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Balanced Ternary, Ternary Group Divisible and Nested Ternary Group Divisible Designs

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SUMMARY

This paper is concerned with the recursive construction of balanced ternary (BT), ternary group divisible (TGD) and nested ternary group divisible (NTGD) designs through a set of balanced incomplete block (BIB) designs. An illustrative example in each case has been added separately. The efficiency of BT design has also been computed.

Keywords: Balanced incomplete block design (BIBD), Balanced Ternary design (BTD), Ternary group divisible (TGD) design, Nested ternary group divisible (NTGD) design, Intercropping experiments.

1. INTRODUCTION

Balanced *n*-ary designs were introduced by Tocher (1952) as a generalization of balanced incomplete block (BIB) design. In this design, the incidence matrix can take any value of n out of possible values, often 0, 1, 2… $(n - 1)$. If $n = 3$, we get a ternary block design. Billington (1984, 1989) have extensively given results on balanced ternary designs. These designs may not exist for all parametric combinations or even if exist may require a large number of replications. In the present paper, we provide a new method of construction of BT, TGD and NTGD designs using a set of balanced incomplete block designs.

2. DEFINITION AND NOTATIONS

2.1 Balanced Ternary Design

A balanced *n-ary* design with parameters V, B, R, K, Λ and incidence matrix $\mathbf{N} = ((n_{ij}))$ is an arrangement of V treatments in B blocks, each of cardinality $K (K \le V)$ such that (*i*) the i^{th} treatment appears n_{ij} times in the j^{th} block where n_{ij} can take any of the values 0, 1, 2, … $n - 1$. (ii) each treatment occurs R times, and (iii) $\Sigma_{j=1}^{j=B} n_{ij} n_{i'j} = \Lambda$ for all $i \neq i' = 1, 2, \ldots V$. Note that $\Sigma_{j=1}^{j=B} n_{ij} = R$ for all i and $\Sigma_{j=1}^{j=V} n_{ij} = K$ for all j. For $n = 2$ (binary) the design is called a BIB design with the usual coincidence number $\lambda = \Lambda$. When $n = 3$ we use the term "balanced ternary design". Thus, a balanced ternary design is a collection of B blocks, each of cardinality K $(K \le V)$, chosen from a set of size V in such a way that each of the V treatments occurs R times altogether, each of the treatments occurring once in precisely in Q_1 blocks and twice in precisely Q_2 blocks, and with incidence matrix having inner product of any two rows Λ is denoted by BTD $(V, B, Q_1, Q_2, R, K, \Lambda)$. It is to be noted that $Q_1 + 2Q_2 = R$ (Gupta *et al*. 1995, Sarvate and Seberry 1993).

2.2 Ternary Group Divisible Design

Let A be a set of V treatments arranged in B blocks. Let $\{G_i | i = 1, 2, \ldots, m\}$ be a partition of A into m distinct

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sets each of size n, called groups. The groups define an association scheme on A with two classes; two treatments are first associates if they belong to the same group and are second associates otherwise. A TGD design with parameters ($V = mn$, B, Q_1, Q_2, R, K, Λ_1, Λ_2) is defined to be an incidence structure satisfying

$$\sum_{j=1}^{B} n_{ij} = R \text{ for each } i = 1, 2, \ldots, V$$

$$\sum_{i=1}^{V} n_{ij} = K \text{ for each } j = 1, 2, \ldots, B$$

and $\sum_{k=1}^{B} n_{ik} n_{jk} = \Lambda_1$ if treatments i & j ($i \neq j$) are first associates

$= \Lambda_2$ otherwise

where n_{ij} = number of times i^{th} treatment occurs in j^{th} block, $n_{ij} = \{0, 1, 2\}$, $i = 1, 2, \ldots, V$; $j = 1, 2, \ldots, B$.

Each treatment occurs with multiplicity '1' in Q_1 blocks and with multiplicity '2' in Q_2 blocks.

For a TGD, the following two results are true

$VR = BK$

$\Lambda_1 (n - 1) + \Lambda_2 (V - n) = R(K - 1) - 2\, Q_2$.

