

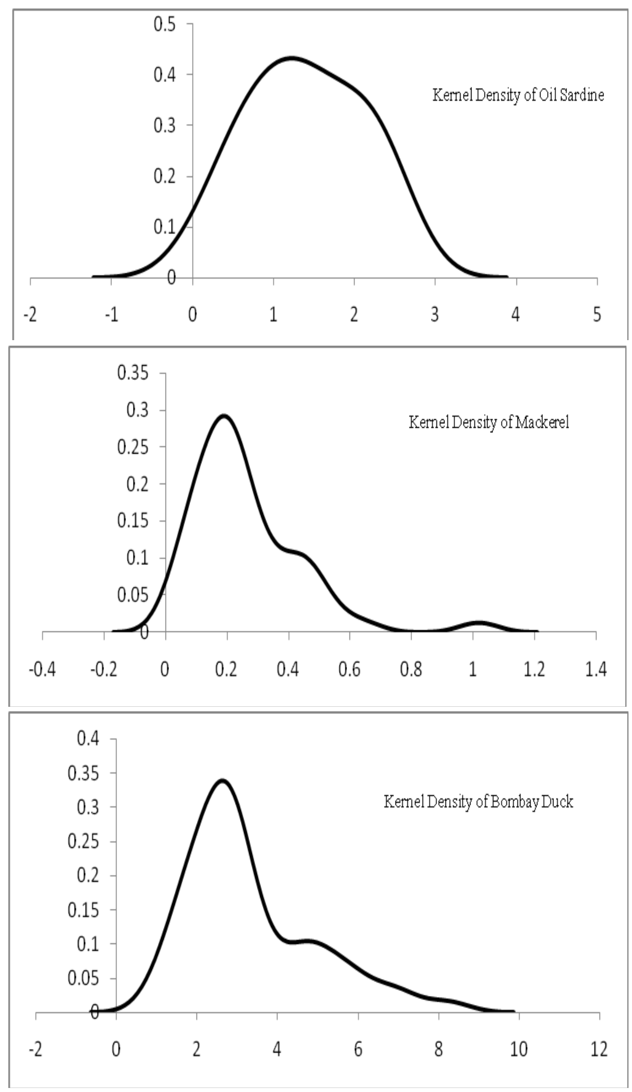

Fig. 1. Kernel density of the three datasets

of Kerala, Karnataka and Maharashtra states of India respectively, from 1961 to 2008 are considered for illustration purpose. These are obtained from Central Marine Fisheries Research Institute (CMFRI), Kochi, India. We use the first 45 data points corresponding to fish landings for the period 1961 to 2005 for building the model and the remaining 3 data points are used for validation purpose. In the first instance, Kernel density estimates of three fish landings data are obtained and exhibited in Fig. 1.

It is observed that all the densities exhibit non-Gaussianity. Oil sardine data have a fairly concentrated centre. In case of Mackerel landings data, the density exhibits short tail behaviour, whereas for Bombay duck landings data, density is positively skewed with long tail. Obviously, conventional ARIMA modelling approach for the given three landings time-series data may not be able to describe these datasets satisfactorily (Fan and Yao 2003).

Further, graph of the landings data in the conventional and the reversed time order exhibit a periodic-like fluctuation. The time-series data donot show the similar look while moving in the forward and backward directions with respect to time. This indicates that the statistical properties of the data are not invariant under reversibility of time direction. Therefore, the landings data are not realization of static transformation of a linear Gaussian random process.

4.2 Test of Linearity

In order to test for presence of nonlinearity in the dataset, Keenan's test, Tsay's test and likelihood ratio test are performed. It is seen that the data points depend on their lags in a nonlinear manner. It is also seen that the likelihood ratio test statistic attains the maximum value at $d = 2$; therefore the delay parameter in this situation may be kept as 2.

The preliminary data analysis justifies the application of ESTAR nonlinear time-series model to describe Oil sardine, Mackerel and Bombay duck landings time-series data. The PSO technique is applied for parameter estimation. Accordingly, several ESTAR models were fitted to the data and the best model was identified on the basis of minimum NAIC criterion. The number of particles generated initially was 100, and the inertia weight was initially set as 0.95 and reduced to 0.05, and $b_1 = b_2 = 1$. The maximum iteration was taken as 500. The accuracy or stopping criteria of search procedure for optimum value is kept as 10^{-5}. The best ESTAR models for Oil sardine, Mackerel and Bombay duck landings time-series data are respectively obtained as

$$Y_t = 4345.79 + 0.43Y_{t-1} + 0.03Y_{t-2} + [2826.99 + 0.44Y_{t-1} + 0.01Y_{t-2}][1 - \exp\{-0.5(Y_{t-2} - 1.01)^2\}]$$

$$Y_t = 1853.64 + 0.32Y_{t-1} + 0.05Y_{t-2} + [983.54 + 0.21Y_{t-1} + 0.07Y_{t-2}][1 - \exp\{-0.6(Y_{t-2} - 0.83)^2\}]$$

$$Y_t = 763.35 + 0.69Y_{t-1} + 0.09Y_{t-2} + [486.34 + 0.38Y_{t-1} + 0.32Y_{t-2}][1 - \exp\{-0.5(Y_{t-2} - 1.21)^2\}]$$