2.3 Nested Ternary Group Divisible Design

A nested ternary group divisible design with parameters $V = mn$, B_1, B_2, Q_1, Q_2, R_1, R_2, K_1, K_2, Λ_1, Λ_2) is an arrangement of V treatments each replicated R_1, R_2 times with two systems of blocking such that:

(i) the second system is nested within the first with each block from the first system (subsequently referred to as whole block) contained exactly $(k + 1)!/k!$ blocks from the second system (sub-block);

(ii) ignoring the second system leaves a TGD design with B_1 blocks each of K_1 units and with Λ_{11} and Λ_{12} concurrences;

(iii) ignoring the first system leaves a NTGD design with B_2 blocks each of K_2 units and with Λ_{21} and Λ_{22} concurrences, and

(iv) $\sum_{k=1}^{B} n_{ik} n_{jk} = \Lambda_1$ if treatments i & j ($i \neq$ j) are first associates

$= \Lambda_2$ otherwise

Thus, $VR_1 = B_1K_1$, $VR_2 = B_2K_2$, $\Lambda_{11} (n - 1) + \Lambda_{12} (V - n) = R_1(K_1 - 1) - 2Q_{12}$, $\Lambda_{21} (n - 1) + \Lambda_{22} (V - n) = R_2(K_2 - 1) - 2Q_{22}$ where Q_{12} and Q_{22} are multiplicities of '2' in TGD and NTGD design respectively.

3. CONSTRUCTION

Theorem 3.1 The existence of a BIB design with parameters $V = 2k + 1$, b, r, k, λ with $b = 3r - 2\lambda$ implies the existence of balanced ternary design (BTD) with following parameters $V = 2(k + 1)$, $B = 2b(b - 1)$, $Q_1 = b(b - 1)$, $Q_2 = b(b - 1)/2$, $R = 2b(b - 1)$, $K = 2(k + 1)$, $\Lambda = (b+2r)(b - 1)$.

Proof: With the existing BIB design, a self complementary BIB design with the parameters $V = 2(k + 1)$, $b' = 2b$, $r' =$ b, $k' =$ k + 1, $\lambda' = r$ can be constructed (Mitra and Mandal 1998). Then BTD are constructed by taking the combination of two blocks of the self complementary design together at a time.

Hence $\binom{2b}{2} = b(2b - 1)$. But b blocks are such that consist of all **1'**s. Therefore, the total number of blocks is $B = b(2b - 1) - b = 2b(b - 1)$.

The parameters V, B, and K need no explanation. Remaining parameters are explained below:

Q_1: Let us consider a block containing a particular treatment x. This block will consist of $(b - 1)$ blocks containing treatment x with multiplicity '1'. Thus, $Q_1 = b\,(b - 1)$.

Q_2: The total number of replications is b. For multiplicity '2' we have to consider $\binom{b}{2}$ and thus $Q_2 = b(b - 1)/2$.

R: Replication number R for treatment x is $R = Q_1 + 2\, Q_2$. Hence $R = 2b(b - 1)$.

K: $2(k+1)$

Λ: This parameter will consist of (2, 2) (2, 1) (1, 2) and (1, 1) ordered pair of treatments. For the ordered pair (2, 2) we consider 2's of the total λ's. Since total λ's is r. Hence for ordered pair (2, 2) we have $\binom{r}{2} = \frac{r(r-1)}{2}$. For ordered pair (2, 1) and (1, 2), the total λ's will consist of $(b - r)$ blocks. Hence for ordered pair (1, 2) and (2, 1) we have $(b - r)r$. For the ordered pair (1, 1), let us consider the following statement.

There are b number of replications out of which r comes from the existing design and $(b - r)$ from complementary design. Hence (1, 1) occurs r times in r number of replications and $(b - r)$ times in $(b - r)$ number of replications.

Therefore the ordered pair (1, 1) occurs in $r \times r + (b - r)(b - r)$ but in b blocks, there are all 1's.