To get a visual idea, fitted ESTAR models along with data points are exhibited in Fig. 3.1-3.3. A perusal of the graphs shows that fitted models are able to capture properly the cyclical behaviour present at various time epochs. Further, the values of Ljung-Box Q are computed, using maximum lag as 10, and they are all found to be less than the tabulated values. This

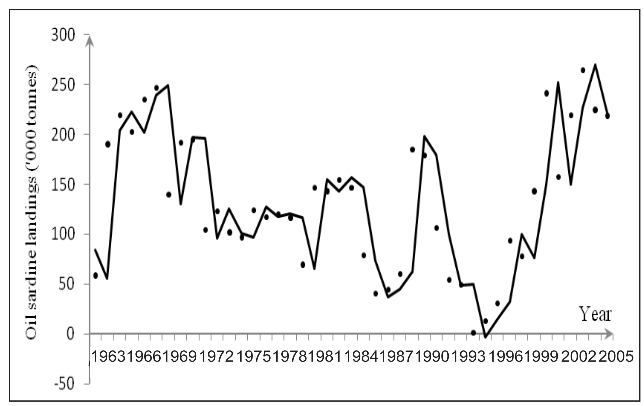

Fig. 3.1 The fitted ESTAR model along with Oil sardine landings data points

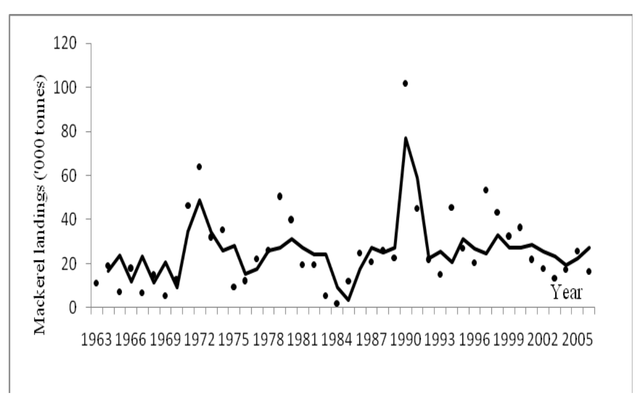

Fig. 3.2 The fitted ESTAR model along with Mackerel landings data points

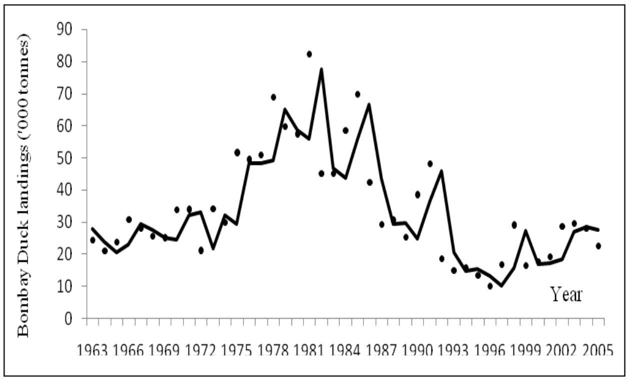

Fig. 3.3 The fitted ESTAR model along with Bombay duck landings data points

indicates that the fitted ESTAR models are properly specified.

4.3 Simulation

Simulation study is carried out to validate the proposed method of estimation. First, we produce a sample from the ESTAR model. Then, we generate 200 replications of samples of size n=45 for some values of the parameters. Applying the method developed, we estimated the parameters. For each of the parameters, the mean and the variance of the estimates of 200 replications were taken as the estimated value and the estimated variance of the corresponding estimator. The performance is shown in Table 1.

Table 1. Results of Simulation study

ϕ_0	ϕ_1	ϕ_2	$\hat{\phi}_0$	$\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{Var}(\hat{\phi}_0, \hat{\phi}_1, \hat{\phi}_2)$	γ	$\hat{\gamma}$	$\hat{Var}(\hat{\gamma})$
–2	–1	–1	–1.88	–0.92	–1.2	(0.043, 0.075, 0.056)	0.15	0.16	0.069
–2	–1	–0.5	–2.02	–1.03	–0.51	(0.091, 0.037, 0.038)	0.25	0.22	0.021
–1	–0.5	0.5	–0.92	–0.48	0.46	(0.054, 0.029, 0.061)	0.35	0.31	0.043
–1	–0.5	1	–0.94	–0.47	0.94	(0.049, 0.038, 0.064)	0.45	0.43	0.028
1	0.5	–1	1.11	0.53	–0.97	(0.048, 0.029, 0.054)	0.55	0.51	0.062
1	0.5	–0.5	0.92	0.52	–0.52	(0.069, 0.073, 0.044)	0.65	0.66	0.082
2	1	0.5	1.84	1.04	0.43	(0.053, 0.066, 0.023)	0.75	0.72	0.054
2	1	1	2.15	1.03	1.12	(0.045, 0.049, 0.063)	0.85	0.83	0.063

θ_0	θ_1	θ_2	$\hat{\theta}_0$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{Var}(\hat{\theta}_0, \hat{\theta}_1, \hat{\theta}_2)$	c	$\hat{c}$	$\hat{Var}(\hat{c})$
–2	–1	–1	–2.21	–1.31	–1.16	(0.032, 0.054, 0.076)	0.7	0.61	0.041
–2	–1	–0.5	–1.86	–0.92	–0.41	(0.054, 0.051, 0.042)	0.8	0.74	0.032
–1	–0.5	0.5	–0.92	–0.52	0.48	(0.058, 0.084, 0.071)	0.9	0.96	0.051
–1	–0.5	1	–1.21	–0.42	1.19	(0.049, 0.039, 0.029)	1.0	0.97	0.071
1	0.5	–1	1.11	0.54	–0.92	(0.084, 0.051, 0.062)	1.1	1.21	0.043
1	0.5	–0.5	0.92	0.47	–0.42	(0.065, 0.050, 0.043)	1.2	1.26	0.037
2	1	0.5	2.09	0.91	0.44	(0.076, 0.065, 0.061)	1.3	1.18	0.032
2	1	1	1.92	1.13	0.93	(0.081, 0.032, 0.070)	1.4	1.29	0.097

4.4 Comparison of ESTAR and ARIMA

In this section, the fitted ESTAR models are compared with ARIMA models from modelling and forecasting point of view. To this end, for the datasets under consideration ARIMA models were fitted. On the basis of minimum NAIC criterion, the ARIMA (2) model were selected and the results obtained are reported for all the three datasets in Table 2.

Table 2. Estimates of parameters

Parameters	Oil sardine	Mackerel	Bombay duck
Constant	4345.40	1853.53	763.60
AR 1	0.77	0.38	0.65
AR 2	–0.08	–0.08	0.12

The Normalized Akaike information criterion (NAIC), Bayesian information criterion (BIC) and Mean square error (MSE) are computed to evaluate the goodness-of-fit of the fitted ESTAR and ARIMA models. The forecast performance of fitted models was compared on the basis of Mean square prediction error (MSPE), Mean absolute prediction error (MAPE) and Relative mean absolute prediction error (RMAPE).

Fitted ESTAR are throughout found to be lower than the corresponding ones for fitted ARIMA models as reported in Table 3 and 4.

Table 3. Goodness-of-fit performance

Criterion	Oil sardine		Mackerel		Bombay duck	
	ARIMA	ESTAR	ARIMA	ESTAR	ARIMA	ESTAR
NAIC	19.27	17.89	6.32	5.99	5.32	4.81
BIC	274.91	266.81	258.70	217.87	237.43	212.62
MSE	312.76	289.67	289.26	249.97	268.56	251.45

Table 4. Forecast performance

Criterion	Oil sardine		Mackerel		Bombay duck	
	ARIMA	ESTAR	ARIMA	ESTAR	ARIMA	ESTAR
MAPE	32.49	16.76	9.13	7.51	1.89	1.61
MSPE	1282.77	348.88	145.71	133.93	7.05	6.76
RMAPE	13.59	6.66	24.10	18.76	7.72	6.98

In view of the above, superiority of ESTAR model over ARIMA model for fitting as well as forecasting purposes for the datasets under consideration is demonstrated.

6. CONCLUDING REMARK

In this paper, the estimation of parameters of ESTAR model through a powerful optimization technique of PSO is described. It is suggested that, for modelling and forecasting of cyclical time-series data, researchers should make use of this model rather than the ARIMA model. As future work, effort can also be directed towards considering more parsimonious ESTAR models.

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