Hence the ordered pair (1, 1) $= r^2 + (b - r)^2 - b$

Thus,

$$\Lambda = 4r(r - 1)/2 + 4r(b - r) + r^2 + (b - r)^2 - b = (b + 2r)(b - 1)$$

Hence the theorem.

Corollary 3.1: The existence of a BIB design with parameters $V = b = 4t - 1$, $r = k = 2t - 1$, $\lambda = t - 1$, implies the existence of a BTD with parameters, $V = 4t$, $B = 4(4t - 1)(2t - 1)$, $Q_1 = 2(2t - 1)(4t - 1)$, $Q_2 = (2t - 1)(4t - 1)$, $R = 4(4t - 1)(2t - 1)$, $K = 4t$, $\Lambda = 2(2t - 1)(8t - 3)$.

Corollary 3.2: The existence of a BIB design with parameters $V = 2t - 1$, $b = 4t - 2$, $r = 2t - 2$, $k = t - 1$, $\lambda = t - 2$, implies the existence of a BTD with parameters, $V = 2t$, $B = 4(2t - 1)(4t - 3)$, $Q_1 = 2(2t - 1)(4t - 3)$, $Q_2 = (2t - 1)(4t - 3)$, $R = 4(2t - 1)(4t - 3)$, $K = 2t$, $\Lambda = 2(12t^2 - 20t + 9)$.

Theorem 3.2. The addition of two BIB designs with parameters $V = 2k$, $b_1 = 2r$, r_1, k_1, λ_1 and v, b_2, r_2, $k + 1$, λ_2 will provide the existence of a BTD with following parameters

$$V = 2k + 1,\ B = \binom{b_1 + b_2}{2},\ Q_1 = (r_1 + r_2)b_2,$$

$$Q_2 = \binom{r_1 + r_2}{2},\ R = Q_1 + 2Q_2,\ K = 2(k + 1),$$

$$\Lambda = 4\binom{\lambda_1 + \lambda_2}{2} + 4(\lambda_1 + \lambda_2)r_2 + (\lambda_1 + \lambda_2)b_2 = (\lambda_1 + \lambda_2)[2(\lambda_1 + \lambda_2 - 1) + b_2 + 4r_2]$$

Proof: BTD are constructed with the addition of two BIB designs in which first BIB consists of the condition that $V = 2k$, $b = 2r$. Hence the number of treatments in BTD is obviously $V = 2k + 1$, and number of blocks is $B = \binom{b_1 + b_2}{2}$. The other parameters are explained below:

Q_1**:** Total number of replications is $r_1 + r_2$. For the multiplicity '1' the total number of replications should be multiplied by b_2, the block size of second BIB designs. Hence, $Q_1 = (r_1 + r_2)\, b_2$.

Q_2**:** It is obvious to take $\binom{r_1 + r_2}{2}$.

R: Replication number R for treatment x is $R = Q_1 + 2\,Q_2$.

K: $2(k + 1)$

Λ: This parameter will consist of (2, 2) (2, 1) (1, 2) and (1, 1) ordered pair of treatments. For the ordered pair (2, 2) we consider 2's of the total λ's. Since total λ's is $\lambda_1 + \lambda_2$. Hence for ordered pair (2, 2) we have $\binom{\lambda_1 + \lambda_2}{2}$. For ordered pair (2, 1) and (1, 2), the total λ's will be multiplied by r_2 blocks. Hence for ordered pair (1, 2) and (2, 1), we have $(\lambda_1 + \lambda_2)\, r_2$. For the ordered pair (1, 1) we have $(\lambda_1 + \lambda_2)\, b_2$.

Thus, $\Lambda = 4\binom{\lambda_1 + \lambda_2}{2} + 4(\lambda_1 + \lambda_2)\, r_2 + (\lambda_1 + \lambda_2)b_2$.

Hence the theorem.

One major limitation of this design is that it provides design with larger number of replications.

Theorem 3.3. The existence of group divisible designs with parameters $V_1 = 2(k + 1)$, $b_1 = 2(b - 1)$, $r_1 = b - 1$, $k_1 = k + 1$, $\lambda_1 = r - 1$, $\lambda_2 = r$, $m = 2$, $n = k + 1$, constructed by BIB design and with its complements having parameters $V = 2k + 1$, b, r, k, λ with $b = 3r - 2\lambda$, implies the existence of ternary group divisible (TGD) design with following parameters

$V = 2(k + 1)$, $B = 4(b - 1)$, $Q_1 = 2(b - 1)$, $Q_2 = b - 1$, $R = 4(b - 1)$, $K = 2(k + 1)$, $\Lambda_1 = 4(b - r) + 6\lambda_1$, $\Lambda_2 = 4r + 2\lambda_2$, $m = 2$, $n = k + 1$.

Proof: Choose any block in BIB design and its complement. Let us consider the association of remaining blocks with these two blocks (*i.e.*) in first we have $2(b - 1)$ and in second, $2(b - 1)$. Hence total number of blocks is $4(b - 1)$.

Q_1**:** The number of replications is $(b - 1)$ in group divisible. Hence the multiplicities of '1' will be $2(b - 1)$ because two blocks are associated with the remaining blocks.

Q_2**:** It is obvious to take $(b - 1)$.

R: Replication number R for treatment x is $R = Q_1 + 2\,Q_2 = 4(b - 1)$.

K: $2(k + 1)$

Λ_1**:** Consider a treatment pair (x, y) where x and y are first associates. To each of the λ_1 blocks of semi-regular GDD, containing treatment pair (x, y), if pair (x, y) is added we get λ_1 blocks in which the treatment pair (x, y) occurs 4 times. Therefore, first term in the expression for Λ_1 is $4\lambda_1$. There are $(b - \text{r})$ pairs of first associate treatment which contain treatment x which occurs with multiplicities '2' and y occurs with multiplicity '1'. Similarly, if x occurs with multiplicity '1' and y occurs with '2' then we get $4(b - \text{r})$ blocks in which treatment pair occurs twice.

The treatment pair occurs once *i.e.,* (1, 1) is given by expression $2\,\lambda_1$. Thus $\lambda_1 = 4(b - r) + 4\lambda_1 + 2\lambda_1 = 4(b - r) + 6\lambda_1$.

Λ_2**:** There are r pairs of second associate treatments which contain treatment x which occurs with multiplicity '2' and y occurs with '2', similarly, if x occurs once and y occurs twice then we get $4r$ blocks in which treatment pair occurs twice. There are (x, y) pairs which occurs once (*i.e.*) (1, 1) is given by $2\lambda_2$.

Thus, $\Lambda_2 = 4r + 2\lambda_2$.

It is to be noted that the resulting TGD design has the same association scheme as that of original group divisible design.

Corollary 3.3: The existence of a BIB design with parameters $V = b = 4t - 1$, $r = k = 2t - 1$, $\lambda = t - 1$, implies the existence of a TGD design through group divisible design with parameters, $V = 4t$, $B = 8(2t - 1)$, $Q_1 = 4(2t - 1)$, $Q_2 = 2(2t - 1)$, $R = 8(2t - 1)$, $K = 4t$, $\Lambda_1 = 4(5t - 3)$, $\Lambda_2 = 12t - 5$.

Corollary 3.4: The existence of a BIB design with parameters $V = 2t - 1$, $b = 4t - 2$, $r = 2t - 2$, $k = t - 1$, $\lambda = t - 2$, implies the existence of a TGD design through group divisible design with parameters, $V = 2t$, $B = 4(4t - 3)$, $Q_1 = 2(4t - 3)$, $Q_2 = (4t - 3)$, $R = 4(4t - 3)$, $K = 2t$, $\Lambda_1 = 2(15t - 9)$, $\Lambda_2 = 2(6t - 5)$.

Theorem 3.4. The existence of group divisible designs constructed through BIB design having parameters $V = 2k + 1$, b, r, k, λ with $b = 3r - 2\lambda$ implies the existence of nested ternary group divisible design provided that the block size of NTGD design is $2k + 1$, with following parameters

$$V = 2(k + 1),\ B = 2(b - 1)(k + 1),\ Q_1 = (b - 1),$$
$$Q_2 = (b - 1)k,\ R = (2k + 1)(b - 1),\ K = 2k + 1,$$
$$\Lambda_1 = 4\lambda_1(n - 1),\ \Lambda_2 = 4\lambda_2(n - 1).$$

Proof: This GDD is regular if $r > 2\lambda + 1$. The parameter $V = 2(k + 1)$ is obvious. There are $2(b - 1)$ blocks which inflates $(k + 1)$ times to get the number of blocks. Hence $B = 2(b - 1)(k + 1)$.

Q_1**:** $(b - 1)$ blocks will provide the blocks with multiplicity '1'.

Q_2**:** $(b - 1)\,k$ blocks will provide blocks with multiplicity' '2'.

R: Replication number R for treatment x is $R = Q_1 + 2\,Q_2 = (2k + 1)(b - 1)$.

K: $2(k + 1)$

Λ_1**:** $4\lambda_1 (n - 1)$ on the basis of the arguments of the previous theorems.

Λ_2**:** $4\lambda_2 (n - 1)$

Corollary 3.5: The existence of a BIB design with parameters $V = b = 4t - 1$, $r = k = 2t - 1$, $\lambda = t - 1$, implies the existence of a NTGD design with parameters, $V = 4t$, $B = 4t(4t - 2)$, $Q_1 = 4t - 2$, $Q_2 = (2t - 1)(4t - 2)$, $R = 2(2t - 1)(4t - 1)$, $K = 4t - 1$, $\Lambda_1 = 8(t - 1)(2t - 1)$, $\Lambda_2 = 4(2t - 1)(2t - 1)$.

Corollary 3.6: The existence of a BIB design with parameters $V = 2t - 1$, $b = 4t - 2$, $r = 2t - 2$, $k = t - 1$, $\lambda = t - 2$, implies the existence of a TGD design through group divisible design with parameters, $V = 2t$,

$B = 2t(4t-3)$, $Q_1 = 4t-3$, $Q_2 = (4t-3)(t-1)$, $R = (4t-3)(2t-1)$, $K = 2t-1$, $\Lambda_1 = 4(t-1)(4t-3)$, $\Lambda_2 = 8(t-1)(t-1)$.

4. ILLUSTRATIVE EXAMPLE

Example 4.1 Let us consider BIB design with parameters $V = b = 3$, $r = k = 1$, $\lambda = 0$. It implies self complementary BIB design with parameters $V = 4$, $b' = 6$, $r' = 3$, $k' = 2$, $\lambda' = 1$ and hence a group divisible design with parameters $V = 4$, $b_1 = 4$, $r_1 = 2$, $k_1 = 2$, $\lambda_1 = 0$, $\lambda_2 = 1$, $m = 2$, $n = 2$.

Applying Theorem 3.1, it may be developed as BTD with parameters $V = 4$, $B = 12$, $Q_1 = 6$, $Q_2 = 3$, $R = 12$, $K = 4$, $\Lambda = 10$

We get the following blocks

B_1: (1, 4, 2, 4)	B_2: (1, 4, 3, 4)	B_3: (1, 4, 1, 3)
B_4: (1, 4, 1, 2)	B_5: (2, 4, 3, 4)	B_6: (2, 4, 1, 2)
B_7: (2, 4, 2, 3)	B_8: (3, 4, 1, 3)	B_9: (3, 4, 2, 3)
B_{10}: (1, 3, 1, 2)	B_{11}: (1, 3, 2, 3)	B_{12}: (1, 2, 2, 3)

Efficiency of this design is 0.833.

Example 4.2 Let us consider first BIB design with parameters $V = 4$, $b = 6$, $r = 3$, $k = 2$, $\lambda = 1$ and another BIB design with parameters $V = b = 4$, $r = k = 3$, $\lambda = 2$

Applying Theorem 3.2, we get the BTD with following parameters $V = 5$, $B = 45$, $Q_1 = 24$, $Q_2 = 15$, $R = 54$, $K = 6$, $\Lambda = 60$.

Thus, we have the following blocks:

B_1: (1,4,5,2,4,5)	B_2: (1,4,5,3,4,5)	B_3: (1,4,5,1,3,5)
B_4: (1,4,5,1,2,5)	B_5: (1,4,5,2,3,5)	B_6: (1,4,5,1,2,3)
B_7: (1,4,5,2,3,4)	B_8: (1,4,5,1,3,4)	B_9: (1,4,5,1,2,4)
B_{10}: (2,4,5,3,4,5)	B_{11}: (2,4,5,1,3,5)	B_{12}: (2,4,5,1,2,5)
B_{13}: (2,4,5,2,3,5)	B_{14}: (2,4,5,2,3,4)	B_{15}: (2,4,5,1,2,3)
B_{16}: (2,4,5,1,3,4)	B_{17}: (2,4,5,1,2,4)	B_{18}: (3,4,5,1,3,5)
B_{19}: (3,4,5,1,2,5)	B_{20}: (3,4,5,2,3,5)	B_{21}: (3,4,5,1,2,3)
B_{22}: (3,4,5,2,3,4)	B_{23}: (3,4,5,1,3,4)	B_{24}: (3,4,5,1,2,4)
B_{25}: (1,3,5,1,2,5)	B_{26}: (1,3,5,2,3,5)	B_{27}: (1,3,5,1,2,3)
B_{28}: (1,3,5,2,3,4)	B_{29}: (1,3,5,1,3,4)	B_{30}: (1,3,5,1,2,4)
B_{31}: (1,2,5,2,3,5)	B_{32}: (1,2,5,1,2,3)	B_{33}: (1,2,5,2,3,4)
B_{34}: (1,2,5,1,3,4)	B_{35}: (1,2,5,1,3,4)	B_{36}: (2,3,5,1,2,3)
B_{37}: (2,3,5,2,3,4)	B_{38}: (2,3,5,1,3,4)	B_{39}: (2,3,5,1,2,4)
B_{40}: (1,2,3,2,3,4)	B_{41}: (1,2,3,1,3,4)	B_{42}: (1,2,3,1,2,4)
B_{43}: (2,3,4,1,3,4)	B_{44}: (2,3,4,1,2,4)	B_{45}: (1,3,4,1,2,4)

Efficiency of this design is 0.926.

Example 4.3 Let us consider first BIB design with parameters $V = b = 3$, $r = k = 1$, $\lambda = 0$ in addition to its complementary BIB design. Thus we have self complimentary with parameters $V = 4$, $b = 6$, $r = 3$, $k = 2$, $\lambda = 1$ and then we have group divisible design with parameters $V = b = 4$, $r = k = 2$, $\lambda_1 = 0$, $\lambda_2 = 1$ and then we have TGD design with following parameters

$V = 4$, $B = 8$, $Q_1 = 4$, $Q_2 = 2$, $R = 8$, $K = 4$, $\Lambda_1 = 8$, $\Lambda_2 = 6$.

Thus, we have the following blocks:

B_1: (1, 4, 2, 4)	B_5: (2, 4, 2, 3)
B_2: (1, 4, 3, 4)	B_6: (3, 4, 2, 3)
B_3: (1, 4, 1, 3)	B_7: (2, 3, 1, 3)
B_4: (1, 4, 1, 2)	B_8: (2, 3, 1, 2)

Example 4.4 Let us consider group divisible design constructed through BIB design with parameters $V = b = 3$, $r = k = 1$, $\lambda = 0$. Applying Theorem 3.4, we have nested TGD design with following parameters

$V = 4$, $B = 8$, $Q_1 = 2$, $Q_2 = 2$, $R = 6$, $K = 3$, $\Lambda_1 = 0$, $\Lambda_2 = 4$.

Thus, we have the following blocks:

B_1: (2, 4, 2)	B_5: (1, 3, 1)
B_2: (2, 4, 4)	B_6: (1, 3, 3)
B_3: (3, 4, 3)	B_7: (1, 2, 1)
B_4: (3, 4, 4)	B_8: (1, 2, 2)

Example 4.5 The blocks given in Example 4.3 and Example 4.4 can be used for conducting intercropping experiments when the intercrops are sub-divided into various groups based on agronomic practices. We construct design for experiments where each plot consists of two main crops and eight intercrops, such that each of these intercrops is selected from a group of intercrops following Rao and Rao (2001).

Now let us consider an intercropping experiment using two main crops and eight intercrops where the intercrops are partitioned into four groups S_1, S_2, S_3 and S_4 with 2 in each group *viz.,* S_1 = [1, 2,], S_2 = [3, 4,], S_3 = [5, 6,] and S_4 = [7, 8]. Let us designate the symbols 0, 2 of first row of TGD design with intercrops 1, 2 of S_1, second row with intercrops 3, 4 of S_2, third row with intercrops 5, 6 of S_3 and fourth row with intercrops 7, 8 of S_4. Taking into consideration the column of the array as the plots of the intercropping experiment in addition to two main crops in each plot, the resulting intercropping experiment will consists of the following 8 plots on the basis of the blocks given in Example 4.3. The similar eight plots can also be generated based on the blocks which are given in Example 4.4 assuming the symbols 1, 2 of first row of NTGD design with intercrops 1, 2 of S_1, second row with intercrops 3, 4 of S_2, third row with intercrops 5, 6 of S_3 and fourth row with intercrops 7, 8 of S_4.

$(m_1,m_2, 5,8)$; $(m_1,m_2,3,8)$; $(m_1,m_2, 2,3)$; $(m_1,m_2 ,2,5)$

$(m_1,m_2,1,4)$; $(m_1,m_2,1,6)$; $(m_1,m_2, 6,7)$; $(m_1,m_2,4,7)$

It is to be noted that the constructed blocks can provide intercropping design with two main crops and 8 intercrops divided into four groups of two intercrops each. Rao and Rao (2001) has developed intercropping design for eight intercrops with one main crop and four intercrops, while this design consists of 8 intercrops merely in eight plots having two main crops and two intercrops in each plot. It may be suggested that this design is superior in terms of reduction of size of blocks.

In the context of an actual example of intercropping experiment, Takim (2012) have used the different mix-proportions and planting patterns of maize (*Zea mays* L.) and cowpea (*Vigna unguiculata* L.) for the comparison of sole cropping of each crop during 2010 and 2011 growing seasons under the southern Guinea savanna conditions in Nigeria. The experiment comprised of 6 treatments: sole maize (51,282 plants ha^{-1}), sole cowpea (61,538 plants ha^{-1}) and 4 maize-cowpea intercropping mix-proportion: 100 maize:100 cowpea, 50 maize:50 cowpea, 60 maize:40 cowpea and 40 maize:60 cowpea using randomized complete block design with three replications. Evaluation of the intercropping patterns was performed on basis of several intercropping indices. The study revealed that the mix-proportion of 50 maize: 50 cowpea gave a similar grain yield compared to other intercropped plots. The study also revealed that intercropping systems could be an eco-friendly approach for reducing weed problems through non-chemical methods, mix-proportion of 50 maize: 50 cowpea planted on alternate rows could be a better intercropping pattern.

In another example of intercropping experiment, Pandey *et al.* (2003) have studied the effect of maize (*Zea mays* L.) based intercropping system on maize yield as main crop and six intercrops *viz.,* pigeonpea, sesamum, groundnut, blackgram, turmeric and forage *meth* by conducting an experiment during the rainy seasons of 1998 and 1999 at the research farm of Rajendra Agricultural University, Pusa, Samastipur (Bihar). The experiment consisted of six intercrops with one main crop was conducted in randomized complete block design with four replications. Maize was grown at a spacing of 75 cm. Row spacing in sole as well as in intercropping on 26 and 22 June respectively in the first and second year of experimentation. One row of pigeon pea at a distance of 75 cm and 2 rows of other intercrops at 30 cm distance were accommodated between two rows of maize. The intra row spacing of 30, 30, 10, 15, 10 and 15 cm were maintained by thinning for six intercrops.

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